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Quantifying Investment Risk in Pension Funds

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Abstract

The concept of investment risk is generalised, which allows the quantification of the investment risk associated with any given investment strategy to provide for a pension. Case studies, using historic market data over the long term, estimate the investment risk associated with different investment strategies. It is shown that a few decades ago, when bond markets only extended in depth to 20-year maturities, the investment risk of investing in equities was of the same order of magnitude as the investment risk introduced by the duration mismatch from investing in bonds for immature schemes. It is shown that now, with the extension of the term of bond markets and introduction of strippable bonds, the least risk portfolio for the same pension liability is a bond portfolio of suitable duration. It is argued that investment risk voluntary undertaken in defined benefit pension plans has grown markedly in recent decades at a time when the ability to bear the investment risk has diminished. Investment risk in pension funds is quite different to investment risk of other investors, which leads to the possibility that current portfolios are not optimised – that is, there exists portfolios which increase the expected surplus without increasing risk. The formalising of our intuitive concept of investment risk in pension saving is a first step in the identification of more efficient portfolios.

Keywords: Investment risk, defined benefit pension funds, investment strategies, actuarial investigations.

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1. Introduction

If a quantity is not measured, it is unlikely to be optimised. Despite its importance, investment risk in pension funds is not routinely quantified. Indeed, no consensus on how to measure investment risk in pension funds has emerged yet, so pension savers to date have relied on more qualitative assessments of investment risks, often assessed against several competing objectives simultaneously.

This chapter, extending Whelan (2007), proposes a definition of investment risk which formalises our intuitive concept. We develop in a more technical setting ideas first presented in Arthur and Randall (1989) and provide, using historic data on the UK, US, and Irish capital markets, an empirical assessment of the magnitude of risk entailed by different investment strategies and relative to different objectives. The analysis, through a series of case studies, leads to a rather simple conclusion: sovereign bond portfolios (of appropriate duration and index-linked/nominal mix) are the least risk portfolio for pension savers, irrespective of the age of pension saver, irrespective of currency of the pension and, within a reasonable range, irrespective of the precise investment objectives of the pension saver. The analysis allows us quantify the risks in all investment strategies and we provide figures for the risks inherent in investing in equities, conventional long bonds, cash, and the closest matching bonds by duration.

Investment risk is defined in the next section and some of its properties considered. From the definition, one can quantify the investment risk inherent in any given investment strategy and thereby identify the strategy with the lowest investment risk. Section 3 reports the results of case studies that quantify the investment risk for pension savers from various different investment strategies. This analysis shows that the relative risk inherent in different strategies appear to be very similar over different time periods and different national markets and reasonably robust when the objective is to provide pensions in deferment increasing in line with wages or increasing in line with inflation subject to a nominal cap. We get an important insight from this analysis: even conventional long bonds are not long enough to match the liabilities of young scheme members, and investing in such bonds can be as risky as investing in equities, but without the expected rewards. We conclude that just as much care must be exercised in matching liabilities by duration as by matching liabilities by asset type. Section 4 demonstrates that the fallaciousness of the argument that the risk of equity investment dissipates with time so that, at some long investment horizon, equities are preferable over other asset classes by any rational investor). This argument, generally known as the 'time diversification of risk', does not hold in that strong a form. True, the expected return from equities might well be higher than other asset classes but, on some measures, so too is the risk and this remains true over all time horizons. We conclude that the most closely matching asset for pension fund liabilities is composed mainly of conventional and index-linked bonds, which, if history is any guide, has a lower expected long term return than a predominantly equity portfolio.

Our analysis does not allow us to suggest one investment strategy is preferable to another. Investors, including pension providers, routinely take risks if the reward is judged sufficiently tempting. However, pension providers should appreciate the risks involved in alternative strategies and, at a minimum, seek to ensure that the investment portfolio is efficient in the sense that risk cannot be diminished without diminishing reward.

2. Defining Investment Risk

There would be no concept of risk if all expectations were fulfilled: risk arises from a clash between reality and expectations. Accordingly, one first needs to formulate and make explicit future expectations before risk can be quantified. Note that future expectations at any point in time are dependent to an extent on the experience up to that time, as past experiences influence future expectations.

Our intuitive notion of investment risk is that it measures the financial impact when the actual investment experience differs from that expected, holding all other things equal. In this section, we formalise this notion. Once investment risk is properly defined, it is straightforward (in theory at least) to measure and attempt to minimise it.

The task of formally setting down future expectations when it comes to investing to generate a series of future cashflows is often known as a 'valuation' (e.g., the actuarial valuation of defined benefit schemes). We adopt this terminology and call the desired series of cashflows the ‘liabilities’.

Let t = 0 represent the present time and t > 0 be a future time. Let A_t denote the forecast cashflow from the assets at time t and L_t be the forecast liability cashflow at time t. We shall assume, for convenience, that the investment return expected over each unit time period in the future is constant, denote it *i* and term it the ‘valuation rate of interest’. It will be clear that allowing *i* to vary with the time period poses no theoretical issues. The reported valuation result at time 0, expressing the surplus (if positive) or deficit (if negative) of assets relative to liabilities, is denoted X_0. Thus

$$X_0 = \sum_{t=0}^{\infty} (A_t - L_t)(1+i)^{-t} \qquad (1)$$

Consider X_0. We shall assume that this is a number.[1] So, under this simplifying assumption, X_0 is a constant, representing the surplus at the present time identified by the specified (deterministic) valuation methodology.

Let *p* be the time that the next valuation falls due. Let X_p^0 represent the results of the next valuation at time *p*, using the same underlying assumptions as used in the valuation at time *0*. Then the relationship between X_0 and X_p^0 is:

$$\begin{aligned} X_0 &= \sum_{t=0}^{\infty} (A_t - L_t)(1+i)^{-t} \\ &= \sum_{t=0}^{p} (A_t - L_t)(1+i)^{-t} + \sum_{t=p}^{\infty} (A_t - L_t)(1+i)^{-t} \\ &= \sum_{t=0}^{p} (A_t - L_t)(1+i)^{-t} + (1+i)^{-p} \sum_{t=p}^{\infty} (A_t - L_t)(1+i)^{-t+p} \qquad (2) \\ &= \sum_{t=0}^{p} (A_t - L_t)(1+i)^{-t} + (1+i)^{-p} \sum_{s=0}^{\infty} (A_t - L_t)(1+i)^{-s} \\ &= \sum_{t=0}^{p} (A_t - L_t)(1+i)^{-t} + X_p^0 (1+i)^{-p} \end{aligned}$$

[1] If this is allowed be a non-constant random variable then we call the valuation methodology used stochastic otherwise the valuation approach is said to be deterministic. Note that a stochastic valuation is representing some part of the assets and/or liabilities as a non-trivial random variable at time 0. We shall discuss only deterministic valuation methods in the sequel to simplify the analysis but, as should be clear, the results carry through (with relatively straightforward extensions) when applied to stochastic valuation approaches.

If we make the further assumption that the experience in the inter-valuation period is exactly in line with that assumed at time 0, as well as the assumptions underlying the valuation at time p are also the same, then the valuation result at time p will be $X_0(1+i)^p$, i.e., $X_p^0 = X_0(1+i)^p$. This can readily be seen, as the cashflow in the inter-valuation period will be invested (or disinvested) at the valuation rate of interest, accumulating at time p to $(1+i)^p \sum_{t=0}^{p}(A_t - L_t)(1+i)^{-t} = \sum_{t=0}^{p}(A_t - L_t)(1+i)^{p-t}$ and this amount is to be added to the discounted value of all the yet unrealised asset and liability cashflows at time p, namely X_p^0. The total value at time p is then $\sum_{t=0}^{p}(A_t - L_t)(1+i)^{p-t} + X_p^0$, which is just the right-hand side of equation (2) multiplied by $(1+i)^p$, whence the result.

It is generally possible to form a reasonable apportionment of the difference of the valuation result at the next valuation date from that expected from the valuation at time 0 (i.e., $X_0(1+i)^p$) into that due to either

(i) the actual experience over the inter-valuation period differing from that assumed, or,

(ii) that due to a changed valuation method or basis applied at time p.

In particular, it is possible to form a reasonable assessment of the financial impact of the actual investment experience relative to that expected, other things being held the same.

Let X_{0-p}^i denote the result of the valuation at time 0, under the same methodology and assumptions as underlying the valuation result, X_0, at time 0 but now reflecting the actual investment experience in the inter-valuation period. Then $X_{0-p}^i - X_0$ measures the financial impact at time 0 of how the actual investment experience up to time p differed from that assumed in the original valuation at time 0. Obviously, if it turns out that the actual investment experience bears out the assumed experience in the inter-valuation period then $X_{0-p}^i = X_0$, so $X_{0-p}^i - X_0$ takes the value zero. We shall call $X_{0-p}^i - X_0$ the 'investment variation' up to time p.

Investment variation, so defined, is a non-trivial concept. It measures the financial impact at time 0 created when the actual investment experience up to time *p* differs from the investment assumptions underlying the valuation at time 0. This key concept deserves a definition.

Definition of Investment Variation (up to time p): The financial impact at time 0 created when the actual investment experience up to time *p* differs from the investment assumptions underlying the valuation at time 0, all other things being equal. In the notation introduced earlier, the investment variation is denoted $X_{0-p}^{i} - X_0$.

Investment variation up to time *p* can generally only be measured at time *p*, before that it may be modelled as a random variable with an associated distribution. Viewed in this way, the investment variation at time 0, up to time *p*, is a random variable. Investment variation at time 0 can be viewed as a stochastic process, $X_{0-p}^{i} - X_0$, indexed by *p*.

