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DEFINING AND MEASURING INVESTMENT RISK IN DEFINED BENEFIT PENSION FUNDS

By S. F. Whelan

ABSTRACT

A formal definition of investment risk in actuarial investigations is given. Case studies estimating the investment risk associated with different investment strategies for defined benefit pension funds using historic market data are presented. It is shown that a few decades ago, when bond markets only extended in depth to 20-year maturities, the investment risk of investing in equities was of the same order of magnitude as the investment risk introduced by the duration mismatch from investing in bonds for immature schemes. It is shown that now, with the extension of the term of bond markets and introduction of strippable bonds, the least risk portfolio for the same pension liability is a bond portfolio of suitable duration. It is argued that investment risk voluntarily undertaken in defined benefit pension plans has grown markedly in recent decades, at a time when the ability to bear the investment risk has diminished. Investment risk in pension funds is quite different to investment risk for other investors, which leads to the possibility that current portfolios are not optimised — that is, there exist portfolios which increase the expected surplus without increasing risk. The formalising of our intuitive concept of investment risk in actuarial applications is a first step in the identification of more efficient portfolios.

KEYWORDS

Investment Risk; Defined Benefit Pension Funds; Investment Strategies; Risk in Actuarial Applications

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1. Introduction

Exley *et al.* (1997) assume that the markets are complete (i.e. there exists a portfolio of assets which perfectly matches the liabilities) or, equivalently, there exists a zero risk investment strategy for defined benefit pension funds. In incomplete markets, there is a need to define the concept of investment risk so it can be measured. In this paper, a definition of investment risk in actuarial investigations is proposed. It is applied to give an empirical measure of the investment risk of different investment strategies for defined benefit pension funds.

The formalisation of our intuitive concept of investment risk in pension

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funds allows us to identify two developments which were significant in heightening voluntary investment risk in defined benefit pension funds over the last few decades:

(1) The defined benefit pension has evolved from an incomplete contract (i.e., benefits largely discretionary) to a more complete contract, as regulation has established certain minimum entitlements for early leavers and minimum benefits to be paid on scheme termination. Further, regulation has demanded a level of asset-backing to the pension promise, with standards of pre-funding now in place.

(2) Simultaneously, and especially in the last decade or so, the capital markets have introduced new securities which allow pension guarantees to be better hedged. Key developments have been the issue of inflation-linked bonds in many currencies and the extension of the maturities of conventional bonds from 20 years out to half a century and more.

The first of these developments, by more explicitly defining the pension contract, allows us to better define investment risk. Before this clarification, it could reasonably be argued that the guaranteed part of the pension promise had so little value in relation to the accumulated fund that it posed no practical constraint to investing. The second development, elaborated on in this paper, allows us to measure, on a market consistent basis, the investment risk involved in pursuing any given investment strategy, as there are now traded assets matching or near-matching the defined liabilities.

This fundamental idea of defining investment risk for pension funds presented here is based on ideas first presented in Arthur & Randall (1989), which was developed in a more technical setting in Whelan (2004).

We report, from case studies, that the ranking of the main asset classes by investment risk is similar across time periods, is similar across national markets studied, and is similar in each of the two types of actuarial investigations studied. We conclude, in agreement with Exley *et al.* (1997), that a duration-matching bond-based investment strategy minimises investment risk for defined benefit schemes. However, for immature defined benefit pension funds, it is shown that an equity-based strategy historically produced a risk no greater than for bonds with a term to maturity of 20 years.

The paper concludes with a brief discussion on what these findings imply for past and current investment strategies of defined benefit schemes and the on-going controversy within the actuarial profession subsequent to Exley *et al.* (1997).

