Journal of the Statistical and Social Inquiry Society of Ireland
Vol. XXXV, pp. 236-246
(read before the Society on 27th April 2006 in Belfast)

Dr Shane Whelan, FFA, Seconding Vote of Thanks to Trends in Mortality Statistics in Northern Ireland, by Dermot O'Reilly

Introduction

It gives me pleasure to second Chris Shaw's Vote of thank to Dr Dermot O'Reilly for such an excellent paper.

Mortality is obviously an important topic – important to the individual, the family, and the community. A measure of its relative significance is estimated in the construction of the Economist Intelligence Unit's Quality of Life Index, which gives life expectancy at birth a weighting of 19.0%, marginally higher than GDP per person (Economist (2005)). The United Nation's Human Development Index gives it an equal weight (at 33%) with material well-being (United Nations Development Programme (2004)). Ireland and the UK currently rank 1st and 29th respectively in the Quality of Life Index (out of 111 nations) and 8th and 15th respectively in the Human Development Index (out of 159 nations). Despite the high ranking of our nations, the performance in these indices would have been improved further if life expectancy in the UK or Ireland was in line with the EU-15 average. Below I set out in tabular form the expectation of life at birth in 2002 in the EU-15 nations, subdividing the UK into England, Northern Ireland, Scotland and Wales.

Table 1: Expectation of Life at Birth, 2002, in EU-15 Countries plus Northern Ireland, England, Wales, and Scotland (Ranked by Person)

	Male	Female	Person
Sweden	77.7	82.1	79.9
Italy	76.8	82.9	79.9
Spain	75.7	83.1	79.4
France	75.6	82.9	79.3
Austria	75.8	81.7	78.8
EU-15	***75.8***	***81.6***	***78.7***
England	*76.2*	*80.7*	*78.5*
Germany	75.6	81.3	78.5
Netherlands	76.0	80.7	78.4
UK	***75.9***	***80.5***	***78.2***
Finland	74.9	81.5	78.2
Luxembourg	74.9	81.5	78.2
Belgium	75.1	81.1	78.1
Greece	75.4	80.7	78.1
Northern Ireland	***75.6***	***80.4***	***78.0***
Wales	*75.6*	*80.2*	*77.9*
Ireland	***75.1***	***80.3***	***77.7***
Denmark	74.8	79.5	77.2
Portugal	73.8	80.5	77.2
Scotland	*73.5*	*78.9*	*76.2*

Sources: CSO (2004), Table 5; Interim Life Tables 2001-03 from Government Actuary's Department Website (UK) at www.gad.gov.uk; www.mortality.org.

Given its importance and the evident scope for improvement, population mortality should be researched and managed as much as the economy. However this is manifestly not the case and so Dr O'Reilly's paper is particularly welcome.

Population mortality data on this island has been only lightly studied since it was continuously collected from 1864. For instance, Northern Ireland's mortality data was only graduated to produce life tables for Life Tables 1926 (using the mortality experience in the years 1925-27), and Life Tables 1950-52. While Ireland has produced a graduated life table coincident with each census (so there are now 14 life tables in all), the method of graduation used is still King's Method – a method devised early in the twentieth century to overcome the then significant problem of age rounding (King (1909)).

Dr O'Reilly's paper must be put in this context. He gives a remarkably comprehensive and accessible overview of the trends in mortality in Northern Ireland by underlying cause of death, making us aware of the limitations of this sub-division. He provides examples of how the statistics have informed policy decisions to target adverse trends by way of information campaigns (e.g., dangers of smoking, the 'back-to-sleep' campaign that dramatically reduced Sudden Infant Death Syndrome) or legislation (e.g., the compulsory wearing of safety-belts in 1983, limits on alcohol consumption before driving). Any one of these informed policy actions surely justifies the cost of producing the statistics.