$X_{0-p}^{i} - X_0$, when viewed at time 0, is a random variable, so it has an associated distribution. The mean of this distribution captures the bias in the original investment assumptions – a positive mean implies that the original investment assumptions were conservative (as, on average, the experienced conditions turn out better than originally forecast).

Note that if the valuation is testing the adequacy of the existing portfolio, and future prescribed contributions, to generate future cashflows to meet targeted pension payments then other expectations (e.g., on future mortality) must also be embedded in the liability cashflows. In the definition of investment variation these non-investment expectations are held constant, so only the impact of the variation in the investment experience is measured. The actual scale of the resultant figure for the observed investment variation is, though, a function of these other expectations.

Some prefer to give a single number to capture the notion of riskiness in a distribution, often using some parameter that measures the spread of the distribution, such as its standard deviation, its semi-variance, or its inter-quartile spread. Often this summary measure is called 'investment risk'. Alternatively, one can apply some other measures such as the value below which there is a specified low probability of the outcome falling (the so-called 'Value-at-Risk').[2] The key point to be made is that the distribution of $X^{i}_{0-p} - X_0$ is a more foundational concept and maintains more information than any summary spread statistic. We do not enter on the wider discussion of the most appropriate measure to apply to the investment variation distribution to capture our intuitive notion of risk but adopt the generally accepted measure of standard deviation. So we identify, to a first order approximation, investment risk as the standard deviation of the investment variation distribution.

Definition of Investment Risk (up to time p): A measure of the spread of the (ex ante) investment variation distribution. For concreteness, we shall use the standard deviation as our measure of investment risk in the sequel.

If the valuer was known to have perfect foresight then the investment assumptions would be perfectly in line with the future investment experience, and so the investment variation distribution would be a degenerate constant, with a standard deviation of zero. More uncertainty about the investment variation implies a greater spread of the (ex ante) distribution, which corresponds to a greater investment risk under the above definition.

If we have perfect matching of assets to liabilities[3] then any valuation method will always report the investment variation to be a degenerate distribution (i.e., a constant) and, accordingly, the investment risk to be zero. This can be seen as, by perfect matching, $\sum_{t\geq 0}(A_t - L_t)(1+i)^{-t} = \sum_{t\geq 0} 0.(1+i)^{-t} = 0$. Thus, while the present value of the assets at time 0 (i.e., $\sum_{t\geq 0} A_t(1+i)^{-t}$) might vary with the investment assumptions, it must

[2] Of particular importance in the probability distribution is its extreme left tail behaviour, which gives the probability of a reduction to the surplus of any given large amount. Such an event might cause sudden and severe financial strain that undermines the whole saving objective. Measures for such extreme risks include, for symmetric distributions, the kurtosis or higher even moments if they exist.

[3] In the technical sense that $A_t = L_t$, for all t, independent of any investment assumptions.

vary in direction proportion to $\sum_{t \geq 0} L_t(1+i)^{-t}$. Hence, in aggregate, a gain (loss) on the assets relative to that expected is exactly offset by an increase (decrease, respectively) in the value of the liabilities relative to that expected. In short, perfect matching of asset and liability cashflows has zero investment variation, irrespective of the experienced or the assumed investment conditions.

Let us assume that (i) assets are to be valued at market value, and (ii) there exist a portfolio of assets that perfectly matches the liabilities. Note, from earlier, we know that if the matching asset portfolio was held at time 0 then the investment variation would be 0 (irrespective of what happened in the inter-valuation period). Also, at time p, the present value of the future liabilities must equal to the market value of the matching asset at that time (by the definition of matching asset). Hence the experienced valuation rate in the inter-valuation period can now be seen as the market return on the matching asset over the inter-valuation period. We see immediately from this that investment variation is positive only if the increase in the market value of the actual assets held exceeds the increase in the market value of the matching asset.[4] The upshot is that the investment variation is the present value of the extent to which the increase in the value of the assets exceeds the increase in the liabilities over the inter-valuation period, discounted at the rate of return on the matching asset over the period.[5]

Appendix I draws attention to a major limitation of our definition of investment variation (and the associated investment risk) for pension investors.

3. Case Studies Estimating Investment Risk

Estimating investment risk has been identified in the last section with estimating the standard deviation of the (*ex ante*) investment variation distribution. Let us assume

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[4] Or, as expressed in Arthur and Randall (1989), "the Main Guiding Principle merely reaffirms an earlier fundamental principle, namely that if you are mismatched and you get your forecasts wrong then you have to pay the penalty" (Section 2.5).

[5] This expresses, in more technical terms, the 'Main Guiding Principle' of Arthur and Randall (1989) that states "that if there is a rectifiable mismatch, a relative change in market values of the matched and mismatched assets should be reflected in the valuation result" (Section 5.1).

that the *ex post* investment variation is a reasonable proxy for the *ex ante* investment variation, i.e., make the commonplace assumption that historical experience can be used to assess realistic *ex-ante* expectations.

This section presents two case studies designed to explore the relative investment risk of different investment strategies for those attempting to provide a pension. However, before delving in the case studies proper, we begin with by considering the case of a person aged 55 years or over attempting to provide a pension – in real or nominal terms – from age 65 years. This provides some insights to identifying the least risk portfolio for pension savers at all ages which, as it turns out, is confirmed by the case studies.

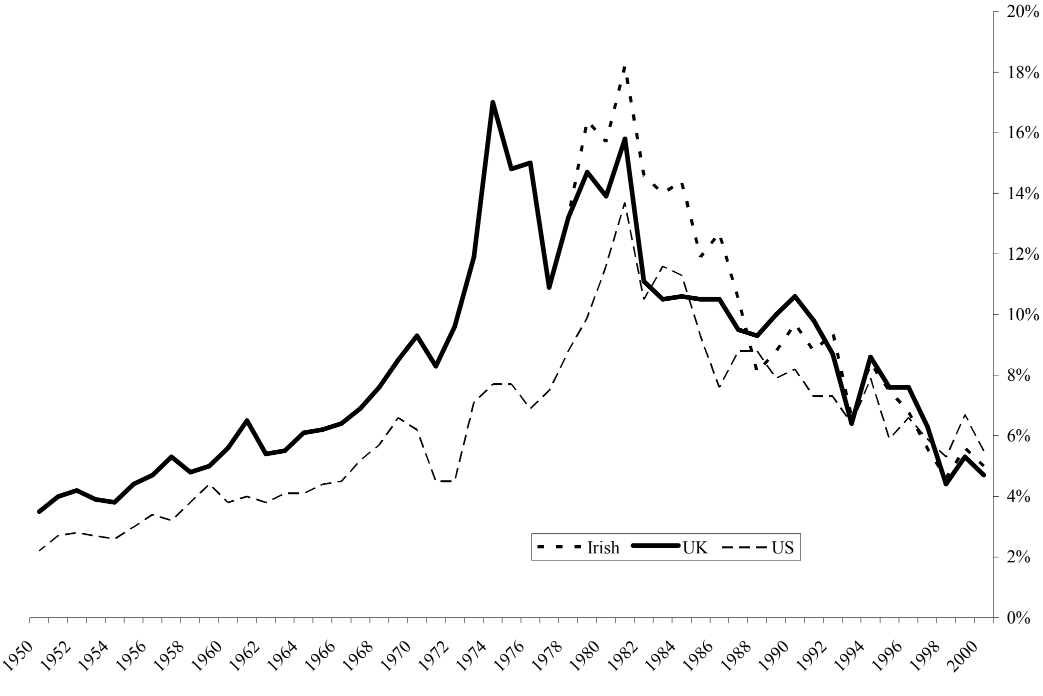
The case studies determine the historic investment risk for a pension saver attempting to provide a pension by investing in, be in, alternatively, a broad equity index, a 20 year conventional bond, a 30 year bullet bond, and short-term cash instruments in (a) the UK markets, (b) the US markets and (c) the Irish markets. We give several descriptors of the investment variation distribution from the historic data – including the key measures of its geometric mean and its standard deviation (or investment risk). These latter two summary measures give an illustration of the relative rewards of the different strategies and, to a first approximation, the risks associated with the strategies.

The first case study takes a relatively low value of the targeted pension, by assuming that the pension before vesting escalates at inflation subject to a nominal cap. This corresponds to the liability that a defined benefit scheme in Ireland has on termination to contractual pension promises under current regulations. In the second case study, we assume that the pension prior to vesting will increase in line with wage increases, reflecting the pension liability for final salary defined benefit schemes on an on-going basis. We treat, in both cases, the position of a 40 and a 30 year old with a pension due from their 65th birthday.

A picture of the *ex post* investment variation distribution associated with investing in the various asset classes are computed in the following manner. At the valuation date, it is assumed that the market value of the assets equals the value of the liabilities on a market consistent basis. Investment over the year subsequent to the valuation is assumed to be alternatively in each different asset class. Each investment strategy for each of the two case studies at each age generates n data points where n is number of years in the historic period studied. Each data point gives the present value of the surplus or deficit arising over the year, expressed as a percentage of the market value of assets at time 0 (termed the 'standardised investment variation'). From these data key summary statistics of the empirical investment variation distribution ($p=1$) for each investment strategy are tabulated, such as the mean, median, geometric mean, the standard deviation (which equates to the investment risk up to one year), and higher moments.

