Discussions of Papers Already Published

"Equity Risk Premium: Expectations Great and Small," Richard A. Derrig and Elisha D. Orr, January 2004

Shane F. Whelan*

Derrig and Orr provide a comprehensive overview of current estimates of the equity risk premium (ERP), carefully distinguishing between the many different definitions in common use that lead to much confusion. Their survey concentrated almost entirely on the U.S. markets, with the statistical analysis based primarily on the 77 annual returns over the period 1926–2002 (inclusive) given in Ibbotson Associates (2003), with occasional reference to a longer series of annual returns from 1871 to 2002 in Wilson and Jones (2002) or data over the years 1802–2001 in Siegel (2002). In this discussion I address two issues:

1. I draw attention to another strand of research in this area that demonstrates that returns from capital markets are not a stationary series. If returns are nonstationary, then this undermines the direct use of simple historical averages or estimating the future ERP based on projections from stationary models fitted to the data. As the approach outlined by the authors as well as many of those surveyed assumed returns are stationary, this is a particularly devastating critique.
2. I explore a little further the alternative way of viewing the historical market returns suggested by the authors (Section 6), when the U.S. experience is treated as just one realized path of the grand stochastic process that is the capital markets. The past performance of other national capital markets traces other paths, which, though perhaps neither independent nor equally likely, can be used to shed light on the process of asset price formation and the evolving market price of risk. To provide added contrast to the Derrig and Orr study, I treat the experience of the smallest national market with a history as long as the U.S. market: the Irish capital markets. The Irish experience reinforces the earlier remarks on the nonstationarity of the ERP.

Is the Path Traced by the U.S. Equity Risk Premium Weakly Stationary?

Derrig and Orr do provide some tests for the ERP being weakly stationary (Sections 6–9) but fail, in my opinion, to interpret them correctly. First, they report that equality of the sample variances over two subperiods can be rejected at the 1% significance level under a standard F-test (footnote 16). This is evidence, insofar as the normality assumption under the F-test is tenable, that the annual ERP does not form a stationary series and, in particular, cannot adequately be modeled as independent and identically distributed as suggested (Sections 6 and 9). Second, the t-test they employ to test equality of means in Table 6 (or more strictly, that the mean of the subperiod 1960–2002 equals the mean of the total period 1926–2002) is questionable in light of the reported difference of variances. However, even if the variances were equal and best estimated with just the 1960–2002 data, the test they employ has such low power that it could not reject the null at the 5% critical level if the true ERP in the 1926–1959 fell anywhere in the range –5.7% to +16.3%.[1] As this range encompasses all reasonable values for the ERP, the failure to reject the null of constancy of the ERP is really saying more

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[1] The t-test in Table 6 of the paper is that the mean of the second subperiod equals that of the total period. The test statistic can therefore be decomposed into the mean of the second subperiod weighed by the number of data points plus the mean of the first subperiod weighed by its number of data points, i.e., $\frac{5.27\% - (43 \times 5.27\% + 31\bar{x}_{1926\text{–}1959})/77}{15.83\%/\sqrt{43}}$, which, as applied, is assumed to follow a t-distribution with 42 degrees of freedom. Using critical values of the statistic, it is straightforward to solve for the range of values that $\bar{x}_{1926\text{–}1959}$ can take without rejecting the null. The result is somewhat larger than their reported confidence intervals.

about the paucity of data than about the structure of the data.

If we move from annual return to monthly return data, then the data set increases 12-fold, and statistical testing can be more discriminating between alternative hypotheses. I am unaware of studies testing properties of the monthly ERP, but a considerable literature exists on the properties of monthly returns from both bonds and equities. Loretan and Phillips (1994) is a particularly pertinent study as it demonstrates that U.S. monthly stock returns (from January 1834 to December 1987) are not weakly stationary (even when allowance is made for the well-documented seasonality in means, second moment dependencies, and failure of the fourth moment of the unconditional return distribution). This finding is especially general as it rules out many classes of models popularly used to characterize return data, such as the ARMA suite, ARCH and GARCH processes where the unconditional second moment is constant, and many types of regime-switching models (where the unconditional model found from integrating over all possible regimes is stationary). As the ERP is the difference between volatile stock returns and less volatile cash (or bond) returns, one would expect the ERP series to inherit noncovariance stationarity from the stock return series.