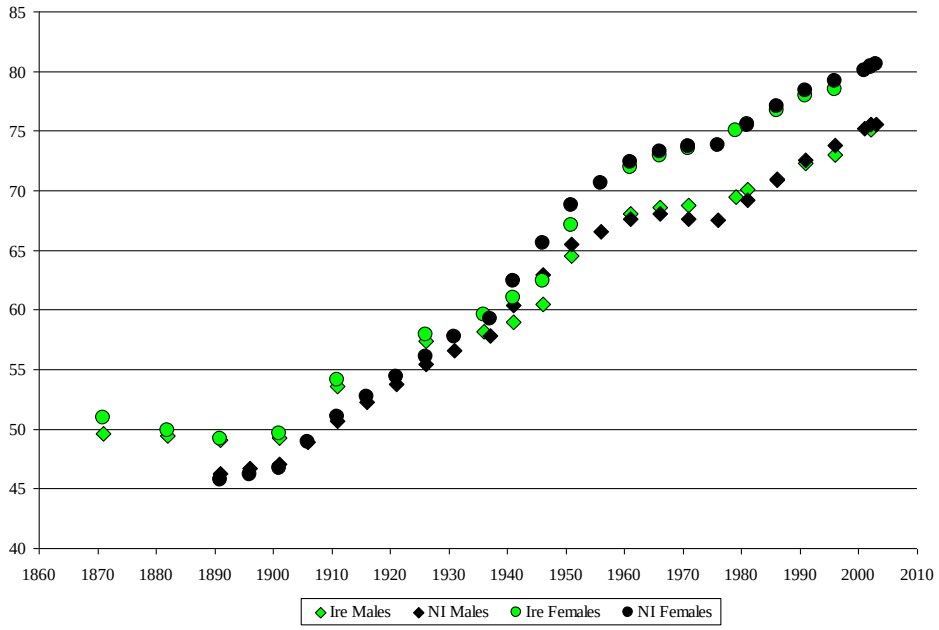
The paper raises a concern regarding the accuracy of cause-of-death classifications. Dr O'Reilly shows that some cause-of-death classifications in the past such as 'old age' contain little information. More alarmingly, he highlights that cause-of-death descriptions might be inaccurate in one-third of cases. Accordingly, a decomposition by cause-of-death might be too fine a division to produce reliable results when comparing mortality rates over time or between one region and another.

Comparative View of Northern Ireland Mortality Statistics

In this section, I outline a complementary method to compare mortality statistics in Northern Ireland over time and with other regions. The method is applied to the overall mortality experience of males in Northern Ireland since 1926 to highlight overall trends and compare the trends with those in England & Wales and Ireland. The method involves modelling the level and shape of the mortality curve by an eight parameter formula.

Graph 1 highlights how the expectation of life at birth in Northern Ireland evolved over the long term past and compares the expectation with that of Ireland (that is, the Republic of Ireland). The graph tells a story of convergence. Expectation of life at birth was determined more by region than gender in the latter part of the nineteenth century and early part of the twentieth century but, from the second half of the twentieth century, gender came to dominate. By the start of the twenty-first century, the expectation of life in Northern Ireland and Ireland were very similar, and both record a gender gap of about 5 years.