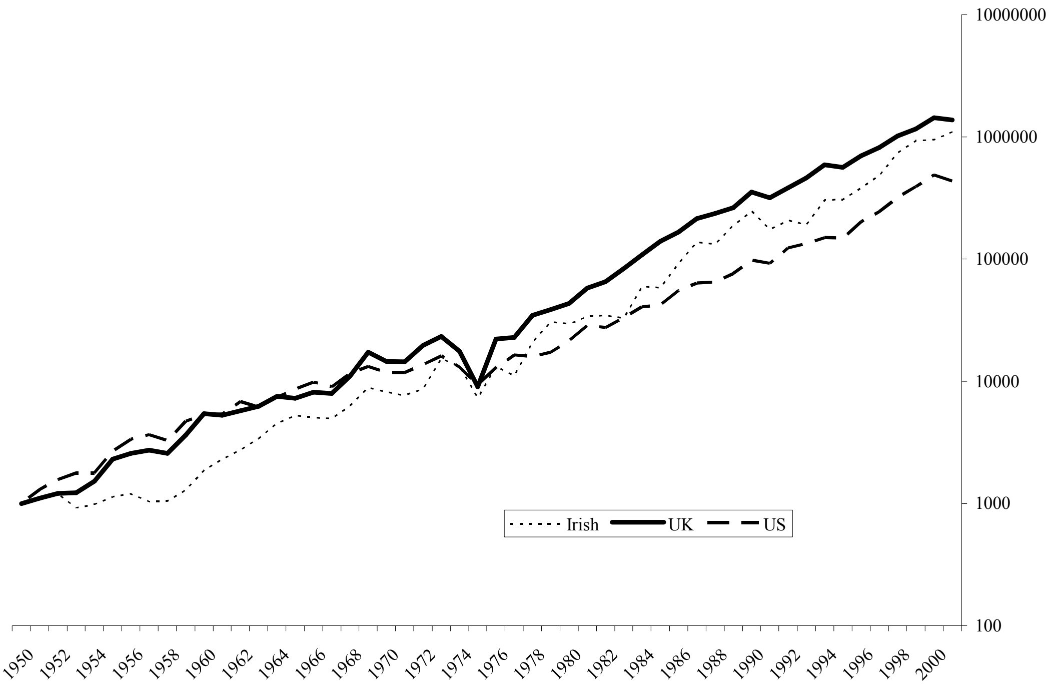
Annual returns and yields from the UK, US and Irish bond, equity and cash markets were sourced from *Barclays Capital Equity Gilt Study 2003,* Dimson, Marsh and Staunton (2004), Mitchell (1988), and Whelan (2004). Figures 1 and 2 display, respectively, the 20 year sovereign bond yield and a broad-based equity index, from each national market over the second half of the twentieth century.

Figure 1: Long Bond Gross Redemption Yield, US, UK and Ireland, Year Ends, 1950-2000 (inclusive).



Note that prior to 1978 the yield on Irish long bonds was almost identical to UK long bonds because of the currency link.

Figure 2: Equity Market Total Return Indices, US, UK, and Ireland, Year Ends, 1950-2000 (Log Scale)



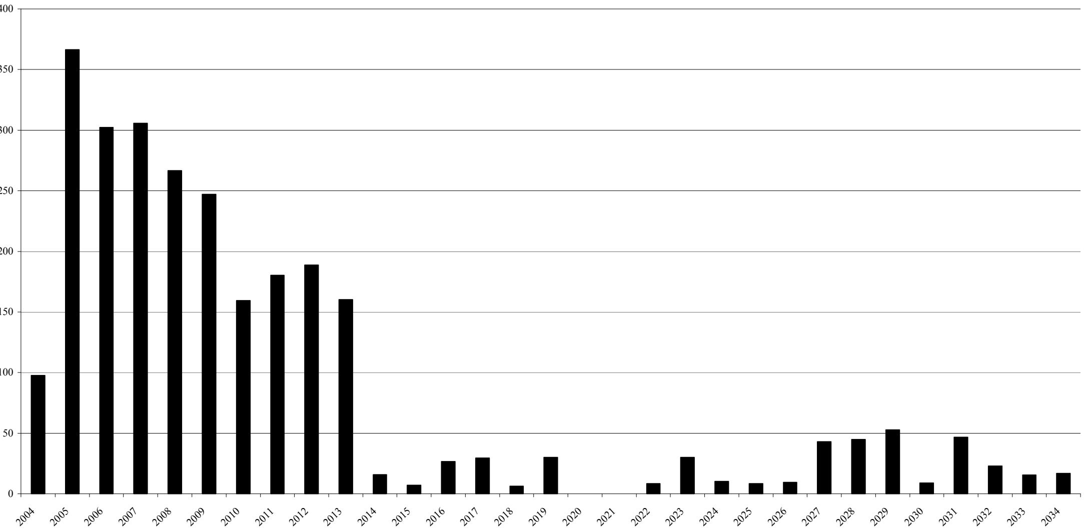
3.1 Pension Saving, Person Aged 55 Years and Over

Consider a person aged 55 years targeting a pension from age 65 years, the pension subject to either inflation-linked or fixed rate increases both prior to retirement and while in payment. For concreteness, we shall make the demographic assumption that the person will die on his 85th birthday. Accordingly, the liability in this case is a series of real or nominal amounts falling in a regular pattern, beginning in 10 years' time and ending in 30 years' time.

From our definition of investment variation and investment risk earlier, it is clear that to minimise the investment risk requires investing in an asset portfolio that provides an income that most closely matches this liability stream. Whether these liabilities are nominal or real in sterling, euro, dollar, there is arguably a sufficiently deep market in conventional and index-linked sovereign bonds so that a near-perfect matching portfolio can be constructed.

First, consider the case that the liability cashflows are all nominal (that is, not linked to inflation). The maturity profile of euro-denominated sovereign debt markets is shown below.

Figure 3: Outstanding Nominal Amount of Euro-denominated Government bonds over 1 year, by Calendar Year of Maturity, € Billions (as at September 2003)



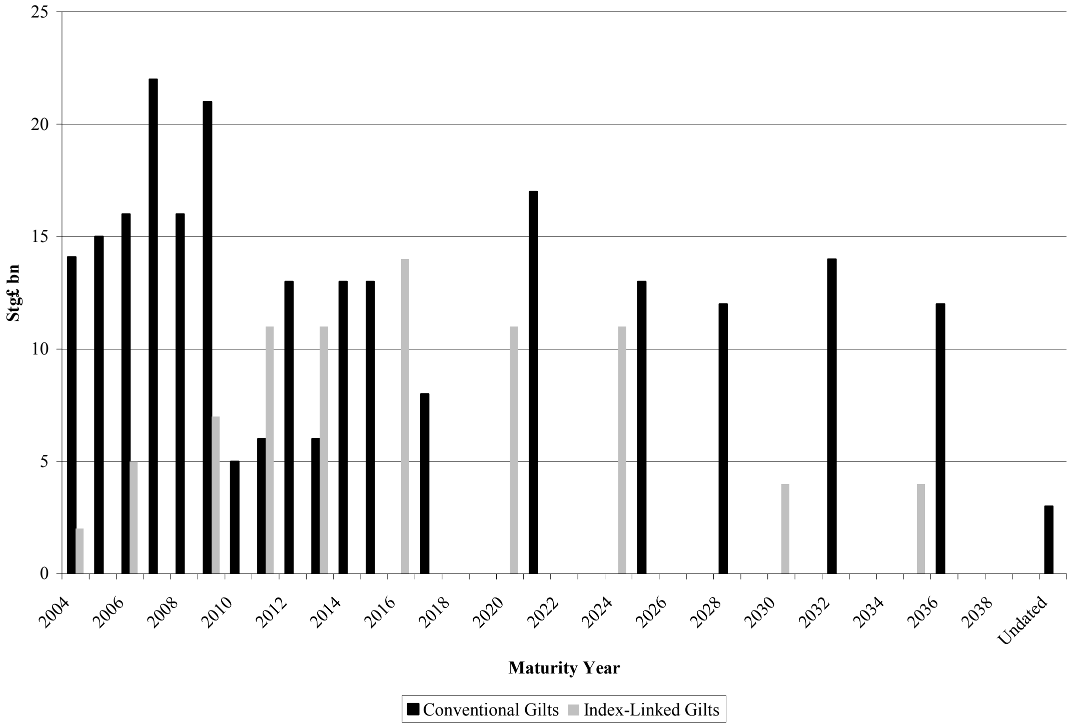
The graph indicates that a pattern of fixed amounts in euros falling due anywhere within the next three decades can adequately be matched by euro-denominated sovereign bonds, especially now that that many such bond issues are strippable.[6] Similar remarks hold for sterling, dollar and yen bond markets. It follows that we can identify a bond portfolio closely matching a nominal pension liability in these currencies for the 55 year old person.

Now, consider the case that the liability cashflows are real in nature - subject, to say, wage increases prior to retirement and inflationary increases thereafter. In order to estimate the payments falling due after 10 years' time now requires an estimate of the person's wage increases over the next decade. This problem can be decomposed into estimating (a) the general rate of inflation over the next decade and (b) the real rate of wage increase. The latter might be estimated to a reasonable accuracy leaving us to allow for the rate of inflation over the next decade. The development of the index-linked bond markets allow for a portfolio to be constructed that match a pattern of

[6] Stripping means trading each coupon or principal payment of the bond as a separate asset – each a bullet bond. The sovereign euro bonds are generally strippable, with France issuing such bonds since 1991, Germany since 1997, followed by many others (including Ireland) in more recent years.

such real payments in the UK, Eurobloc and US economies up to, again, three decades years into the future. Figure 4 illustrates the maturity profile of the sterling sovereign debt market in both nominal and index-linked bonds.