2. Historical Evolution of Asset Allocation of Defined Benefit Pension Funds

Prior to the 1950s, pension funds invested proportionately little in

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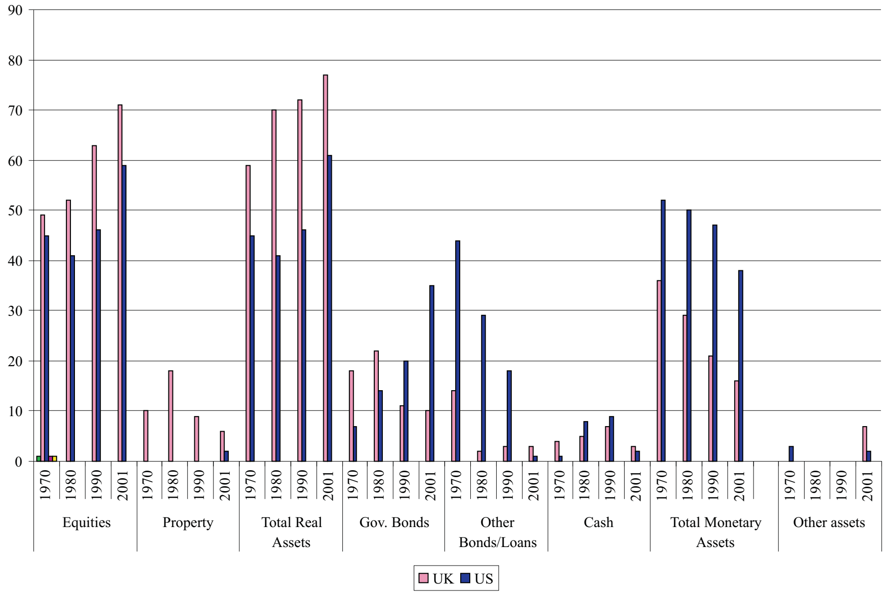
equities, preferring debt securities. The British actuary Ross Goobey brought in the 'cult of the equity' by highlighting the case for equities, and, materially, switching the holdings of the Imperial Tobacco pension fund to equities in the 1950s (see Sutcliffe (2005) for an excellent overview).

Figure 1 highlights the evolution of the average asset allocation of pension funds in the United Kingdom and the United States of America from 1970 to 2001. The continued trend over the period towards real assets, especially equities, in these two economies is also discernible in all countries where the 'prudent man principle' underpins investment policy (see Whelan, 2003).

The declining proportion invested in bonds and monetary assets generally over this period was in the context of arguments advocating that a 100% bond investment strategy is optimal for defined benefit pension funds, made by, amongst others, Black (1980), Tepper (1981), from adapting an argument originally made by Modigliani & Miller (1958). We attempt to provide an explanation for the high equity proportion of pension fund assets in our conclusion.



Source: See Whelan (2003)

Figure 1. Evolution of average asset allocation (%) of pension funds in the U.K. and the U.S.A., 1970 to 2001

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3. Definition of Investment Risk in Pension Funds

What is meant by 'investment risk' in actuarial investigations? Our intuitive concept is that it captures the impact on the results of the valuation when the actual investment experience differs from that expected in the valuation. So defined, the concept applies, no matter what method of valuation is used — discounted cashflow, market-based approach, or other. Significantly, the measure of investment risk depends of the valuation method and the valuation basis. We formalise our intuitive concept of investment risk in the following couple of definitions:

— *Definition of investment variation (up to time p).* This is the financial impact at time zero created when the actual investment experience up to time p differs from the investment assumptions underlying the valuation at time zero, all other things being equal.

— *Definition of investment risk (up to time p).* This is a measure of the spread of the (*ex ante*) investment variation distribution. For concreteness, we shall use the standard deviation as our measure of investment risk in the sequel.

In order to estimate investment variation and investment risk, we need one more principle: the principle of *consistency* in valuations. This is no more than the no-arbitrage principle of financial economics:

— *Definition of consistency in valuations.* A valuation method is said to be consistent if the present value of a cashflow of a given amount at time t is the same up to a change in sign, whether the cashflow is positive (an asset) or negative (a liability).

Note, in the above definition, it is assumed that the cashflows are dependent on the same contingencies, i.e. they have the same embedded risks. We limit our treatment to consistent valuations.

So, investment variation affects both sides of the actuarial balance sheet in general — the present value of both assets and liabilities — as some assumptions used to value the assets can also affect the value of the liabilities. If we have perfect matching of assets to liabilities, then any consistent valuation method will always report the investment variation to be a degenerate distribution (i.e. a constant), and, accordingly, the investment risk to be zero. Equally, if the actuary has perfect foresight, then the investment assumptions would be perfectly in line with the future investment experience, and so the investment variation distribution would again be degenerate. More uncertainty about the investment variation implies a greater spread of the distribution, which corresponds to a greater investment risk under the above definition.