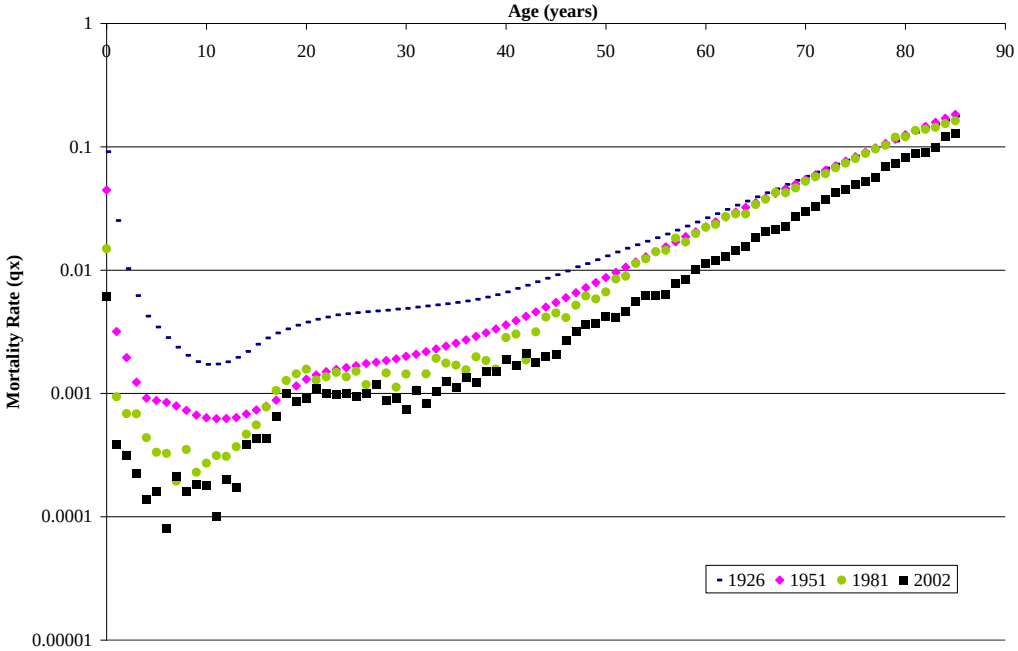
Graph 1: Expectation of Life at Birth, Males & Females, Ireland & Northern Ireland, 1870-2002.



Sources: Data for Northern Ireland (NI) from Table K of the Annual Report 1995 of the Registrar General for Northern Ireland; Data for Ireland (Ire) from Table 3 in CS0 (2004).

Mortality curves – that is, how mortality rates vary with age – have a similar shape in most developed economies. Graph 2 shows the shape of the curve for males in Northern Ireland in 1926, 1951, 1981 and 2002.

Graph 2: Shape of Mortality Curve, Northern Ireland, Males, 1926, 1951, 1981, 2002.



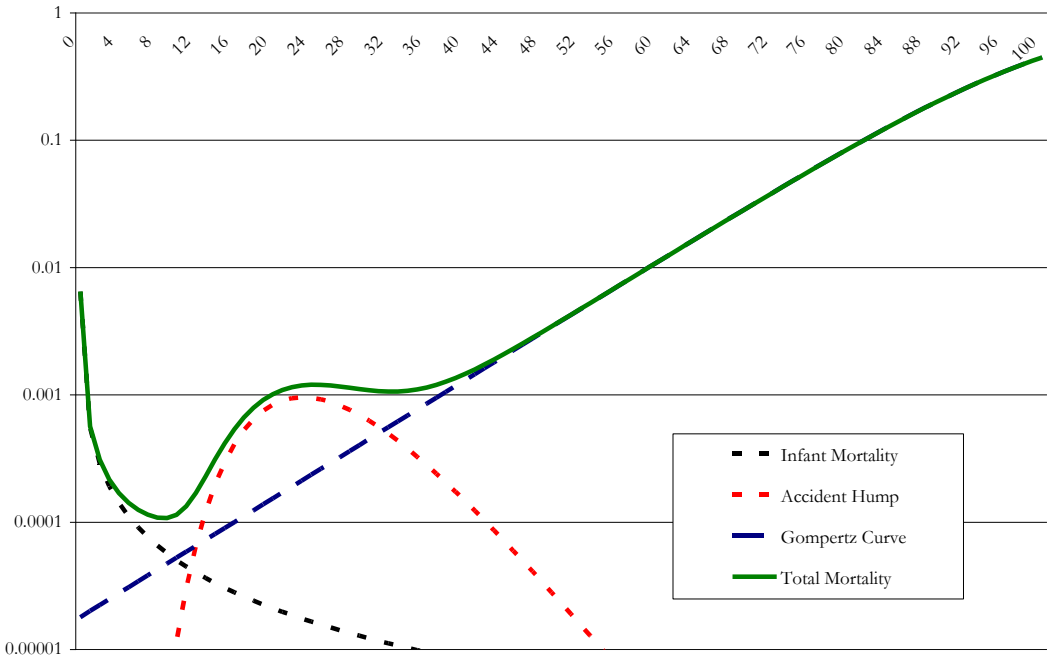
Sources: The Registrar-General's Review of Vital Statistics of Northern Ireland and Life Tables (1926); ditto 1950-52; Interim Life Tables for Northern Ireland 1980-82 and 2001-03 from UK Government Actuary's Department (www.gad.gov.uk).