Figure 4: Outstanding Nominal Amount of Sterling-denominated conventional and index-linked (inflation-adjusted) Government bonds, by Calendar Year of Maturity, Stg£ Billions (as at end March 2004)



The above considerations allow us to identify, in general terms, that the most closely matching portfolio to the stylised pension liabilities comprises solely of bonds. In particular, a role for equities has not been identified in the most closely matching portfolio as the proceeds from equities are not known in advance. Clearly a similar procedure applied to finding the closest matching portfolio to the liabilities of persons over age 55 years will again identify portfolios consisting of just bonds (conventional and index-linked).[7]

For persons younger than 55 years, there is no sovereign guaranteed security matching payments falling due after about three decades in the major economies, whether nominal or real. However, the market allows us to provide a nominal amount or

[7] We would not like to give the impression that is always straightforward. It can be non-trivial to estimate the closest matching portfolio for some liabilities, particularly those expressed as the lower of two amounts.

inflation-linked amount in three decades' time and this can be used as a stepping-stone to provide payments falling due after the three decades. Applying this logic entails that solving for the most closely matching asset for nominal or index-linked liabilities after 30 years is perhaps best done by extrapolating the yield curve beyond the present maturity cut-off and price on the basis that longer dated securities at the extrapolated yield exist. This suggests that the investment strategy to allow for these very distant payments would be to invest the estimated required amount in the longest dated bonds available.

Of course, extrapolation of the yield curve introduces another risk, the magnitude of the risk related to the extent of the extrapolation. However, if the weight of the liabilities falling due occurs within the next three decades[8] then this extrapolation technique will produce an acceptable error as a proportion of the total liability. A key question is how much investment risk is increased with the extrapolation technique and the associated investment strategy proposed above. When the liabilities are linked to inflation then we cannot, unfortunately, reliably back-test how well the extrapolation method proposed above would have worked as sovereign index-linked stocks have only been in issue since 1981 in UK, since 1997 in US, and since 1998 in France. However, we can derive the empirical investment variation associated with other different investment strategies over the last century, and this is done in the following case studies.

3.2 Case Study 1: Measurement of Investment Risk in Pension Funds, Termination Liability

Let the pension liability be to a 40 year old who is due a non-escalating pension from age 65 expressed as a fraction of his salary at the time of retirement. Let us further assume that the person will die on his 85th birthday. The minimum reserve that must be held for such a pension liability, as required by current legislation in Ireland, is that amount determined if the pension based on his current salary is to be revalued by the lesser of inflation or 4% in any year, up to vesting at age 65. Let us take this latter approach in valuing the termination liability to this pension.

[8] This is often the case with defined benefit schemes as the liability increases, other things being equal, with the greater the age of the member, the longer the past service and the higher the salary. However, the extent to which it holds true is dependent on the maturity of the scheme.

Given that we want our valuation method to be market-based, then we would take the valuation rate of interest equal to the gross redemption yield on the bond closest in cashflow to the liability – in this case, given the restricted maturities on the bond markets and assuming no index-linked bonds, the yield of a 30 year bullet bond is taken. The annual rate of escalation of the benefit pre-retirement is assumed to be 2½% (this latter assumption is not a material, as discussed later). Finally, we assume at time 0 that the valuation shows that the value of the assets, assessed at market value, is identical to the (discounted) value of the liabilities. We wish to estimate the investment variation when the investment strategy is to invest totally in either (a) the equity market, (b) a conventional 20 year bond, (c) a bullet (or stripped) bond with a single payment in 30 years, or (d) short-term cash. The period between valuations is taken to be a calendar year (i.e., $p = 1$ in our formal definition of investment variation earlier).

From section 2, we know that the investment variation is the present value of the extent to which the increase in the value of the assets exceeds the increase in the liabilities over the year, the rate of discount (or inter-valuation rate of interest) being the rate at which the liabilities increased over the year. In the example, the inter-valuation rate of return, i_v, is given by:

$$i_v = \frac{\dfrac{(1.025)^{65-41}}{(1+i_1)^{65-41}}(1.025)(Pen)\bar{a}_{\overline{20|}}^{@i_1}}{\dfrac{(1.025)^{65-40}}{(1+i_0)^{65-40}}(Pen)\bar{a}_{\overline{20|}}^{@i_0}} - 1$$

where,

i_j is the valuation rate of interest at time j (that is, the gross redemption yield on the 30 year bullet bond at that time)

Pen is the pension on termination at time 0, payable from age 65.

The inter-valuation rate of interest can be seen as the hurdle rate of return that assets must exceed to show a positive investment variation over the year.

Using historic statistics of the UK capital markets, we investigated over each calendar year in the 20th century the *ex post* investment variation, assuming the assets are invested in different asset classes.[9] The result shows the *ex post* investment variation in each calendar year for each investment strategy, standardized by dividing the investment variation by the value of the liabilities at time 0.

Figure 5a: Standardised Investment Variation for 40 Year Old for each Investment Strategy, in each calendar year, UK Market (Case Study 1)

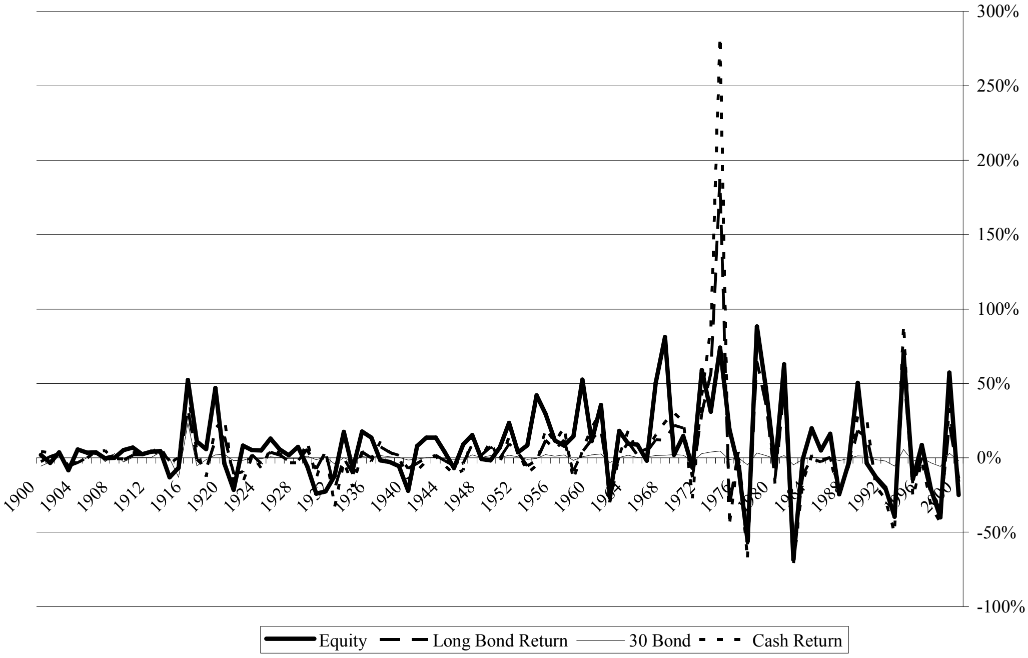
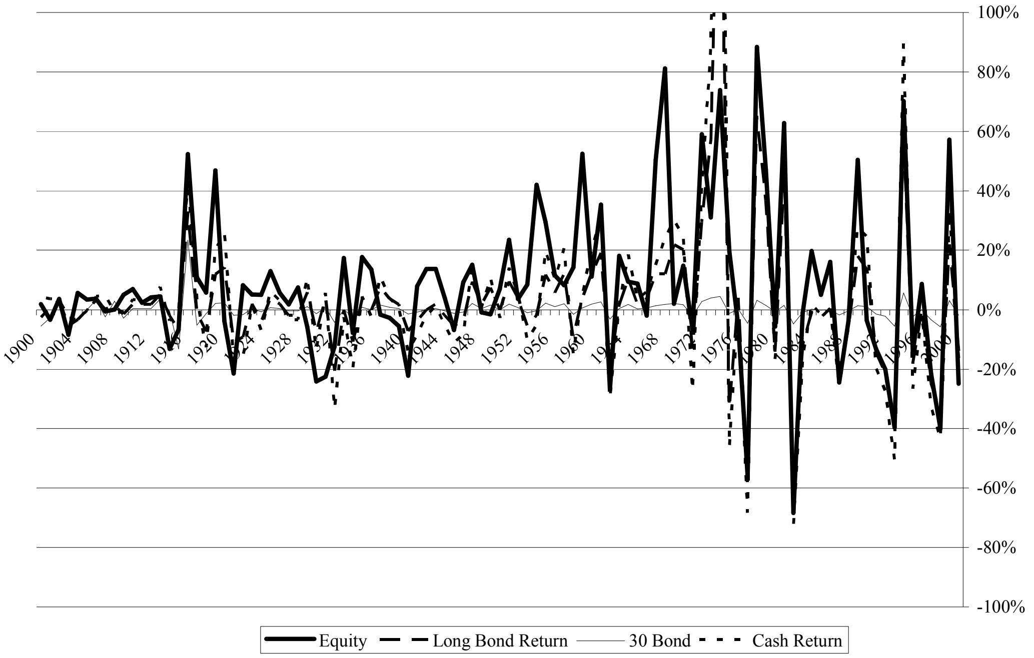


Figure 5b: Standardised Investment Variation for 40 Year Old for each Investment Strategy, in each calendar year, UK Market (Case Study 1) [Rescaled]

9 Returns and yields for the UK market were sourced as follows: 20 year gilt yields and returns and also cash returns were sourced from *Barclays Capital Equity Gilt Study 2003* for the period after 1962. Prior to 1962, yields at the year end and interest rates during the year were sourced from Mitchell (1988) and the return on a notional 20 year bond and cash calculated as outlined in Whelan (2004). The annual UK equity market returns were sourced from Dimson, Marsh and Staunton (2004). We assume that the yield on the 30 year bullet bond is the same as the yield on the long bond.