Let us assume that: (1) assets are to be valued at market value; (2) there exist assets which perfectly match the liabilities; and (3) the valuation method

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is consistent. Then it can be concluded directly from the definition that the investment variation is positive only if the actual return on assets exceeds the return on the matching asset (up to time p). Equivalently, the return on the matching asset is the hurdle rate for the return on the assets to show positive investment variation. As Arthur & Randall (1989, Section 2.5) put it: "if you are mismatched and you get your forecasts wrong then you have to pay the penalty." More generally, it can easily be shown that the investment variation is the present value of the increase in the surplus over the period, discounted at the rate of interest equal to the return on the market value of the matching asset. (A proof, if needed, is presented in Whelan, 2004.)

4. Case Studies

Estimating investment risk in actuarial investigations is thus identified with estimating the standard deviation of the (*ex ante*) investment variation distribution. Let us assume that the *ex post* investment variation is a reasonable proxy for the *ex ante* investment variation, i.e. make the commonplace assumption that historical experience can be used to assess *ex ante* expectations.

The rest of the paper presents two case studies designed to explore the relative investment risk of different investment strategies for defined benefit pension schemes.

The first case study assumes that the actuarial investigation is to determine surplus on a notional wind-up. The second case study assumes that the scheme is on-going. In both cases, we assume that the liabilities of an immature pension scheme are adequately represented by the liability to a 40-year-old due to retire at age 65 years, from which time a pension is payable for 20 years. The actuarial valuations are assumed to be market-consistent, so that the assets are taken at market value and the valuation rate of interest for the liability cashflows is set equal to the market yield on government bonds of the same duration. Without loss of generality, the value of the assets is assumed equal to the value of the liabilities at time zero.

Summary descriptive statistics of the investment variation distribution associated with investing in various asset classes are computed in the following manner. Investment over the year subsequent to the valuation is assumed to be in, alternatively: a broad equity index; a 20-year conventional bond; a 30-year zero-coupon bond; and short-term cash instruments. Each investment strategy for each of the two actuarial investigations generates n data points, where n is number of years in the historic period studied. Each data point gives the present value of the surplus or deficit arising over the year, expressed as a percentage of the market value of assets at time zero (termed the 'standardised investment variation'). From these data, key summary statistics of the empirical investment variation distribution ($p = 1$)

for each investment strategy are tabulated, such as the mean, median, geometric mean, the standard deviation (which equates to the investment risk up to one year), and higher moments.

4.1 *Case Study 1: Measurement of Investment Risk in Pension Funds — Termination Liabilities*

A 40-year-old is promised a non-escalating pension, from age 65, of a fraction of his salary at the time of retirement. The termination liability is the pension amount based on his current salary, to be revalued by the lesser of inflation or 4% in any year, up to vesting at age 65 (as required under current legislation in Ireland). Let us further assume that the person will die on his 85th birthday. Given that we want our valuation method to be market-based and consistent, then we would take the valuation rate of interest equal to the gross redemption yield on the bond closest in cashflow to the liability — in this case, given the restricted maturities on the bond markets and assuming no index-linked bonds, the yield of a 30-year zero-coupon bond is taken, and the annual rate of escalation of the benefit pre-retirement is assumed to be 2½% (this latter assumption is not material, as discussed later). Finally, we assume that, at time zero, the valuation shows that the value of the assets, assessed at market value, is identical to the (discounted) value of the liabilities. We wish to estimate the investment variation when the investment strategy is to invest totally in either: (a) the equity market; (b) a conventional 20-year bond; (c) a zero-coupon (or stripped) bond with a single payment in 30 years; or (d) short-term cash. The period between valuations is taken to be a calendar year (i.e., $p = 1$ in our formal definition).

From our earlier definition and observation, we know that the investment variation is the present value of the extent to which the increase in the value of the assets exceeds the increase in the liabilities over the year, the rate of discount (or inter-valuation rate of interest) being the rate at which the liabilities increase over the year. In the example, the inter-valuation rate of return i_v is given by:

$$i_v = \frac{\dfrac{(1.025)^{65-41}}{(1+i_1)^{65-41}}(1.025)(Pen)\bar{a}_{\overline{20}|}{}^{@i_1}}{\dfrac{(1.025)^{65-40}}{(1+i_0)^{65-40}}(Pen)\bar{a}_{\overline{20}|}{}^{@i_0}} - 1$$

where:

i_j is the valuation rate of interest at time j (that is, the gross redemption yield on the 30-year zero-coupon bond); and

Pen is the pension on termination at time zero, payable from age 65.