A law of mortality akin to a law of physics – that is mathematically modelling the aging process – is elusive, despite notable attempts by, *inter alia*, Gompertz (1825), and Makeham (1860) (see, for instance, Olshansky & Carnes (1997) for a review). However, more purely descriptive formulae have been proposed with that of Heligman & Pollard (1980) being one of the more promising. Heligman & Pollard (1980) propose the following formula to model mortality rates from age (x) from 0 to age 100 and over[1]:

$$\frac{q_x}{p_x} = A^{(x+B)^C} + De^{-E[\ln x - \ln F]^2} + GH^x$$

There are three distinct elements of the formula, each modelling mortality rates at a particular stage in life. Graph 3 shows the part of the mortality curve that each element models.

Graph 3: Shape of Mortality Curve, Decomposed into Mortality Rates in Three Stages of Life as Modelled by Heligman & Pollard (1980)



Many parameters of the curve admit of an intuitive interpretation, as described below:

Infant mortality described by the term:

$$q_x \approx \frac{q_x}{p_x} = A^{(x+B)^C}$$, significant when x small

[1] They also propose two nine parameter variations: $q_x = A^{(x+B)^C} + De^{-E[\ln x - \ln F]^2} + GH^x / (1 + KGH^x)$ and $q_x = A^{(x+B)^C} + De^{-E[\ln x - \ln F]^2} + GH^{x^\kappa} / (1 + GH^{x^\kappa})$. We employ only the eight parameter version. See Heligman & Pollard (1980) for further discussion.

where

- A: Almost identical to q_1 (mortality rate at age 1)
- B: Location of q_0 in range q_1 to 0.5 (high value good)
- C: Rate of decline of infant mortality

Young adult accident mortality 'hump' described by the term:

$$q_x \approx \frac{q_x}{p_x} = De^{-E[\ln x - \ln F]^2}$$, for *x* corresponding to young adult ages

where

- D: Magnitude/Severity of mortality hump
- E: Spread of hump
- F: Location of hump (when maximum)

Near geometric progression of rates at later ages modelled by Gompertz-like curve

$$q_x \approx GH^x \Big/ (1+GH^x)$$, for *x* large

where

- G: Level of mortality
- H: Rate of increase of mortality

The above formula has been used to describe concisely the life tables of many countries (e.g., Heligman & Pollard (1980) for Australia; Forfar & Smith (1987) for first 14 English Life Tables covering the period 1841 to 1981; McNown & Rogers (1989) for US Male mortality each year from 1900 to 1985).

We fit the the Heligman & Pollard curve to the life tables for Northern Ireland males for the years 1926, 1951, 1981, 1986, 1991, 1996, and 2002 (sources as per Graph 2, augmented by Interim Life Tables for Northern Ireland 1985-87, 1990-92 and 1995-97 from UK Government Actuary's Department (www.gad.gov.uk)). The eight parameters (A to H) were selected as those parameters that minimise the sum of squares of the relative errors at each age in the range 0 to 85 years inclusive, i.e.,

choosing parameters A to H such that $S^2 = \sum_{x=0}^{x=85} \left(\frac{q_x^{HP}}{q_x^{MR}} - 1 \right)^2$ is minimised