Figures 5a&b are dominated by the large positive investment variation posted by many mismatching investment strategies over the 1970s and early 1980s (coincident with the first and second oil shocks which raised inflation markedly leading, in turn, to large rises in bond yields). In particular, it shows that 1974, regarded as a bad year for UK equity investment because of the market crash was, from the perspective of immature defined benefit schemes, one of the better years, as the rise in long bond yields over the year reduced the present value of the liabilities by a considerably greater amount than equities fell. Figure 5 gives a very different history of the rewards from investing in the different asset classes to the traditional version of historic returns based on annual real or nominal returns on a unit invested.

The spread of the empirical distribution appears non-stationary in the graph – that is, the spread appears to change with time.[10] The implication of this observation for those attempting to forecast the distribution of the investment variation for each asset class is that it is especially challenging and past experience is only a loose guide to the future experience (see Whelan (2005) for further discussion on this point).

Table 1 sets out summary statistics to describe the key features of the empirical investment variation based on historic experience, with figures calculated for the

[10] This is not surprising as there is considerable evidence that returns from capital markets form a non-stationary time series (e.g., Loretan and Phillips (1994)).

whole 20th century, the second half of the 20th century, and those reflecting the experience since 1970.

Table 1: 40 Year Old: Summary Statistics of the Empirical Investment Variation Distribution, UK Markets in 20th Century (Case Study 1)

	Based on an Investment Strategy of 100% in…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
20th Century				
Mean	8.0%	2.9%	0.0%	4.9%
Median	5.0%	0.9%	0.2%	1.5%
Geometric Mean	4.6%	0.2%	-0.1%	-0.1%
Stan. Dev. [Investment Risk]	26.7%	26.5%	3.5%	38.2%
Skew	0.6	3.4	2.3	4.0
Excess Kurtosis	1.5	23.6	20.0	28.0
Since 1950				
Mean	13.0%	5.5%	0.1%	9.4%
Median	8.8%	1.9%	0.3%	2.0%
Geometric Mean	7.2%	0.3%	0.1%	0.1%
Stan. Dev. [Investment Risk]	34.4%	36.5%	2.5%	52.3%
Skew	0.1	2.5	-0.4	2.9
Excess Kurtosis	-0.1	11.7	0.3	14.4
Since 1970				
Mean	8.9%	5.5%	-0.3%	10.2%
Median	-1.0%	-2.5%	-0.2%	-1.6%
Geometric Mean	1.2%	-2.5%	-0.3%	-4.1%
Stan. Dev. [Investment Risk]	39.6%	46.5%	2.9%	66.4%
Skew	0.2	2.0	0.0	2.4
Excess Kurtosis	-0.6	6.9	-0.3	8.8

We can draw the following conclusions from Table 1:

(1) The 30-year bullet bond is, of those tested, the closest match to the liability as the investment variation distribution for this asset type exhibits the lowest standard deviation. Hence the 30-year bullet bond is close to the hedging portfolio. Equities and long bonds have similar investment risk, while cash is considerably higher.

(2) While the figures change whether one looks at the 30 year period, the 50 year period or the whole century, the relative ordering of the different asset classes in terms of this new definition of investment risk is largely unaltered. However, the estimated figure for investment risk is very high and dependent on the sample period for equities, conventional long bonds, and cash. This points to the need for considerable judgement in estimating the future investment risk of the different classes.

(3) Note in particular that a 20 year conventional bond (which, of course, has a weighted average duration lower than 20 years) is a duration mismatch for the 30 year bullet bond (which has a weighted duration of 30 years), and on the historic simulation, this term mismatch has introduced as much risk as equity investment.[11] The implication of this finding is that if pension funds could invest only in conventional, non-strippable bonds with a term to maturity no longer than 20 years, then the investment risk is almost the same for bonds and equities.[12] Equities could then be seen as preferable given their historic outperformance. This justified the cult of the equity and had been received actuarial wisdom until challenged by, *inter alia*, Exley, Mehta and Smith (1997).

(4) One of the assumptions in calculating the figures in Table 1 was that inflation subject to a cap of 4% over the year following the valuation could be approximated with the rate 2½%. The upper limit of possible outcomes is 4% that, if applied, would deduct about 1½% from the mean, median, and geometric mean figures above and leave all the other figures largely unaffected. This shows that the results of our analysis are not particularly sensitive to estimating this figure, once deflation of any severity is considered unlikely.

(5) The skew of the investment variation for the three conventional asset classes has been non-negative, which ensures that the mean exceeds the median. The geometric mean of the data, which corresponds to the annualised rate over the period, is the more relevant average for actuarial investigations. Table 1 shows that, historically, investing in the most closely matching asset of those studied (the 30 year bullet bond) involved a material reduction of the geometric mean only when compared to equity investment.

(6) Note that there is no simple relationship between the geometric mean (or other measures of average return) and the standard deviation (or investment risk) of the standardised empirical investment variation distribution. This entails, materially, that there is not necessarily a compensation for assuming extra risk in pension investing. Accordingly, investment advice can add real value by identifying the idiosyncratic risk of the pension provider (that is the deviation with respect to the hedging or least risk portfolio) and exploiting the uniqueness of this risk measure

[11] Note that the returns from the long bond and the 30-year bullet bond are highly correlated but the variability of the former is much lower than the latter, which leads to the mismatch.

[12] Note that if the class of portfolios is widened to include portfolios with either short-sales or borrowings, then it would have been possible to engineer higher durations.

relative to other investors' risk measures to help select investment strategies where the associated investment variation distributions have the largest geometric means for any given standard deviation.

Table 2 takes the same liability as case study 1 but now gives metrics on the empirical investment variation distribution based on equity, bond, and cash returns and long bond yields in the US and Irish capital markets over the second half of the twentieth century. The figures for the UK are included to aid comparison.

Table 2: 40 Year Old: Summary Statistics of the Empirical Investment Variation Distribution, 1950-2000 (inclusive), US, UK and Irish Experiences, Case Study 1

	Based on Investment Strategy of 100%…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
US Market				
Mean	13.6%	4.2%	0.3%	6.2%
Median	11.7%	1.7%	0.4%	2.8%
Geometric Mean	8.3%	1.5%	0.2%	1.1%
Stan. Dev. [Investment Risk]	34.1%	24.7%	2.5%	33.7%
Skew	0.2	1.2	0.1	0.9
Excess Kurtosis	-0.5	4.4	0.9	2.3
UK Market (from *Table 1*)				
Mean	13.0%	5.5%	0.1%	9.4%
Median	8.8%	1.9%	0.3%	2.0%
Geometric Mean	7.2%	0.3%	0.1%	0.1%
Stan. Dev. [Investment Risk]	34.4%	36.5%	2.5%	52.3%
Skew	0.1	2.5	-0.4	2.9
Excess Kurtosis	-0.1	11.7	0.3	14.4
Irish Market				
Mean	14.6%	6.1%	0.1%	11.2%
Median	0.6%	4.0%	0.6%	5.1%
Geometric Mean	6.7%	0.4%	0.1%	0.3%
Stan. Dev. [Investment Risk]	44.1%	38.6%	2.4%	57.0%
Skew	1.0	2.0	-0.5	2.6
Excess Kurtosis	1.3	7.7	-0.1	10.8

Table 2 reinforces the conclusions drawn from Table 1. In short, across the three markets studied, the 30 year bullet bond is the least risk investment of those studied, conventional long bonds and equities exhibit investment risk of roughly the same order of magnitude and cash tends to be higher still. Equities record materially higher geometric means than any of the other asset classes studied. Similar calculations have been done for a 30 year old person and, albeit at higher investment risks reported for each investment strategy, the results are similar and are summarised in Appendix II. On further investigation, it was found that only for persons aged 50 years was the risk of investing in the 20 year conventional bond below that of investing in equities.

3.3 Case Study 2: Measurement of Investment Risk in Pension Funds, On-going Liabilities

Case Study 1 treated the termination liabilities on the assumption that the scheme is terminated at the valuation date. However, if the scheme remains open, then under the other assumptions in our case study, the liability will increase by

(1) the excess of the increase in salary over the increase in pension in deferment,

(2) the increase in pensionable service,

(3) other factors capturing how the unfolding experience differs from the other financial and demographic assumptions used to estimate the liabilities.

In practice, of course, almost all schemes will continue so, arguably, the investment strategy that is best adopted is not the one that best matches the termination liabilities at one instant but the one that best matches the increase in the termination liabilities assuming the scheme is not wound up.

We investigate each of the investment strategies previously studied under this new scenario. In order to do so we need to make some further assumptions. We make the following additional assumptions:

i) The wage increase in any calendar year is 2% above inflation for that year. Thus the rate of increase of the termination liabilities assuming the scheme is not terminated is (1 plus rate of wage increase) /(1 plus the lower of 4% or the rate of inflation over the year) times the rate of increase of the termination liabilities assuming it is terminated, all other things being equal.

ii) The increasing pensionable service can be accurately allowed for in advance as it deterministic. This creates a factor (greater than unity) that multiplies the liability factor on scheme termination. We ignore this factor as it varies from scheme to scheme and can be estimated in advance.

iii) The experience of the scheme is in line with that assumed in calculating the termination liabilities in all other matters.

Note the similarity between the approach above and the on-going funding plan known as the 'defined accrued benefit method' described and discussed in McLeish and Steward (1987).

We can redo the previous analysis with these new assumptions, which we term Case Study 2.