The inter-valuation rate of interest can be seen as the hurdle rate of return

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which assets must exceed to show a positive investment variation over the year.

Using the historic statistics of the U.K. capital markets, we investigated over each calendar year in the 20th century the *ex post* investment variation, assuming the assets to be invested in different asset classes.[1] The result shows the (*ex post*) investment variation in each calendar year for each investment strategy, standardised by dividing the investment variation by the value of the liabilities at time zero.

Figures 2a and 2b are dominated by the large positive investment variation posted by many mismatching investment strategies over the 1970s and early 1980s (coincident with the first and second oil shocks, which raised inflation markedly, leading, in turn, to large rises in bond yields). In particular, it shows that 1974, regarded as a bad year for equity investment because of the market crash, was, from the perspective of immature defined benefit schemes, one of the better years, as the rise in long bond yields over the year reduced the present value of the liabilities by a considerably greater amount than equities fell. Figure 2 gives a very different history of the rewards from investing in the different asset classes than one based on real or nominal returns.

The spread of the empirical distribution appears non-stationary in the graph — that is, the spread appears to change with time.[2] The implication of this observation for those attempting to forecast the distribution of the investment variation for each asset class is that it is especially challenging, and past experience is only a loose guide to the future experience (see Whelan, 2005, for further discussion on this point).

Table 1 sets out summary statistics to describe the key features of the empirical investment variation based on historic experience, with figures calculated for the whole 20th century, the second half of the 20th century, and those reflecting the experience since 1970.

We can draw the following conclusions from Table 1:

(1) The 30-year zero-coupon bond is the closest match to the liability (of those tested), as the investment variation distribution for this asset type exhibits the lowest standard deviation. Hence the 30-year zero-coupon bond is close to the hedging portfolio. Equities and long bonds have similar investment risk, while cash is considerably higher.

[1] Returns and yields for the U.K. market were sourced as follows: 20-year gilt yields and returns and also cash returns were sourced from Barclays Capital (2003) for the period after 1962. Prior to 1962, yields at the year end and interest rates during the year were sourced from Mitchell (1988) and the return on a notional 20-year bond and cash calculated as outlined in Whelan (2004). The annual U.K. equity market returns were sourced from Dimson *et al.* (2004), with data for individual years kindly provided by the authors. We assume that the yield on the 30-year zero-coupon bond is the same as the yield on the long bond.

[2] This is not surprising, as there is considerable evidence that returns from capital markets form a non-stationary time series (e.g. Loretan & Phillips, 1994).

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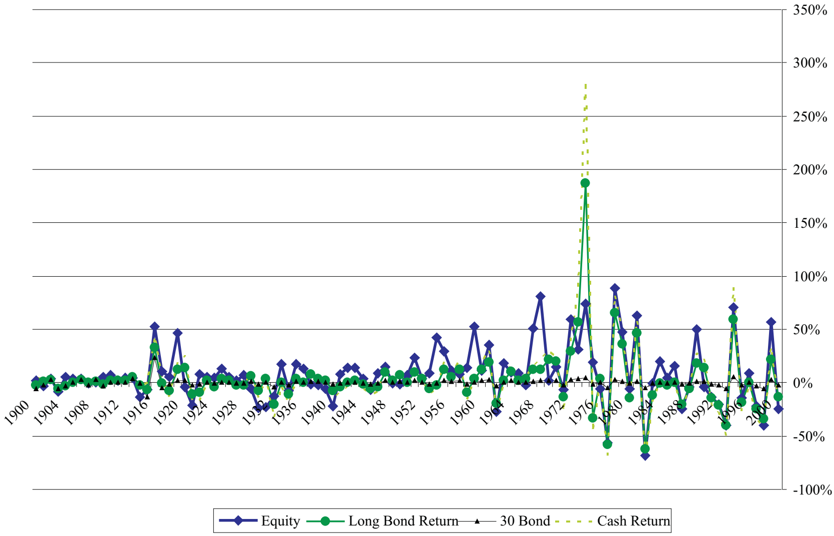