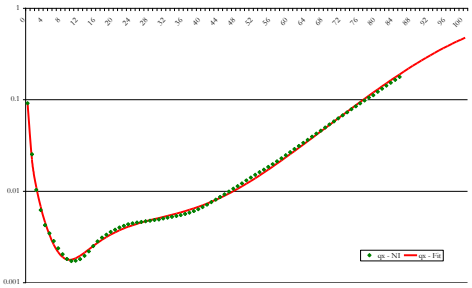
where q_x^{MR} is the mortality rate for that age in the life table and q_x^{HP} is the mortality rate from the Heligman-Pollard formula with parameters specified. In line with other modellers, ages above 85 years have been excluded due to potential problems of reliability at these later ages. The parameters that describe the mortality experience of males in Northern Ireland are given in Table 2.

Table 2: Best Fit Heligman & Pollard Parameters to Mortality Rates for Males in Northern Ireland

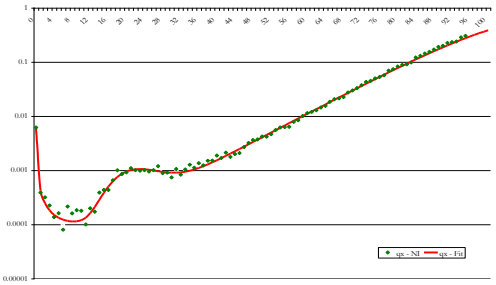
Year	A*10^3	B*10^3	C*10^2	D*10^4	E	F	G*10^5	H	S^2
2002	0.401	19.391	10.906	7.831	8.296	21.942	2.641	1.106	1.823
1996	0.342	9.814	10.022	8.485	8.307	21.641	2.554	1.110	2.364
1991	0.509	10.635	9.702	8.204	8.743	21.095	2.921	1.109	2.088
1986	0.466	0.968	7.501	9.378	11.635	20.746	3.489	1.108	2.226
1981	1.040	12.916	11.333	10.469	9.713	21.001	4.022	1.108	1.881
1951	2.859	5.351	12.370	6.959	7.957	24.256	8.333	1.098	0.079
1926	27.540	212.61	28.627	34.074	1.463	29.944	10.741	1.094	0.263

The fit is reasonable (indicated by a low S^2) when allowance is made for the inevitably increase in S^2 when fit to the ungraduated rates from 1981 onwards. Graphs 4 and 5 illustrate the close adherence of the best fit Heligman & Pollard (H-P) curve to the recorded mortality rates in 1926 and in 2002.

Graph 4: Mortality Rates by Age, Northern Ireland Males, 1926, Graduated Rates and H-P Fit



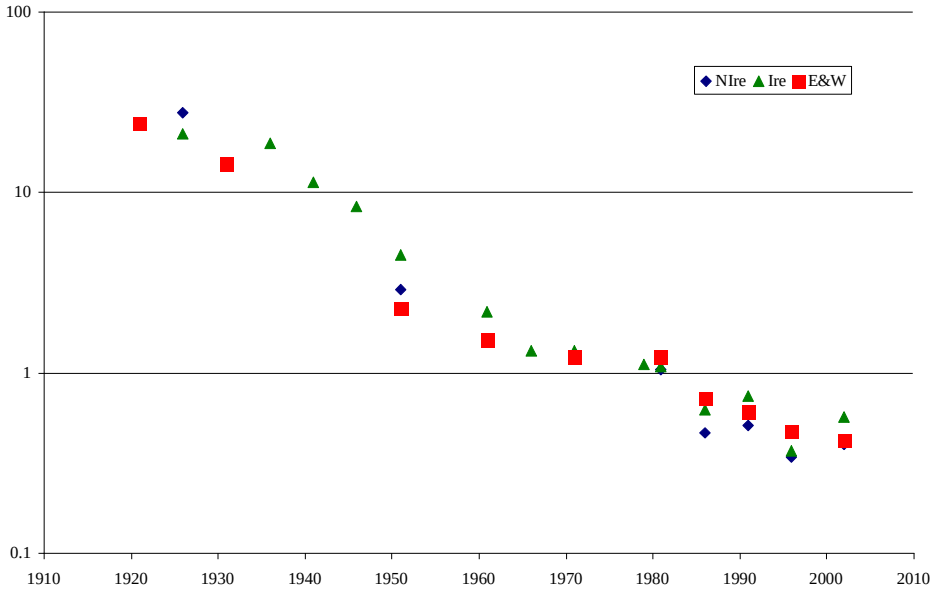
Graph 5: Mortality Rates by Age, Northern Ireland Males, 2002, Ungraduated Rates and H-P Fit