Figure 6: Investment Variation for 40 Year Old for each Investment Strategy, in each calendar year, UK Market (Case Study 2)

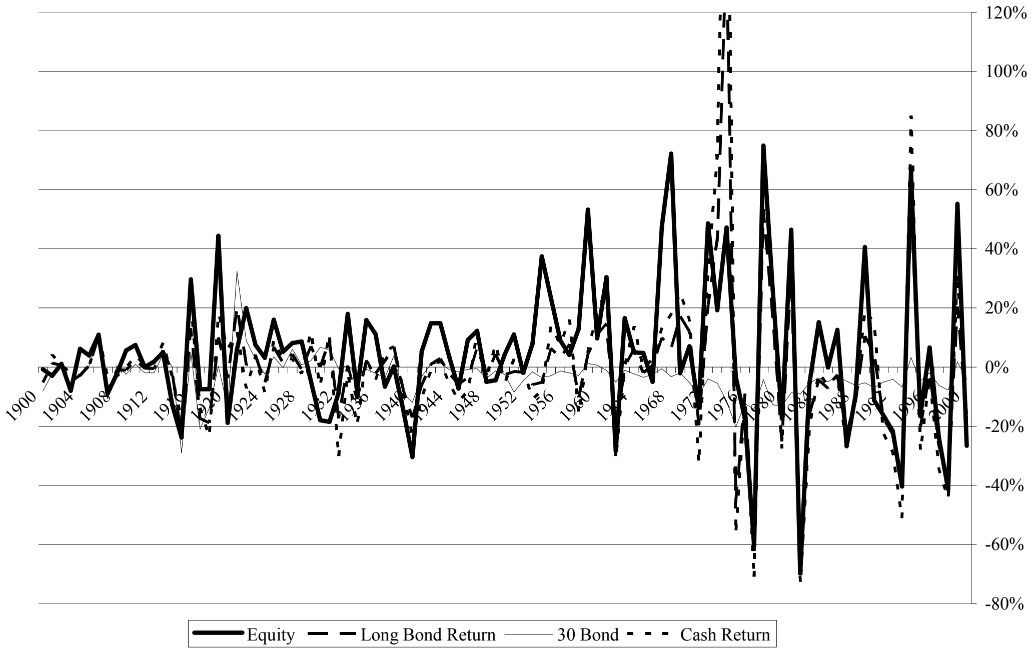


Table 3: 40 Year Old: Summary Statistics of the Empirical Investment Variation Distribution, over 20th Century, Second Half of 20th Century, and since 1970, UK Market, Case Study 2

	Based on an Investment Strategy of 100% in…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
20th Century				
Mean	4.3%	-0.7%	-3.1%	1.0%
Median	4.1%	-0.8%	-2.5%	-0.5%
Geometric Mean	1.1%	-3.1%	-3.4%	-3.4%
Stan. Dev. [Investment Risk]	24.7%	23.0%	7.0%	32.9%
Skew	0.4	2.2	0.5	3.2
Excess Kurtosis	1.5	15.6	7.7	21.1
Since 1950				
Mean	7.0%	-0.4%	-5.0%	3.1%
Median	5.0%	-1.8%	-4.2%	-0.5%

Geometric Mean	1.6%	-4.9%	-5.1%	-5.1%
Stan. Dev. [Investment Risk]	32.1%	31.5%	4.4%	45.2%
Skew	0.1	1.8	-1.1	2.4
Excess Kurtosis	0.0	8.0	2.1	10.7
Since 1970				
Mean	1.4%	-1.9%	-6.7%	2.2%
Median	-5.4%	-9.0%	-5.4%	-9.8%
Geometric Mean	-5.4%	-8.8%	-6.8%	-10.3%
Stan. Dev. [Investment Risk]	36.0%	40.0%	4.7%	57.2%
Skew	0.3	1.6	-0.7	2.0
Excess Kurtosis	-0.4	4.7	1.6	6.6

The 30 year bullet bond is still found, of the strategies assessed above, to entail the least risk, and the ranking of the other asset classes in terms of risk remains the same as the first case study (in fact the figures for investment risk are of the same order of magnitude as those earlier). The means and other measures of the central location of the distribution of the standardised investment variation are altered significantly (as could be expected) but, again, the relative ranking is very similar to that of Case Study 1. Accordingly, a bond-based strategy of suitable duration appears to be the least risk on an on-going as well as on a termination basis.

Further investigations with Irish market data and an explicit Irish wage index over the 20th century are compatible with the run of figures above, the key difference being that risk of equity investment is about one-fifth higher than for the conventional long bond. The higher risk figures on this alternative approach seems to be because wage inflation lags price inflation in any one year (and sometimes across years due to, say, wage controls during the second World War), with wage pressures sometimes released in a large aggregated increment. In short, using 2% above inflation could be regarded as a reasonable proxy for wage pressures, but actual wage increases tend to be somewhat later.

3.4 Summary of Findings

The arguments and evidence in this section leads to a conclusion that the most closely matching portfolio for pension fund liabilities is composed mainly of conventional and index-linked bonds, irrespective of both the age of the pension saver and, within wide bounds, the precise pension cashflows targeted. It also makes clear that there is generally no simple matching asset for pension fund liabilities and some judgement must be used in identifying the closest matching portfolio. We note, in particular, that the above argument leads to a least risk portfolio that, if history is any guide, has a lower expected long term return than a predominantly equity portfolio.

Perhaps the surprise in the results is that equities do not fare better in the risk comparisons, as equities, if a good inflation hedge, could have been expected to match liabilities increasing in line with wage inflation (which, is closely related to inflation). The hypothesis that there is a positive relationship between inflation and nominal return on stocks (so that they both move up and down together) is generally known as the Fisher Hypothesis, after the mathematical economist, Irving Fisher. Equities have not demonstrated themselves an inflation hedge in the US and the major euro equity markets, although there is some evidence to support a weak link in the UK economy. (See Gultekin (1983*)* for international evidence to this effect, covering 26 equity markets capturing more than 60% of the capitalisation of all equities in the world over the period 1947-1979). In short, no consistent positive relationship is evident between equity returns and inflation in most economies.

4. Time Diversification of Risk Argument

The analysis in section 3 compared the actual investment experience with that expected over periods of one year and, from that analysis, reported descriptive statistics for the empirical distribution of the investment variation. A natural question is whether the implications of our empirical investigation would significantly alter if the time period over which the distribution of the empirical variation was assessed increased from one to three or more years. In particular, some have advanced the argument that equity investment is preferable in the long-term but not necessarily the short-term, so if our review period was *p* years, where *p* is a 'large' number, then the equity investment strategy would have better risk and reward characteristics.

The problem in testing this hypothesis empirically is that we have a limited history of capital markets so that as *p* increases the number of non-overlapping intervals quickly decreases. We have only 10 distinct non-overlapping decades in the 20th century, which would give just 10 data-points in the empirical distribution. However, we can resolve the problem with a simple model of the investment variation distribution. We treat one model below but note that the insight it gives applies to a very wide category of models.

The empirical distribution given in the tables earlier was the standardised investment variation over one year, or equivalently, the distribution of the percentage change in the funding level. Let Y be a random variable with this distribution. Then the funding level at time 1 (F_1), given it was 100% funded at time 0 is

$$F_1 = 100(1+Y)$$

A simple model for the funding level at time p (F_p) is

$$F_p = 100(1+Y_1)(1+Y_2)...(1+Y_p)$$

where each Y_i is independent of the others and has same distribution as Y. Now,

$$\ln F_p = \ln 100 + \ln(1+Y_1) + ... + \ln(1+Y_p)$$

Let us further assume $\ln(1+Y)$ is normally distributed with mean μ and variance σ^2. Then $\ln F_p$ is normally distributed and F_p is lognormally distributed. Then, from the well-known parameterisation of the lognormal, we can write

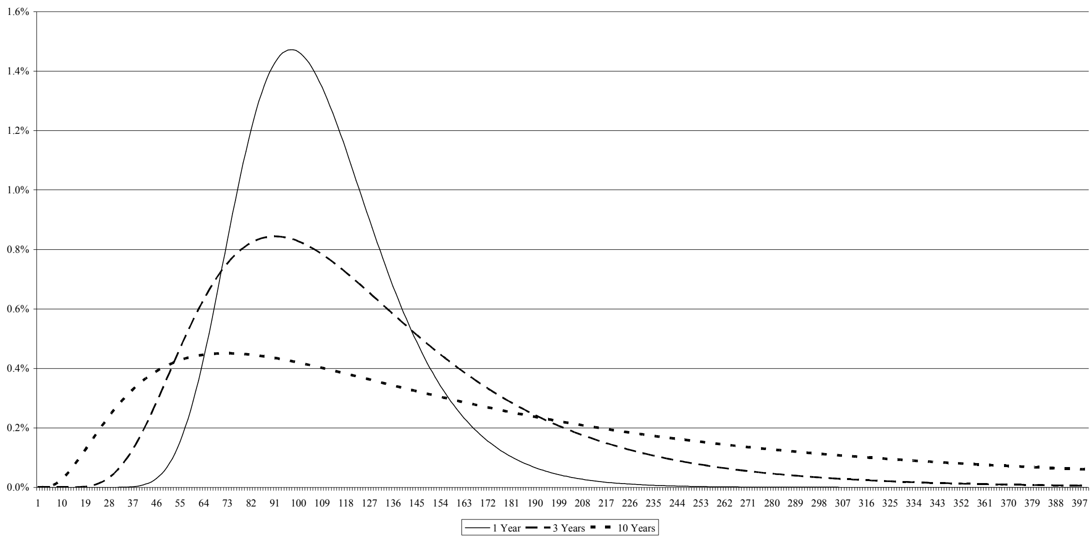
$$E[Y] = e^{\mu+\frac{1}{2}\sigma^2} - 1 \qquad \text{(I)}$$

and

$$Var[Y] = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} \qquad \text{(II)}$$

We have already estimated $E[Y]$ and $Var[Y]$ in the previous section and so can solve the above equations for μ and σ^2. We might assume, for concreteness, that it has a mean of 8% and a standard deviation of 30% for equity investment. Solving (I) and (II) with these parameters gives μ=0.04 and σ=0.27. The density function of the funding level at time *p,* where *p=1, p=3* and *p=10,* is graphed in Figure 7.