Loretan and Phillips's testing procedure reports that the failure of the ERP's being weakly stationary is due to the nonconstancy of the unconditional variance of the return series, so their finding does not preclude the constancy of the unconditional mean of the returns (and thereby the ERP) over the period. However, if the ERP is a premium for assuming equity risk, and equity risk is measured by the volatility of excess returns,[2] then, on economic grounds, one would predict a higher ERP in those times when the equity return series exhibit higher volatility. Hence, we can infer nonconstancy of the ERP from the nonconstancy of the unconditional variance of the stock returns. Since Loretan and Phillips (1994) a number of papers, using different approaches, have appeared that confirm their finding of the nonstationarity of returns from capital markets, although most such studies are based on daily or higher-frequency returns. See, for instance, Ibrahim (2003) for another direct testing procedure that reports failure of weak stationarity in daily returns of the S&P500, or, more indirectly, the very considerable empirical evidence presented in Plerou et al. (1999a, 1999b), and the supporting evidence in Pagan (1996), based on the monthly returns of the S&P Composite Price Index in the period 1928–87, that the fourth moment of the unconditional return distribution of U.S. stocks and stock indices fails—a finding inconsistent with a weakly stationary series where the fourth moment of the innovations exist.

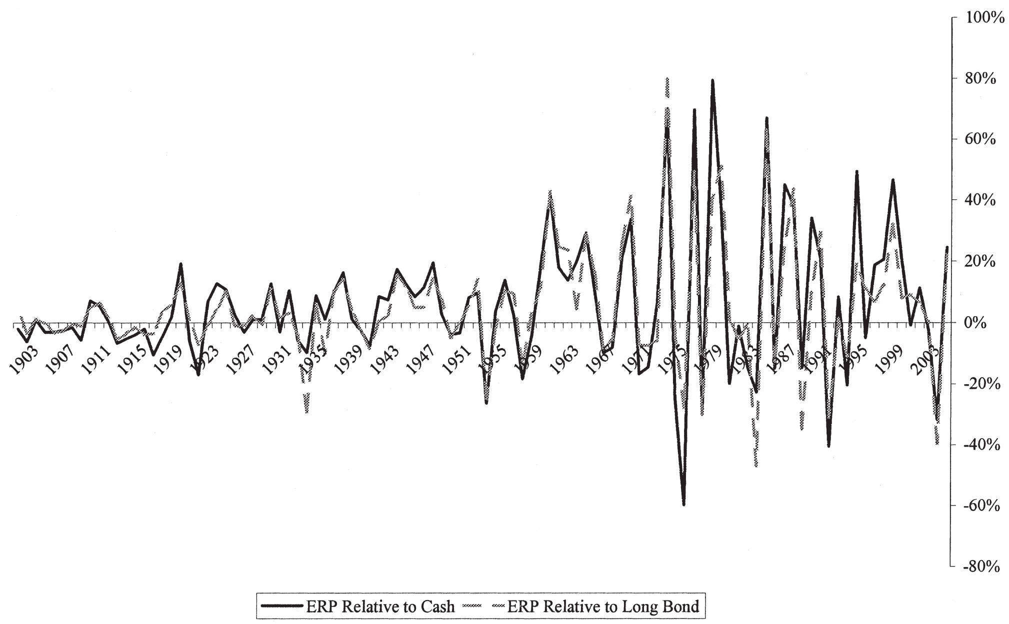
OTHER PATHS

The above considerations point to the conclusion that the path of the ERP in the United States forms a nonstationary series, casting doubt on many of the approaches used to forecast it that Derrig and Orr survey in their paper. Viewing the evolution of the ERP in the United States as just one realized path of a stochastic process as suggested in Section 6, knowledge of the ERP can be augmented by considering other market histories. Consider, for instance, the Irish capital markets, which, though small, have a history of continuous trading as long as that of the U.S. markets.[3] Especially relevant to this discussion is that the path of the ERP in the Irish market reinforces the above remarks on its nonstationarity, as illustrated in Figure 1. Inspection of the figure shows, without the need for formal statistical tests, that the series are obviously nonstationary. The variance of the returns in the latter half of the twentieth cen-