The H-P formula effectively summaries the complete life table with eight parameters, facilitating international and inter-temporal comparisons. We can now compare the mortality trend in Northern Ireland males with males in England & Wales and Ireland over the last eighty years.

Graph 6 shows how parameter A for males in Northern Ireland (corresponding approximately to the mortality rate at age 1 year) has evolved and compares the trend with its trend in England & Wales and Ireland. The graph reinforces the conclusion in Dr O'Reilly paper that infant mortality in all three regions has now essentially converged, with rates in Northern Ireland and Ireland catching up with the lead of England & Wales.

Graph 6: H-P Fit, Evolution of Parameter A: 1921-2002, Northern Ireland, England & Wales, Ireland

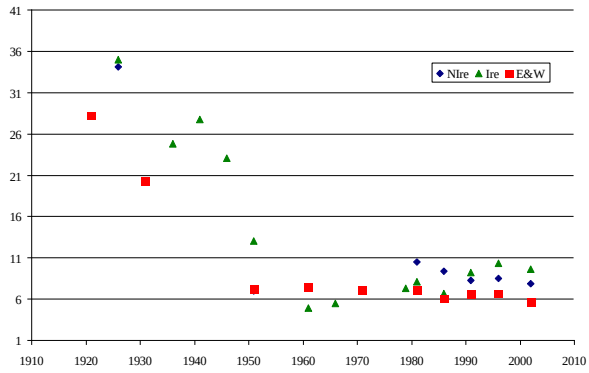


Sources: Forfar & Smith (1987) for England & Wales, 1921-1981, otherwise parameter estimates calculated from interim life tables for England & Wales and from Irish Life Tables 1-14.

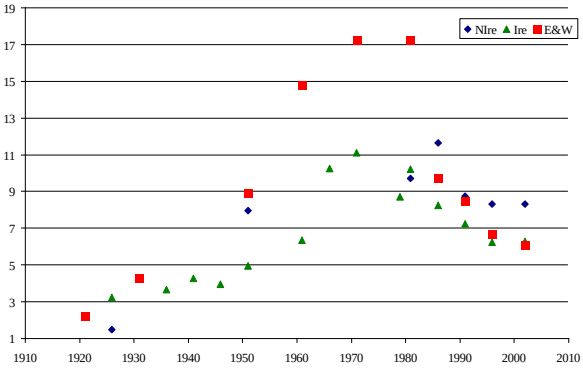
The graphs of parameters B and C add little further insight. The evolution of parameters D to H are shown in Graphs 7 to 11. The broad conclusion is that trends in male mortality in Northern Ireland are very similar to the trends in Ireland and England & Wales over the same period. The relatively small differences in recent years are:

- the accident hump peaks at an earlier age, and is more spread out in Northern Ireland than the other two regions (although the overall intensity of the hump is similar).
- That mortality rates from middle age upwards in Northern Ireland more closely follows that of England & Wales than Ireland.