Figure 2a. Standardised investment variation for a 40-year-old for each investment strategy, in each calendar year, U.K. market (case study 1)

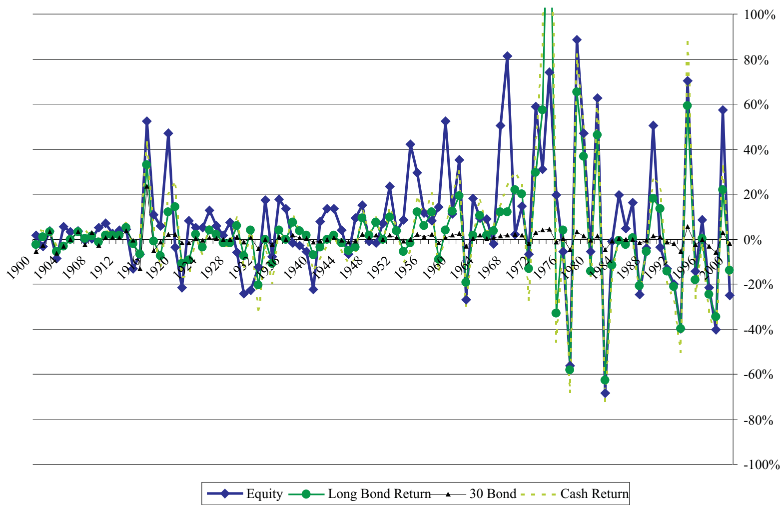

Figure 2b. Standardised investment variation for a 40-year-old for each investment strategy, in each calendar year, U.K. market (case study 1) [rescaled]

Table 1. 40-year-old; summary statistics of the empirical investment variation distribution, U.K. markets in 20th century (case study 1)

	Based on an investment strategy of 100% in:			
	Equity	Long bond	30-year zero-coupon bond	Cash
20th century				
Mean	8.0%	2.9%	0.0%	4.9%
Median	5.0%	0.9%	0.2%	1.5%
Geometric mean	4.6%	0.2%	–0.1%	–0.1%
Stan. dev. [investment risk]	26.7%	26.5%	3.5%	38.2%
Skew	0.6	3.4	2.3	4.0
Excess kurtosis	1.5	23.6	20.0	28.0
Since 1950				
Mean	13.0%	5.5%	0.1%	9.4%
Median	8.8%	1.9%	0.3%	2.0%
Geometric mean	7.2%	0.3%	0.1%	0.1%
Stan. dev. [investment risk]	34.4%	36.5%	2.5%	52.3%
Skew	0.1	2.5	–0.4	2.9
Excess kurtosis	–0.1	11.7	0.3	14.4
Since 1970				
Mean	8.9%	5.5%	–0.3%	10.2%
Median	–1.0%	–2.5%	–0.2%	–1.6%
Geometric mean	1.2%	–2.5%	–0.3%	–4.1%
Stan. dev. [investment risk]	39.6%	46.5%	2.9%	66.4%
Skew	0.2	2.0	0.0	2.4
Excess kurtosis	–0.6	6.9	–0.3	8.8

(2) Note, in particular, that a 20-year conventional bond (which, of course, has a weighted average duration lower than 20 years) is a duration mismatch for the 30-year zero-coupon bond (which has a weighted duration of 30 years), and, on the historic simulation, this term mismatch has introduced as much risk as equity investment.[3]

This result is saying that, if pension funds could invest only in conventional, non-strippable bonds with a term to maturity no longer than 20 years, then the investment risk is almost the same for bonds and for equities.[4] Equities could be seen then as preferable, given their historic outperformance and given that this outperformance was foreseen. This, arguably, has been received actuarial wisdom until challenged by Exley *et al.* (1997), and justified the cult of the equity in pension fund investing.

[3] Note that the returns from the long bond and the 30-year zero-coupon bond are highly correlated, but the variability of the former is much lower than the latter, which leads to the mismatch.

[4] Note that, if the class of portfolios is widened to include portfolios with either short sales or borrowings, then it would have been possible to engineer higher durations.

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(3) While the figures change whether one looks at the 30-year period, the 50-year period or the whole century, the relative ordering of the different asset classes in terms of this new definition of investment risk is largely unaltered. However, the estimated figure for investment risk is very high, and is dependent on the sample period for equities, conventional long bonds and cash. This points to the need for considerable judgement in estimating the future investment risk of the different classes.