[2] Officer (1973) explored the relationship between the standard deviation and other measures of variability, comparing the rolling 12-month standard deviation of returns with the 12-month mean absolute deviation and the 12-month interpercentile range (from the 28th percentile to the 72nd percentile). He reports a reasonably stable relationship between the estimated variability on each of these measures using monthly data from U.S. market from February 1897 to June 1969. Accordingly, defining and estimating risk by other measures of the spread of the return distribution is unlikely to produce significantly different conclusions.

[3] The Irish equity market has a capitalization less than ½% of that of the U.S. market at the present time. The Dublin Stock Exchange was formally constituted in 1799, making it the sixth oldest surviving national stock market in the world according to Goetzmann and Jorion (1999).

Figure 1
Ex Post Equity Risk Premium on Irish Capital Markets Each Year, 1900–2003



Source: Calculated from the annual returns for each market in Whelan (2004), suitably updated.

tury is clearly significantly higher than that of the first half.

As noted by the authors, Dimson, Marsh, and Staunton (2002) provide the most complete synthesis to date of the twentieth-century experience of national capital markets, recording returns from the cash, bond, and equity markets in 16 countries that, in total, cover about 90% of the current world markets by capitalization. No doubt the path traced by the ERP in each of these markets will reinforce the above remarks. However, the domain of study perhaps can be cast even wider than just the paths traced by low-frequency returns of national markets over the long term. Investigations of the statistical properties of the return paths traced by equity markets have shown that many key properties are invariant with respect to a change in the timescale over which returns are measured (e.g., monthly returns exhibit the same patterns as daily or hourly returns), and markets as diverse as those for commodities, currencies, cash, bonds, and equities display remarkably similar properties. Cont (2001) provides an overview of key empirical regularities of the return paths of financial markets, pointing out, aside from their shared property of nonstationarity, that all returns over any timescale exhibit (a) a heavy-tailed distribution, where the variance exists but the kurtosis (fourth moment) does not, (b) a volatility that tends to cluster in time, and the decay from high bouts of volatility tends to follow a characteristic power law, (c) a negative correlation between the current return and future volatility, decaying to zero in a characteristic pattern as the time lag increases, (d) an asymmetry between large positive and negative movement, with the latter more frequent, and (e) a high correlation between volume traded and volatility. The invariance of these properties with respect to time scaling and between markets strongly suggests that the annual returns delivered by the U.S. markets over the long-term past are no different statistically from, say, hourly returns on the dollar-yen over the last few weeks. Modeling with the latter, however, reduces the problems associated with the paucity of data of the former. It is true that estimation of the ERP is based on the difference between two market returns (the risky and riskless), but parallels can be drawn between the ERP and the minimum enticement for market players (in whatever market) to increase their mismatch risks.

IMPLICATIONS OF NONSTATIONARITY

Nonstationarity of the ERP series and, more generally, returns series from capital markets, tell us that past performance is not a reliable guide to future performance. The riskiness of markets, if measured by the standard deviation of returns or other measures of spread, changes with time not just in temporary bouts (as captured in ARCH-type models), but structurally: the whole background volatility of the markets changes level with time. If the underlying risk is a function of time, then the risk premium must also be a function of time, implying, in turn, that simple averages of the historical ex post ERP must be too. To forecast the ERP by fitting a stationary model is therefore unstable in the sense that changing the time period used to calibrate the model will change the forecast ERP.