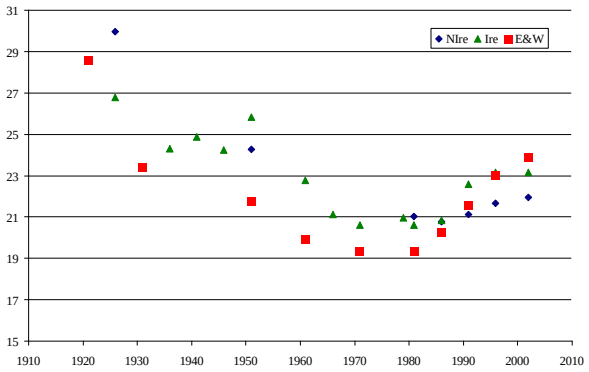
Graph 7: Parameter D: 1921-2002
Northern Ireland, England & Wales, Ireland



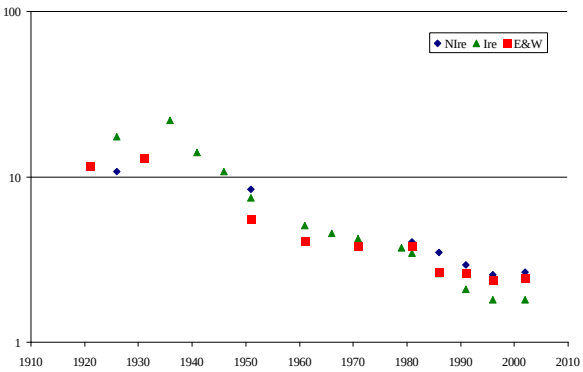
Graph 8: Parameter E: 1921-2002
Northern Ireland, England & Wales, Ireland



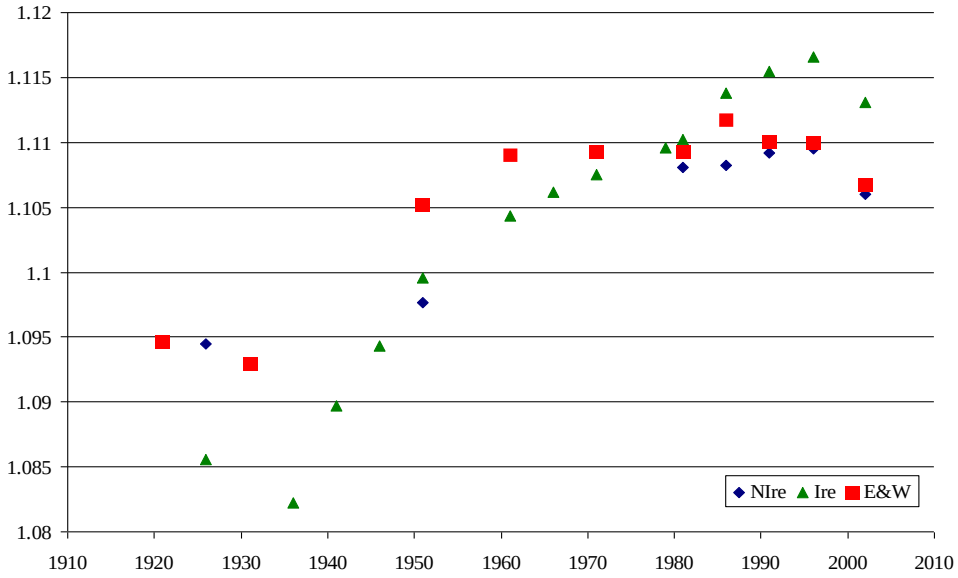
Graph 9: Parameter F: 1921-2002
Northern Ireland, England & Wales, Ireland