Figure 7: Probability Density Function of Funding Level, when Viewed at end of 1,3 and 10 years, assuming Log-Normal Distribution (see above)



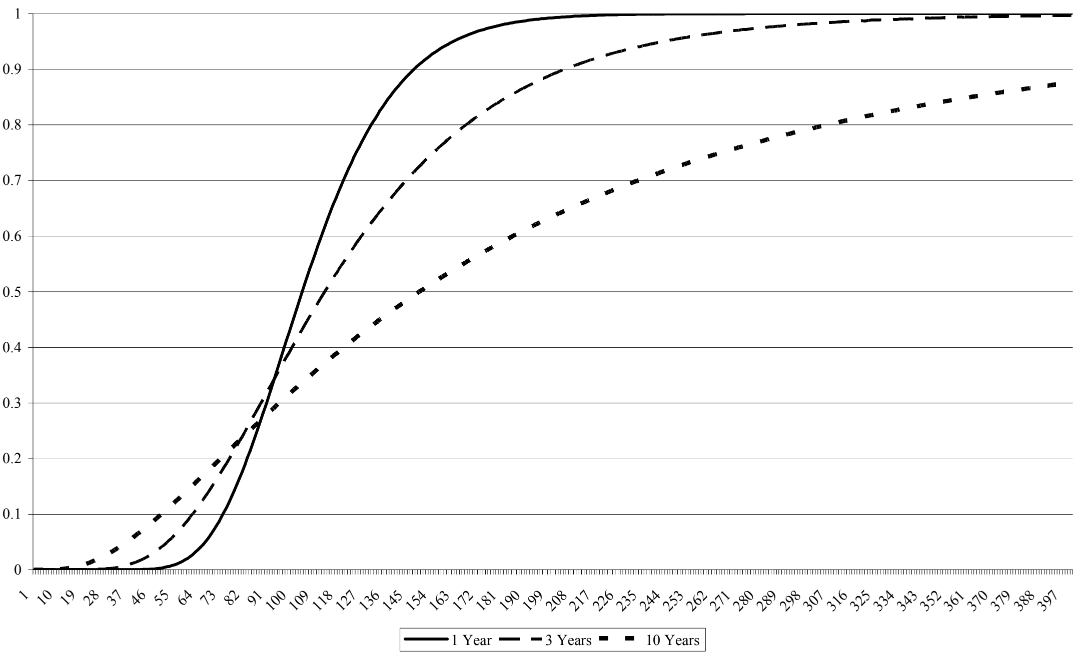
We note that the distribution of possible outcomes is wider when the review term increases ('the expanding funnel of doubt') and, in particular, that the probability of a very low funding level is higher the greater the period between reviews. From the above graph of the funding levels, a rational investor need not necessarily favour the outcome when p=10 (or, more generally, when p is large) over the outcome when p=1. When p=10, the expected value is increased but so too is the probability of an extremely poor outcome. A particularly risk averse investor could conceivably prefer the outcome when p=1 over when p=10.

We can investigate the above remarks in a more formal setting. Given two distributions, the condition that

$$F_1(x) \leq F_2(x), \forall x$$

is described as the first order stochastic dominance (FSD) of $F_1(x)$ over $F_2(x)$, where the $F_i(x)$ are distribution functions. A return distribution that first order dominates another is preferred by any wealth maximiser regardless of their utility function. The distribution functions of the funding levels for each forecast period are graphed in Figure 8.

Figure 8: Cumulative Distribution Function of Funding Level, when Viewed at end of 1,3 and 10 years, assuming Log-Normal Distribution



So, clearly, no distribution for any p stochastically dominates any of the others.

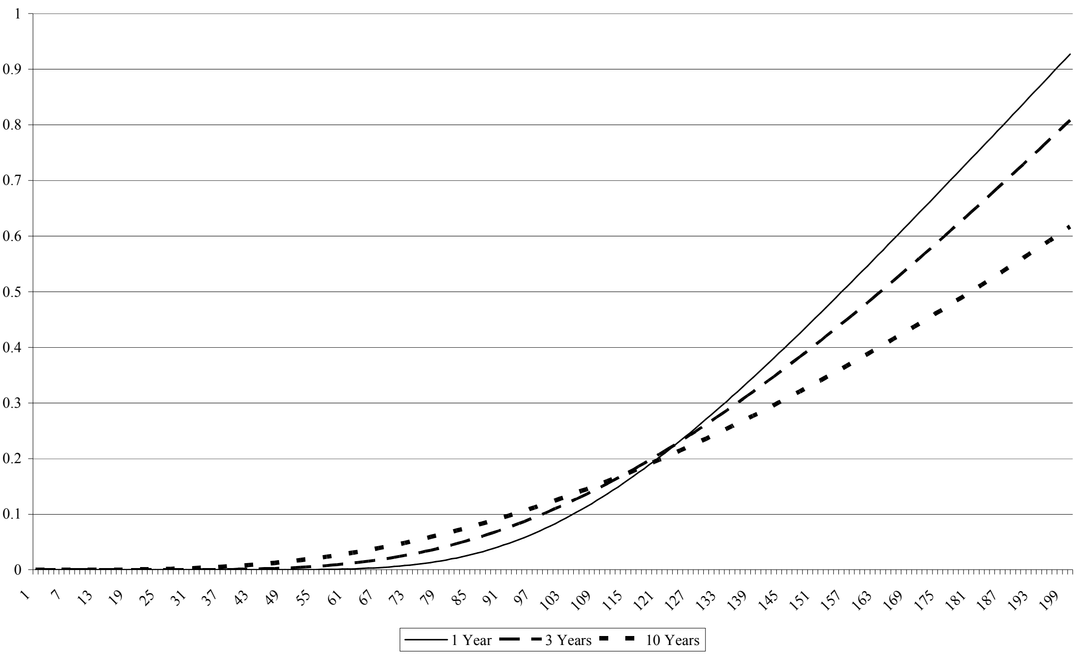
A less stringent condition in comparing two distributions is second order stochastic dominance (SSD), with $F_1(x)$ said to dominate $F_3(x)$ by SSD if and only if

$$\int_{-\infty}^{x} F_1(y)dy \leq \int_{-\infty}^{x} F_3(y)dy, \forall x$$

It can be shown that investors who are both nonsatiated and risk averse can be shown to prefer the payoff of $F_1(x)$ over $F_3(x)$.[13] Again, under our model earlier, we can show that no distribution for any p stochastically dominates to second order any of the others. Figure 9, capturing the area under the distribution functions up to the 400% funding level, demonstrates this.

Figure 9: Area under Cumulative Distribution Function of Funding Level ($\int_{-\infty}^{x} F_p(y)dy$ ***), when p= 1,3 and 10 years, assuming Log-Normal Distribution***

[13] See, for instance, Eichberger and Harper (1997).



5. Conclusion

We defined investment risk in a general context and applied our definition to give an empirical measure of investment risk of different investment strategies for pension providers. Through a series of case studies, we were lead to a conclusion that the most closely matching asset for pension liabilities is composed mainly of conventional and index-linked bonds. The least risk portfolio has, if history is any guide, a lower expected long term return than a predominantly equity portfolio.

Our case studies also show that the equity exposure maintained by pension funds since the 1950s was justified when liabilities were relatively immature and bond markets offered limited duration. In short, the investment risk of investing in equities was of the same order of magnitude of the investment risk introduced by the duration mismatch from investing in bonds, but the rewards were materially higher. With the extension of duration in bond markets in recent times and the innovation of stripping, suitably long bonds now provide the least risk investment strategy even for immature schemes. Alongside the growing ability to manage investment risk, the capacity to bear risk has been eroded over the last couple of decades as regulations have increased the guaranteed part of the pension promise (especially as it related to early leavers or benefits payable on scheme termination) and the surplus has reduced.

Note that our analysis does not allow us to suggest one investment strategy is preferable to another. Investors, including pension savers, are routinely tempted to take risks if the reward (that is, the form of the investment variation distribution) is judged sufficiently tempting. However, pension funds should appreciate the risks involved in alternative strategies and, at a minimum seek to ensure that the investment portfolio is efficient in the sense that risk cannot be diminished without diminishing reward. In particular, it is shown that the idiosyncratic nature of investment risk of pension saver relative to other investors might be exploited to increase expected surplus without increasing risk. In the past, when bond markets offered only limited duration, immature pension schemes exploited this by investing in equities.

To appreciate the risks and ensure that all risks undertaken are reasonably rewarded requires knowledge explicit measuring and monitoring of investment risk. It is hoped that a solid platform to build a consensus on suitable investment strategies for pension funds can be achieved through formalizing our intuitive notion of investment risk in actuarial valuations as outlined in this chapter.

References

Arthur, T.G. and Randall, P.A. 1989. Actuaries, pension funds and investment. *Journal of the Institute of Actuaries* 117: 1-49.

Barclays Capital 2003. *Equity gilt study 2003, 48th edition*. Barclays Capital, 54 Lombard Street, London.

Dimson, E., Marsh, P. and Staunton, M. 2004. *Global investment returns yearbook 2004.* ABN-AMRO and the London Business School.

Eichberger, J. and Harper, I.R. 1997. *Financial economics.* Oxford University Press.

Exley, J., Mehta, S. and Smith, A. 1997. The financial theory of defined benefit pension schemes. *British Actuarial Journal* 3 (IV): 835-939.

Gultekin, N.B. 1983. Stock market returns and inflation: evidence from other countries. *Journal of Finance* 38(1): 49-65.

Loretan, M. and Phillips, P.C.B. 1994. Testing covariance stationarity of heavy-tailed time series. *Journal of Empirical Finance* 1: 211-248.

McLeish, D.J.D. and Steward, C.M. 1987. Objectives and methods of funding defined benefit pension schemes. *Journal of the Institute of Actuaries* 114: 155-199.