We must abandon the stationary assumption of asset returns. One obvious approach to forecasting returns from capital assets is to transform the original return data into a (near) stationary series based on estimates of the unconditional variance at each point in time, forecast the transformed series using standard stationary models, and then apply the inverse transformation to the result to forecast the original returns. Van Bellegem and von Sachs (2004) provide such a development by rescaling time so that the process is "locally" stationary and apply it to forecast daily returns from several markets. Okabe, Matsuura, and Klimek (2002) use another technique to detect the early breakdown of stationarity, claiming that their method can be used to help predict stock-market crashes.[4] Modeling and forecasting allowing for nonstationarity in returns is at an early stage, so it provides, as yet, no reliable guide as to the future evolution of the market price of risk. It does suggest, though, that forecasts from stationary models should be used with circumspection.

Risky assets provide, by definition, an uncertain payoff. Forecasting the equity risk premium must be done in tandem with forecasting the expected course of the riskiness of the asset. But, as this discussion hopes to make clear, the problem is compounded in that we are uncertain of even the riskiness of risky assets.

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VAN BELLEGEM, SEBASTIEN, AND RAINER VON SACHS. 2004. Forecasting

[4] If the unconditional volatility changes significantly as a quantum jump, then one would expect the future ERP demanded to hold risky assets to also change markedly, which necessitates a considerable change in the price of the assets (which might give economic justification for the stylized fact noted earlier that current return is negatively correlated to future volatility). A glance at Figure 1 shows that the volatility of the Irish equity market increases prior to its major crash in 1974. Hence, the ability to forecast changes in the unconditional volatility could reasonably be expected to help in forecasting significant changes in asset values.

Economic Time Series with Unconditional Time-Varying Variance. *International Journal of Forecasting* 20: 611-27.
WHELAN, SHANE 2004. Measuring Investment Risk in Pension Funds. Paper presented to the Society of Actuaries in Ireland, February 24, 2004. Online at www.actuaries.ie.
WILSON, JACK W., AND CHARLES P. JONES. 2002. An Analysis of the S&P500 Index and Cowles Extensions: Price Indices and Stock Returns, 1870–1999. *Journal of Business* 75(3): 505–33.

AUTHORS' REPLY

We welcome Whelan's discussion for highlighting the role that stationarity plays in discussing the theoretical formation of an equity risk premium (ERP) and for the introduction of data from the Irish capital markets. Our reply consists of three points. First, we do not view the assertion of the absence of weak stationarity as a "devastating critique," as Whelan concludes. Second, the principal thrust of our paper was the wide definitional disparity among the many studies of the ERP puzzle for the U.S. market and their subsequent expectations for the future, most of which avoided or ignored the question of stationarity in any form. Third, our recommendation to practicing actuaries was to use the Ibbotson-Chen building-block method to forecast the ERP, a tool that could be applied equally well to the Irish and other equity markets and that does not depend on stationarity but does depend on replication of the historical mean for each block, absent a rationale *and an estimate* for a change in the block value.

STATIONARITY

Strictly speaking, a time series is *stationary* if all of the statistical properties remain unchanged when the period of observation is shifted forward or backward, or equivalently, if the distribution functions of all consecutive subseries are independent of time (Kruskal and Tanur 1978, pp. 1168–69; Kendall and Stuart 1976, p. 424). Thus the mean, variance, and all other existing moments will remain the same when the period of observation is shortened or lengthened in a stationary series. *Weakly stationary* generally means that only the first two moments, mean and variance, need to be equal. The ERP puzzle literature we reviewed relates only to the expected mean and only incidentally to the other moments. Whelan discusses our test for stationarity of the Ibbotson 1926–2002 series (Derrig and Orr 2004, pp. 51–52), where we informally define stationarity as a mean value unchanging with time (Kendall and Stuart [1976, p. 424] define a separate "stationary in the mean" as the "customary" definition of stationarity of stochastic processes), in line with the ERP puzzle, and test for equal means for the entire series and the 1960–2002 subperiod of the Ibbotson annual data. We find that the *t*-test supports equal means whether or not the variances are assumed equal or not and that there is also some support for unequal variances: that is, the entire Ibbotson series is not *weakly* stationary. This result is due to the large volatility of the depression years of the 1930s (41.6% versus less than 20% for later decades; see Ibbotson Associates 2004, Yearbook, Table 6-1, p. 110), much as the latter years of the Irish market data appear to be more volatile. (Ibbotson Associates' [2004, Valuation Edition, pp. 85–86] graphic shows the large pre–World War II volatility similar to Whelan's post-1970s Irish market.) Table 1 indicates that beginning the Ibbotson series in 1943 (60 years) would give us an annual ERP series with about the same subperiod (30 years) means but equal variances.