(4) One of the assumptions in calculating the figures in Table 1 was that inflation, subject to a cap of 4% over the year following the valuation, was 2½%. The upper limit of possible outcomes is 4%, which, if applied, would deduct about 1½% from the mean, median, and geometric mean figures in the table, and leave all the other figures largely unaffected. This shows that the results of our analysis are not particularly sensitive to estimating this figure, once deflation of any severity is considered unlikely.

(5) The skew of the investment variation for the three conventional asset classes has been non-negative, which ensures that the mean exceeds the median. The geometric mean of the data, which corresponds to the annualised rate over the period, is the more relevant average for actuarial investigations. Table 1 shows that, historically, investing in the most closely matching asset of those studied (the 30-year zero-coupon bond) has involved a material reduction of the geometric mean only when compared to equity investment.

(6) Note that there is no simple relationship between the geometric mean (or other measures of average return) and the standard deviation (or investment risk) of the standardised empirical investment variation distribution. This entails, materially, that there is not necessarily a compensation for assuming extra risk in actuarial applications. Accordingly, actuarial advice can add real value by identifying the idiosyncratic risk of the scheme (that is the deviation with respect to the hedging or least risk portfolio), and exploiting the uniqueness of this risk measure relative to other investors' risk measures, to help to select investment strategies where the associated investment variation distributions have the largest geometric means for any given standard deviation.

Table 2 takes the same liability as Case Study 1, but now gives metrics on the empirical investment variation distribution based on equity, bond, and cash returns and long bond yields in the U.S. and Irish capital markets over the second half of the twentieth century. The figures for the U.K. are included to aid comparison.

Table 2 reinforces the conclusions drawn from Table 1. In short, across the three markets studied: the 30-year zero-coupon bond is the least risky investment of those studied; conventional long bonds and equities exhibit

Table 2. 40-year-old; summary statistics of the empirical investment variation distribution, 1950 to 2000 (inclusive), U.S., U.K. and Irish experiences, case study 1

	Based on an investment strategy of 100% in:			
	Equity	Long bond	30-year zero-coupon bond	Cash
U.S. market				
Mean	13.6%	4.2%	0.3%	6.2%
Median	11.7%	1.7%	0.4%	2.8%
Geometric mean	8.3%	1.5%	0.2%	1.1%
Stan. dev. [investment risk]	34.1%	24.7%	2.5%	33.7%
Skew	0.2	1.2	0.1	0.9
Excess kurtosis	−0.5	4.4	0.9	2.3
U.K. market (from Table 1)				
Mean	13.0%	5.5%	0.1%	9.4%
Median	8.8%	1.9%	0.3%	2.0%
Geometric mean	7.2%	0.3%	0.1%	0.1%
Stan. dev. [investment risk]	34.4%	36.5%	2.5%	52.3%
Skew	0.1	2.5	−0.4	2.9
Excess kurtosis	−0.1	11.7	0.3	14.4
Irish market				
Mean	14.6%	6.1%	0.1%	11.2%
Median	0.6%	4.0%	0.6%	5.1%
Geometric mean	6.7%	0.4%	0.1%	0.3%
Stan. dev. [investment risk]	44.1%	38.6%	2.4%	57.0%
Skew	1.0	2.0	−0.5	2.6
Excess kurtosis	1.3	7.7	−0.1	10.8

investment risk of roughly the same order of magnitude; and cash tends to be higher still. Equities record materially higher geometric means than any of the other asset classes studied. Similar calculations have been done for a 30-year-old member, and, albeit at higher investment risks reported for each investment strategy, the results are similar (see Whelan, 2004). It is only for members aged 50 years and more that the risk of investing in the 20-year bond becomes systematically below that of investing in equities.

4.2 *Case Study 2: Measurement of Investment Risk in Pension Funds — On-going Liabilities*

Case Study 1 treated the termination liabilities on the assumption that the scheme is terminated at the valuation date. This then allowed us to consider the distribution of the investment variation over a year if different investment strategies were used. However, if the scheme remains open, then, under the other assumptions in our case study, the liability will increase by:

(1) the excess of the increase in salary over the increase in pension in deferment;

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(2) the increase in pensionable service; and
(3) other factors capturing how the unfolding experience differs from the other financial and demographic assumptions used to estimate the liabilities.