Mitchell, B.R. 1988. *British historical statistics*. Cambridge University Press.

Whelan, S.F. 2001. Investing the National Pensions Reserve Fund. *Irish Banking Review,* Spring: 31-47.

Whelan, S.F. 2004. Measuring investment risk in pension funds. Unpublished paper delivered to *Society of Actuaries in Ireland*, 24th February.

Whelan, S.F. 2005. Discussion on 'equity risk premium: expectations great and small, Derrig, R.A. and Orr, E.D'. *North American Actuarial Journal* 9 (1): 120-124.

Whelan, S.F. 2007. Defining and measuring risk in defined benefit pension funds, *Annals of Actuarial Science* II (1): 54-66.

Appendix I: Limitations of Proposed Definition of Investment Variation (and the associated Investment Risk)

The definition of investment variation (and the associated investment risk) has some limitations. Limitations arise from the fact that the definition ignores the important relationship between the wealth and income generating power of the pension provider (e.g., sponsoring employer, individual) and the investment strategy pursued. A full treatment of the problem would model, not just the distribution of the difference between the value of the assets and that of the liabilities at any point in time, but also the coincidence of risk between a shortfall being revealed at any future date and the ability (and, if possible to model, the willingness) of the pension sponsor to fund the shortfall under the circumstances at that time.

We can make some general points on this limitation. First, as a hypothetical case, consider a defined benefit pension fund with a high exposure to the business of the sponsoring employer. Such an investment strategy increases significantly the twin risk of a shortfall in the value of the assets over the liabilities just when the sponsoring employer is unable to make up the shortfall.[14] In fact, in this case, members might lose their jobs and part of their pension entitlements if the employer fails. Now, in a less extreme case, the performance of an equity-based portfolio could be correlated to some degree with the fortunes of the sponsoring employer. Consider, for instance, the difficulties faced by a small company in the high-technology sector, sponsoring a pension fund over the couple of years since March 2000. Here we have similar underlying factors creating financial strain in the pension fund and to the profitability of the sponsoring employer. This is an instance of a significant fall in the value of the portfolio occurring at an inopportune time for the employer – once again adversely affecting the security of the members' pension entitlements just when those pension assets could be called upon.[15] The extent to which these points are material to any particular scheme and sponsoring employer depend, *inter alia*, on the relative surplus of the value of scheme assets over the value of its liabilities (as, other things being

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[14] For this reason, the regulation typically impose limits on the level of 'self-investment' (as this practice is called) allowed by approved pension schemes.

[15] Indeed, with the new disclosures demanded of companies under the accounting standard FRS 17, a deficit revealed in the pension fund could precipitate a financial crisis for the employer (say, by reducing their credit rating) and, if the deficit was caused by a sudden collapse of equity values, this is likely just at a time when equity capital is expensive and difficult to raise.

equal, the greater the relative surplus the less likely a deficit will be revealed) and the volatility of the employer's profits. A bond-based portfolio of suitable maturity profile ensures that the twin risks of a deficit revealed in the pension funds and, at the same time, the employer is particularly financially constrained are largely independent or perhaps even negatively correlated with one another.

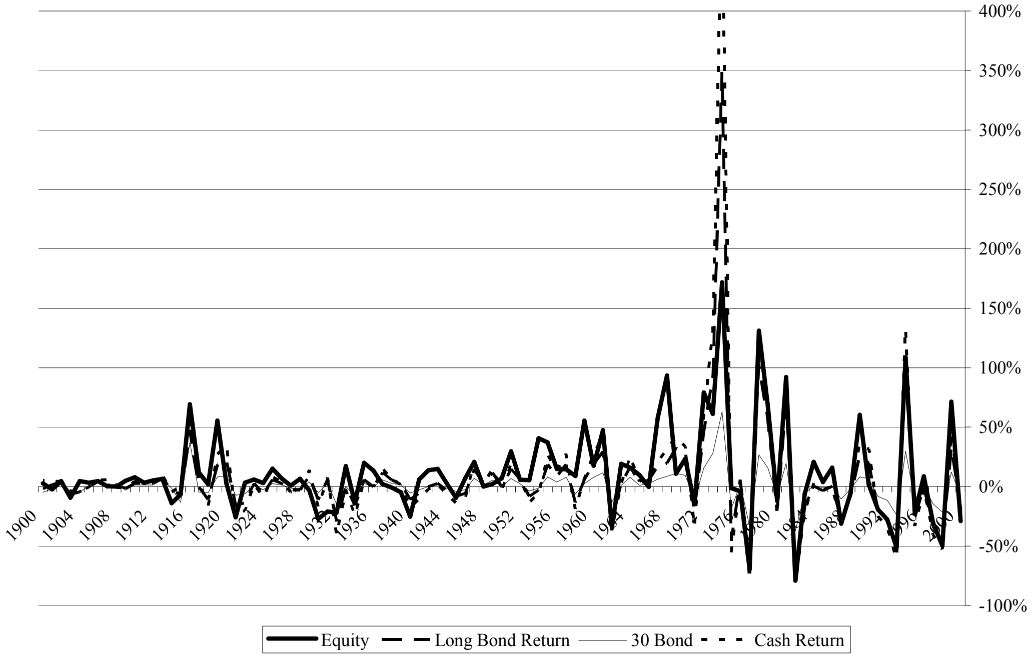
A case can perhaps be made that pension funds to date have not fully exploited asset types or investment strategies that are uncorrelated or negatively correlated with the financial health of the sponsoring employer. Whelan (2002) treats the case of the National Pensions Reserve Fund in Ireland, outlining an argument that the Fund should underweight its exposure to indigenous Irish industries and those sectors of the world equity market in which the Irish economy has already a high exposure (such as the pharmaceutical and technology sectors).

The general point made in this appendix that the very same portfolio could have quite different risk characteristics depending on the nature of the business of the sponsoring employer or the human capital of the individual pension saver. Account should properly be taken of this relationship in a more satisfactory definition of investment risk.

Appendix II: Case Study 1 when Pension Liability due to 30 Year Old

We can apply the very same mode in section 3.2 to a 30 year old. The results are as follows, in graphical and tabular form.

Figure II.1: Investment Variation for 30 Year Old for each Investment Strategy, in each calendar year (Case Study 1)



Note the 30 year bullet bond – the longest available on the market – is not long enough to match the liability so we witness investment variation arising from the term mismatch. The fluctuations in investment variation for the 30 year bullet bond tend, as is apparent from Table II.1, to be lower than that of the other asset classes.

Table II.1: 30 Year Old: Summary Statistics of the Empirical Investment Variation Distribution, over 20th Century, second half of the 20th century, and 1970-2000 (inclusive), Case 1, UK Market

	Based on an Investment Strategy of 100% in…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
20th Century				
Mean	10.8%	6.2%	0.9%	9.3%
Median	4.7%	1.5%	1.1%	2.5%
Geometric Mean	4.8%	0.4%	0.1%	0.1%
Stan. Dev.	37.3%	44.0%	12.6%	60.4%
Skew	1.4	4.9	1.1	5.6
Excess Kurtosis	4.1	36.8	6.8	43.2
Since 1950				
Mean	18.1%	11.5%	1.6%	17.5%

Median	9.8%	2.9%	1.3%	3.8%
Geometric Mean	7.5%	0.5%	0.3%	0.3%
Stan. Dev.	49.0%	60.8%	16.2%	83.2%
Skew	0.8	3.5	0.7	4.0
Excess Kurtosis	1.2	18.7	3.6	22.3
Since 1970				
Mean	15.0%	13.6%	0.4%	21.4%
Median	-1.1%	-3.4%	-1.1%	-2.5%
Geometric Mean	0.0%	-3.6%	-1.5%	-5.2%
Stan. Dev.	59.4%	77.6%	20.3%	106.0%
Skew	0.8	2.8	0.8	3.2
Excess Kurtosis	0.4	11.2	1.9	13.6

We note that equities appear better than 20 year conventional bonds as the risk is lower but the reward is higher. As one would have expected from the earlier discussion, the risk of all asset types studied in meeting the termination liability is increased when compared with that of the 40 year old.

Appendix III: Case Study 2 when Pension Liability due to 30 Year Old

We can apply the very same mode in section 3.3 to a 30 year old. The results are summarises in tabular form.

Table III.1: 30 Year Old: Summary Statistics of the Empirical Investment Variation Distribution, over 20th Century, second half of 20th century and from 1970-2000 (inclusive), Case Study 2, UK Market

	Based on an Investment Strategy of 100% in…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
20th Century				
Mean	7.1%	2.6%	-2.8%	5.7%
Median	3.9%	0.1%	-1.0%	0.3%
Geometric Mean	0.8%	-3.8%	-3.7%	-4.5%
Stan. Dev.	36.3%	42.8%	13.0%	58.9%
Skew	1.1	4.6	-0.4	5.4
Excess Kurtosis	3.4	34.8	3.3	42.4
Since 1950				
Mean	12.5%	5.9%	-4.0%	11.9%
Median	6.7%	-1.0%	-2.7%	0.5%
Geometric Mean	1.0%	-6.1%	-5.4%	-7.2%
Stan. Dev.	48.1%	59.3%	15.8%	81.6%
Skew	0.7	3.4	-0.1	3.9
Excess Kurtosis	0.8	17.7	2.2	21.6
Since 1970				
Mean	7.9%	6.5%	-6.6%	14.3%
Median	-10.9%	-8.5%	-8.3%	-10.5%
Geometric Mean	-7.9%	-12.1%	-8.7%	-15.0%
Stan. Dev.	57.9%	75.8%	19.5%	104.1%
Skew	0.8	2.7	0.3	3.2
Excess Kurtosis	0.2	10.7	0.9	13.2