Whelan cites studies of monthly return data (not ERPs) that show that U.S. and Irish equity returns are not weakly stationary. But in an equity premium world such as CAPM, one would not expect the total return series to be stationary given the history of wide-ranging nominal and real risk-free rates. That is precisely why the ERP, rather than total returns, is of prime interest. (The modeling of the risk-free rate series has fared no better than that of ERPs, leading to a *risk-free rate puzzle.*) His Figure 1 shows graphically an Irish Capital Market ERP series for 1900–2003 with a changing variance in later years,

Table 1
Equity Risk Premium Variability

Ibbotson Annual Data		
Data Period	Mean	Variance
1926–1959	0.1182	0.0600
1960–2002	0.0527	0.0250
1926–2002	0.0817	0.0410
1943–1972	0.1186	0.0279
1973–2002	0.0527	0.0292
1943–2002	0.0856	0.0292

ruling out weak stationarity but not stationarity in the mean, the object of our paper. Indeed, Finnerty and Leistikow (1993) tested for trend and mean reversion in the Ibbotson ERP series and concluded that the ERP series trended downward over time and, therefore, was not stationary in the mean. That conclusion was later refuted by Ibbotson and Lummer (1994) with the original authors "admit[ing] that the alleged decline is not statistically significant" in a reply (Finnerty and Leistikow 1994). The working assumption of stationarity in the mean was reasserted by Ibbotson and Lummer (1994, p. 99) and continues in the current Ibbotson Associates yearbook (2004, Valuation Edition, p. 75). Additionally, long-run annual ERPs are more valuable to actuaries, as opposed to investment traders, precisely because the annual results smooth the monthly and daily results. Actuarial models of equity and other returns have been built for "scenarios" to be used in sensitivity testing for solvency, pricing, and other actuarial problems. They also adopt practical ERP assumptions, similar in rigor to our stationarity of the mean, to reach meaningful models for practical use such as testing the reasonability of assumptions about the future (see, e.g., Wilkie 1995; Ahlgrim et al. 2003).

ERP STUDIES

Our review of ERP estimates covered a wide variety of techniques (see Derrig and Orr 2004, Appendix B, for a listing of 25 studies and their methodologies reviewed), most of which were concerned with forecasting a long-run average ERP for some future period as long as 75 years. The majority of the puzzle research studies employed analyses about dividend or earnings series relative to price over different time periods to support various theories about the relation of the future market ERP to the past. Other studies reported surveys of academics and educated guesses by professionals and managers, many concentrating on the next 10 years (2000–2009) and not the long-run average. Whelan asserts that nonstationarity of the U.S. ERP series is "casting doubt on many of the approaches used to forecast it surveyed in the paper." But stationarity was only an implicit consideration for these analyses of the so-called market fundamentals to reach conclusions that the ex ante ERP is *not* equal to the historical average, implying nonstationarity (see Derrig and Orr 2004, Appendix B ERP Estimates, and Appendix C, ERP Estimates adjusted to a common definition). Like many of these studies, Whalen makes a verbal argument about the equity risk. He asserts that risk is measured by the "high volatility of excess returns" and, hence, higher ERP should correspond with higher volatility as in the U.S. depression or the post-1970s Irish data and nonstationarity follows. Absent a theory of ground-up overall returns, this statement is only a plausible working assumption like many of the others. It would be helpful to show whether it conforms to the data. It would support, however, stationarity in the mean for the 1943–2002 series in Table 1.