Graph 10: Parameter G: 1921-2002
Northern Ireland, England & Wales, Ireland



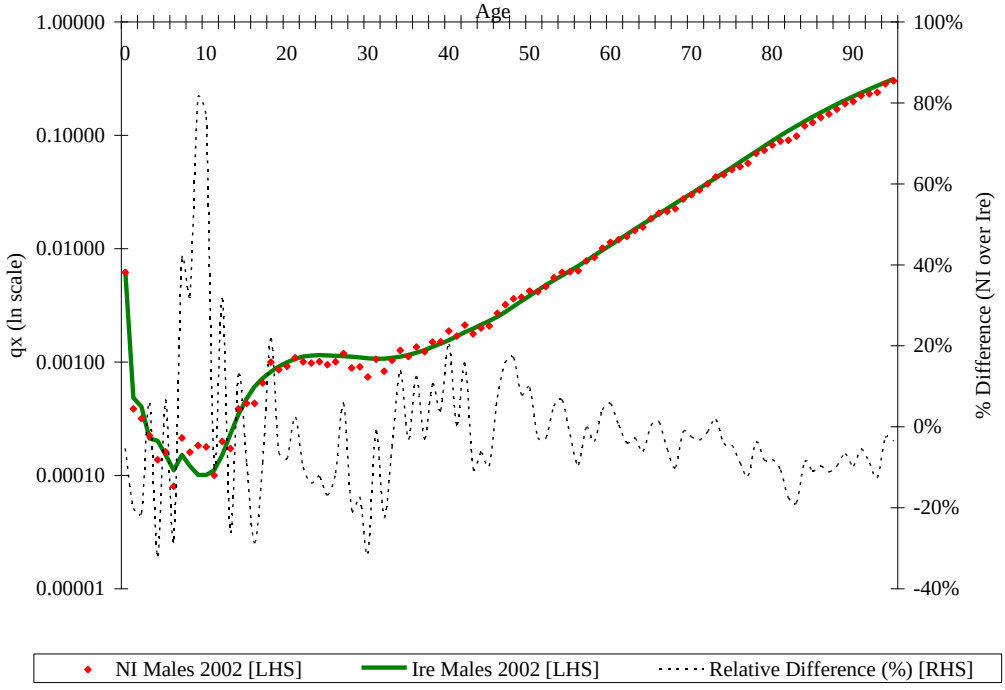
Graph 11: Parameter H: 1921-2002
Northern Ireland, England & Wales, Ireland



Conclusion

The overall conclusion from this complementary and partial analysis of mortality trends in Northern Ireland is that the age-incidence of mortality broadly follows that in Ireland and England & Wales over the last eighty years. In particular, the life table of Northern Ireland males closely resembles that of Ireland in 2002 as shown in Graph 12.

Graph 12: Comparison of Mortality Curves for Males, Northern Ireland and Ireland, 2002



This suggests that similar actions are needed to improve mortality rates in both regions. Co-operation seems sensible, in both research, and policy action. Dr O'Reilly's paper and the insights it brings has broader scope than just Northern Ireland.

References

CSO (2004) *Irish Life Tables No. 14, 2001-2003.* Central Statistic Office, Dublin.

Economist, The (2004) *The World on 2005.* The Economist Newspaper Limited.

Forfar, D.O. & Smith, D.M. (1987) *The Changing Shape of English Life Tables*. Transactions of the Faculty of Actuaries, 40, 98-134.

Gompertz, B. (1825) *On the nature of the function of the law of human mortality and on a new mode of determining the value of life contingencies*. Phil. Transactions of Royal Society, 115, 513-85.

Heligman, L. & Pollard, J.H. (1980) *The Age Pattern of Mortality.* Journal of the Institute of Actuaries, 107, 49-80.

King, G. (1909) *On a New Method of Constructing and of Graduating Mortality and other Tables*. Journal of the Institute of Actuaries, XLIII, 109-184.

Makeham, W.M. (1860) *On the law of mortality*. Journal of the Institute of Actuaries, XIII, 325-58.

McNown, R. & Rogers, A. (1989) *Forecasting mortality: a parameterized time series approach*. Demography, 26, 4, 645-660.

Olshansky, S.J. & Carnes, B.A. (1997) *Ever Since Gompertz*. Demography, 34, 1, 1-15.

United Nations (2005) *Human Development Report 2004: Cultural Liberty in Today's Diverse World.* United Nations Development Programme.