In practice, of course, almost all schemes will continue, so, arguably, the investment strategy which is best adopted is not the one which best matches the termination liabilities at one instant in the past, but is the one which best matches the increase in the termination liabilities, assuming that the scheme is not wound up.

We investigate each of the investment strategies previously studied under this new scenario. In order to do so, we need to make some further assumptions. We make the following additional assumptions:

(1) The wage increase in any calendar year is 2% above inflation for that year. Thus, the rate of increase of the termination liabilities, assuming that the scheme is not terminated, is (one plus rate of wage increase)/(one plus the lower of 4% or the rate of inflation over the year) times the rate of increase of the termination liabilities, assuming that it is terminated, all other things being equal.
(2) The increasing pensionable service can be allowed for accurately in

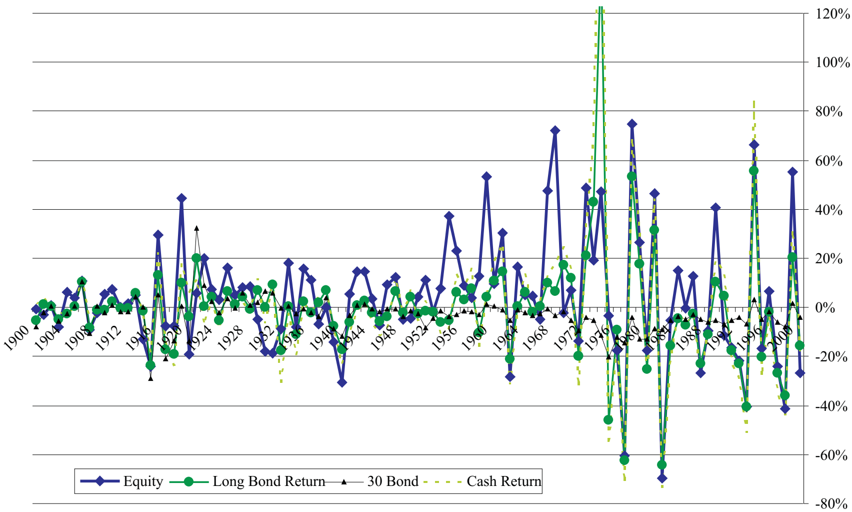

Figure 3. Investment variation for 40-year-old for each investment strategy, in each calendar year, U.K. market (case study 2)

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Table 3. 40-year-old; summary statistics of the empirical investment variation distribution, over 20th century, second half of 20th century, and since 1970, U.K. market, case study 2

	Based on an investment strategy of 100% in:			
	Equity	Long bond	30-year zero-coupon bond	Cash
20th century				
Mean	4.3%	−0.7%	−3.1%	1.0%
Median	4.1%	−0.8%	−2.5%	−0.5%
Geometric mean	1.1%	−3.1%	−3.4%	−3.4%
Stan. dev. [investment risk]	24.7%	23.0%	7.0%	32.9%
Skew	0.4	2.2	0.5	3.2
Excess kurtosis	1.5	15.6	7.7	21.1
Since 1950				
Mean	7.0%	−0.4%	−5.0%	3.1%
Median	5.0%	−1.8%	−4.2%	−0.5%
Geometric mean	1.6%	−4.9%	−5.1%	−5.1%
Stan. dev. [investment risk]	32.1%	31.5%	4.4%	45.2%
Skew	0.1	1.8	−1.1	2.4
Excess kurtosis	0.0	8.0	2.1	10.7
Since 1970				
Mean	1.4%	−1.9%	−6.7%	2.2%
Median	−5.4%	−9.0%	−5.4%	−9.8%
Geometric mean	−5.4%	−8.8%	−6.8%	−10.3%
Stan. dev. [investment risk]	36.0%	40.0%	4.7%	57.2%
Skew	0.3	1.6	−0.7	2.0
Excess kurtosis	−0.4	4.7	1.6	6.6

advance, as it is deterministic. This creates a factor (greater than unity) which multiplies the liability factor on scheme termination. We ignore this factor, as it varies from scheme to scheme, and can be estimated in advance.