THE DERRIG-ORR RECOMMENDATION

We recommend for a best estimation methodology for an ex ante ERP the Ibbotson-Chen six building-block methods as described in the paper and laid out in detail in Appendix D. These methods are related to stationarity but do not depend upon it. They specifically allow for the importation of changes in the historical means of the building blocks, such as inflation, growth in earnings, and reinvestment rates, as those changes can be supported or as the judgment of the practicing actuary wills it. Finally, a simple examination of the ERP numerical series for stationarity or any other property would be a misreading of the message of our paper. Rather, one must go beyond the simple numerical values, as those who created the ERP puzzle did, to attempt to understand the process generating the values, including the behavior of investors.

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"Further Analysis of Future Canadian Health Care Costs," Robert L. Brown and Uma Suresh, April 2004

BEDA CHAN*

THREE-DIMENSIONAL GRADUATION IN POPULATION PROJECTIONS

This excellent paper treats the important topic of health care cost containment in this half century of demographic unfolding in many countries, using Canada as an example.

I view the Lubitz-Scitovsky survivor/decedent costs-split projection (Projection II) as the principal projection. The Denton and Spencer age/sex-specific costs-times-population-pyramid projection (Projection I) is the high variant, since publicly funded health care, when supported, is likely to be utilized as much as it is supported. The advance directive projection (Projection III) is the low variant. The paper by Brown and Suresh, summarized in their Figure 4, is thus a study of a high-principal–low-cost-control environment under a single population projection, which is the best estimate projection for 2001–75 in OSFI's eighteenth actuarial report of 2001. To study the relative importance of population aging scenarios (High Dependency, Best Estimate, Low Dependency of OSFI) versus cost control measures (DS, LS, AD of Brown and Suresh), one can study the 3×3 projections of the high-principal–low-cost-control environment crossed with the High-Dependency–Best-Estimate–Low-Dependency population scenarios.

As variability of health care cost is piggybacked on the variability of population projections, studies on variability of population projections are pertinent to health care cost analyses. In this discussion I would like to offer another high–best estimate–low-population projection over 75 years, in a case where official statistics do not provide high-best-low variants. The case in point is the Hong Kong Special Administrative Region. After 1997 Hong Kong (C&SD 2000, 2002, 2004) caught up with the United Kingdom (ONS 2004, and back every second year for about 40 years) in issuing biannual population projections. The Hong Kong projections, however, do not provide variants in fertility, migration, and mortality and are single-scenario best estimates down 30 years of projections. To keep the discussion brief, I summarize my points in Figure 1.

When the set encompassing high-dependency variant, best estimate, and low-dependency variant is not given in a projection, the later revisions of the projection can be used to construct a high-principal–low-projection band. One can say that based on the June 2004 population projection, 34% of the Hong Kong population will be over age 65 by the year 2063. The high variant would be 39% (mirror image), and the low variant would be 29% (extension of the May 2002 projection). I trusted and used the 2002 projection because it was based on the 2001 full census. The 2000 projection used a high total fertility assumption (1.6 by year 2029) when the observed value was 1.024 for the calendar year 2000. It has since been declining, reaching 0.941 by calendar year 2003. Some technical details are pertinent. The C&SD projections are 30-year projections, and I extended their projections to 75 years by the component method. Since C&SD's fertility, migration, and mortality assumptions stabilized after 15 years, using their last 15 years in five-year intervals allows for graduation and extrapolation

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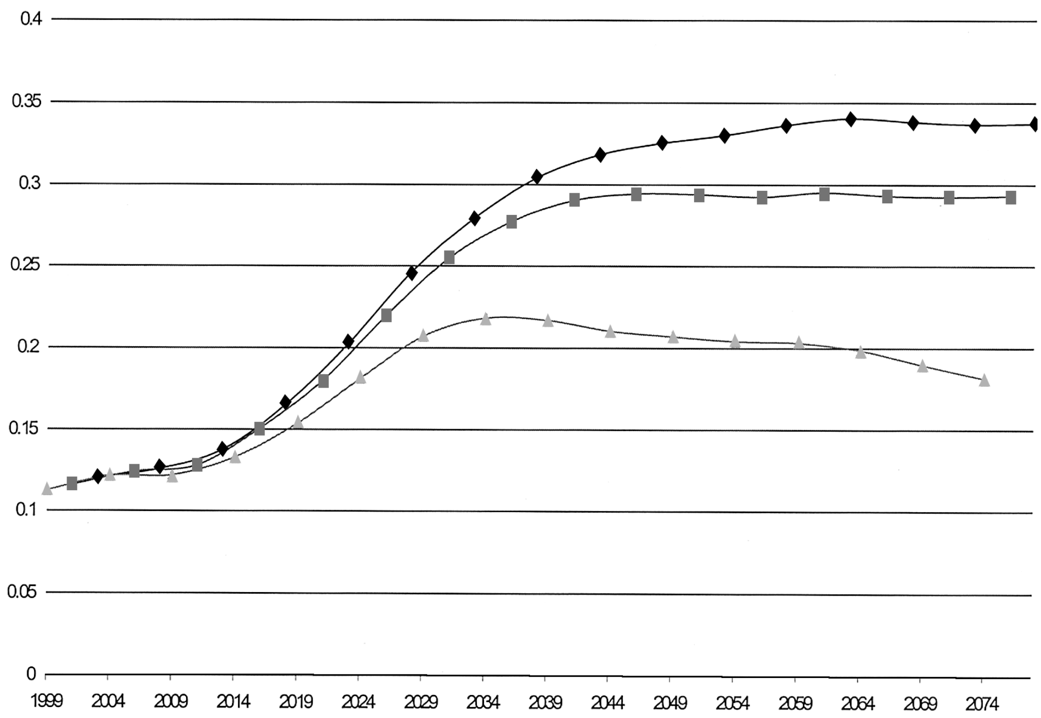
Figure 1
Fraction of population over age 65



to a total of 75 years. C&SD's 2002 and 2004 projections give population figures excluding foreign domestic help every five years but not the 2000 projection. My plot uses population figures excluding foreign domestic help. The 2000 (1999–2074) without domestic help figures are prepared with the 2002 (2001–76) domestic help estimates being graduated and extrapolated back two years. Population figures excluding foreign domestic help are used because domestic help are in the region between ages 20 and 60 and contribute little to health care costs.

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"A Note on the Myers and Read Capital Allocation Formula" Stephen J. Mildenhall, April 2004

HANS U. GERBER*

The purpose of this discussion is to point out some credit to my compatriot Leonhard Euler (1707–1783). In fact, the two Technical Lemmas in the Appendix of the paper are special cases of Euler's Homogeneous Function Theorem, which is available at http://mathworld.wolfram.com/EulersHomogeneousFunctionTheorem.html, as well as in several advanced calculus texts. Furthermore, note the usefulness of Euler's formula, for homogeneous functions, is explained by Cecil J. Nesbitt and Donald A. Jones, in their discussion of John C. Fraser (1962), and by Graham R. McDonald, in his discussion of Raymond L. Whaley (1974).

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"Disruption of a Managed Competition Environment by Low-Ball Premium Bids: The Minnesota State Employees Group Insurance Program," Harry Sutton, Roger Feldman, and Bryan Dowd, April 2004

TIMOTHY M. ROSS*

Market characteristics will affect the degree of relevance to other markets. First, the Minnesota health plan market is a highly concentrated, near-oligopoly. Second, Minnesota health plans are statutorily non-profit, with relatively narrow limits on accumulated surplus. Third, the state health plan constitutes a significant share of the commercially insured market. Readers may find it useful if the authors could provide the market share of the state health plan.

It would be useful to solicit comments from the plan actuary.

The presumed goal of the managed competition was to control state health plan costs. A comparison of aggregated plan costs, with adjustment for plan design, to state or national trends over the same time period would be useful in understanding the effect of managed competition and the disruption.

On a broader note, the authors use "age/gender" factors taken from Milliman USA. This highlights the absence of SOA-sponsored experience studies providing demographic factors in the published literature. The SOA and the Health Section are to be commended for the research sponsored and published in recent years, but much work remains to be done.

As an editorial note, the word "gender" carries a cultural connotation, while "sex" refers to the purely biological (The American Heritage Dictionary 1997). Without enrollment data as to "gender," the actuary can only develop and use "age/sex" factors.

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