(3) The experience of the scheme is in line with that assumed in calculating the termination liabilities in all other matters.

Note the similarity between the approach above and the on-going funding plan known as the 'defined accrued benefit method', described and discussed in McLeish & Stewart (1987).

We can redo the previous analysis with these new assumptions, which we term Case Study 2. The results are summarised in Figure 3 and Table 3.

The 30-year bullet bond is still found, of the strategies assessed above, to entail the least risk, and the ranking of the other asset classes in terms of risk remains the same as the first case study (in fact, the figures for investment risk are of the same order of magnitude as those earlier). The means and other measures of the central location of the distribution of the standardised investment variation are altered significantly (as could be expected), but,

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again, the relative ranking is very similar to that of Case Study 1. Accordingly, a bond-based strategy of suitable duration appears to be the least risky on an on-going as well as on a termination basis.

Further investigations with Irish market data and an explicit Irish wage index over the 20th century are compatible with the figures in Table 3, the key difference being that the risk of equity investment is about one-fifth higher than for the conventional long bond. The higher risk figures on this alternative basis appear to arise because wage inflation lags price inflation in any one year (and sometimes across years, due to, say, wage controls during the Second World War), with wage pressures sometimes released in a large aggregated increment. In short, using 2% above inflation could be regarded as a reasonable proxy for wage pressures, but actual wage increases tend to be somewhat later.

Case study 2 supports the conclusion in Speed *et al.* (2003), viz., the most closely matching asset for pension fund liabilities is composed mainly of conventional and index-linked bonds.[5] It also makes clear that there is generally no simple matching asset for pension fund liabilities, and that some judgement must be used in identifying the closest matching portfolio. We note, in particular, that the above argument leads to a least-risk portfolio which, if history is any guide, has a lower expected long-term return than a predominantly equity portfolio.

5. Conclusion

We defined investment risk in actuarial applications generally, and applied our definition in case studies to give an empirical measure of the investment risk of different investment strategies for defined benefit schemes.

Our case studies show that the equity exposure maintained by pension funds since the 1950s was justified when liabilities were relatively immature and bond markets offered limited duration. In short, the investment risk of investing in equities was of the same order of magnitude as that introduced by the duration mismatch from investing in bonds. With the extension of duration in bond markets in recent times and the innovation of stripping, suitably long bonds now provide the least risk investment strategy, even for immature schemes. Alongside the growing ability to manage investment risk, the capacity to bear risk has been eroded over the last couple of decades, as the guaranteed part of the pension promise (especially as it related to early leavers or benefits payable on scheme termination) rose to reduce the surplus.

[5] Our different arguments hopefully overcome the objections in Hill (2003) to the conclusions in Speed *et al*. (2003).

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Exley *et al.* (1997) sparked a debate within the actuarial profession a decade ago. Initially the debate centred on how to value pension liabilities and invest pension funds, but it has since grown to polarise the actuarial community into advocates of financial economics and advocates of the traditional actuarial approach. To caricature the debate, the 'financial economist' group argues that pension funds should invest primarily in bonds, while 'traditional actuaries' defend the common high equity exposure. The proceedings of a symposium in Vancouver in 2003, 'The Great Controversy: Current Pension Actuarial Practice in Light of Financial Economics Symposium' overviews the issues, with Gordon & Jarvis (2003) giving its history and Day (2003) giving a synopsis of the current state of the debate.

Our analysis does not allow us to suggest that one investment strategy is preferable to another. Investors, including pension funds, can adopt a mismatch investment policy if the payoff is judged sufficiently tempting (that is, the investment variation distribution is attractive). However, pension funds should appreciate the investment risks involved in alternative strategies, and, at a minimum, seek to ensure that the investment portfolio is efficient, in the sense that investment risk cannot be diminished without diminishing reward. To appreciate the risks and ensure that all risks undertaken are reasonably rewarded requires knowledge of the investment variation distribution and, in particular, the least risk or near-hedging portfolio.

It is hoped that a solid platform on which to build a consensus on suitable investment strategies for pension funds can be achieved through formalising our intuitive notion of investment risk in actuarial valuations, as outlined here. In particular, it is shown that pension schemes have an idiosyncratic definition of investment risk relative to other investors, which might be exploited to increase expected surplus without increasing risk. In the past, immature pension schemes exploited this by investing in equities.

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