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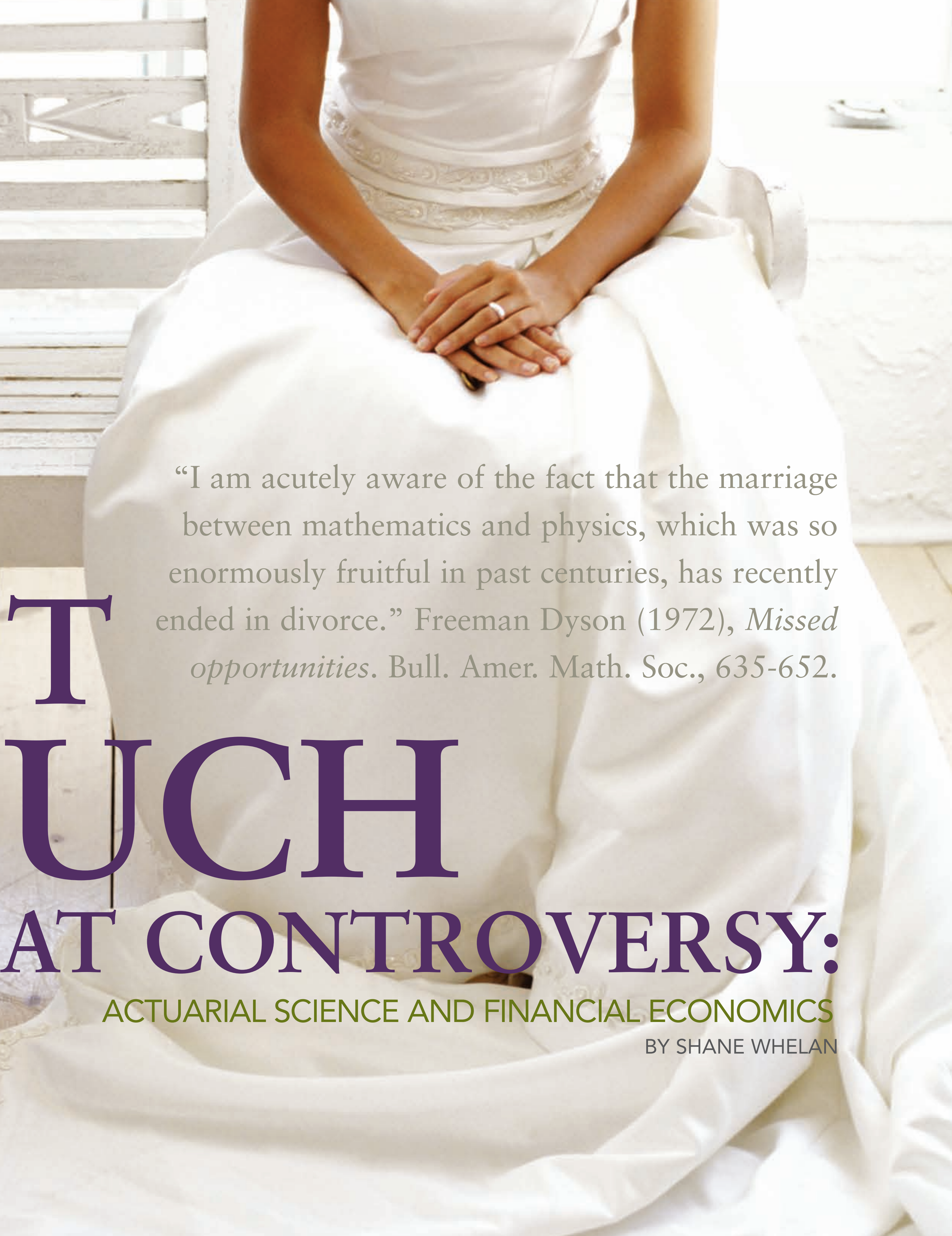

> "I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has recently ended in divorce." Freeman Dyson (1972), *Missed opportunities*. Bull. Amer. Math. Soc., 635-652.

T UCH AT CONTROVERSY:

ACTUARIAL SCIENCE AND FINANCIAL ECONOMICS

BY SHANE WHELAN

Tension

is often observed between the theoretical and the applied branches of a science. While the subject of study is the same, the theoretician and practitioner have differing motivations, take different approaches and judge from different aesthetics. One is deductive and the other inductive. The theoretician seeks simplifying and unifying models while the experimenter or practitioner values models with high fidelity to the underlying data generating process.

This tension has been manifest between financial economists and the financial services industry from the earliest of times. Perhaps it reached its peak when the financial economist Paul Samuelson spelled out what the then dominant orthodoxy amongst theoreticians, the Efficient Market Hypothesis, implied for fund managers' careers: "But a respect for evidence compels me to be inclined toward the hypothesis that most portfolio decision makers should go out of business—take up plumbing, teach Greek. ... Even if this advice to drop dead is good advice, it obviously is not counsel that will be eagerly followed. Few people will commit suicide without a push." (Paul Samuelson (1974), "Challenge to Judgment," *The Journal of Portfolio Management*, (Fall).)

But the passage of a few decades has reduced tensions and a more enlightened cooperation is now observed between finance academics and the fund management industry.

The observed controversy between actuaries and financial economists over the last decade or so is just another incidence of that tension. Financial economists have perhaps been too dismissive of actuarial science, but recent work by Geoffrey Poitras and others is attempting to redress this oversight, arguing that "the techniques in actuarial science have been both mathematically rigorous and supported by careful empirical studies," and that because financial economics has paid so little attention to fixed income analysis and immunization theory that "… the important intellectual and historical connection to actuarial science has been ignored" (Poitras (2006)). Equally, actuaries have not been altogether fair to financial economists. This article briefly traces the different perspective actuarial science has had on the major insights of financial economics. It shows that the key developments in financial economics have not excited our profession as much as could be expected given our common interests and our shared emphasis on mathematical modeling. Actuaries, like other practitioners, value a theory that fits the facts closer than the current theories in financial economics and so have been somewhat dismissive of financial economists. However, as argued here, both the history and philosophy of science suggest that the criticism levied by actuaries is too harsh and their requirements too demanding: theory does not come with ready-to-apply utility so actuaries must learn to adapt and apply the theoretical insights.

The Difficulty of Applying Theoretical Insights

The genesis of financial economics as a separate discipline is conventionally dated from Louis Bachelier's Ph.D. thesis of 1900, some half a century after the actuarial profession was established. Thus it can come as little surprise to learn that 19th century actuaries can claim priority on a number of discoveries in pricing and valuing securities such as the constant growth dividend discount model in pricing equities, the first known British bond valuation table, and even a reasonable rule-of-thumb to price options before Bachelier's seminal thesis.[1]

It is erroneously maintained in economists' circles that Bachelier's work in modeling the evolution of asset prices was lost until rediscovered in the 1950s by Savage and Samuelson.[2] But Bachelier's work was, in fact, quickly disseminated in a manner accessible to actuaries, including in a book, *Théorie et Pratique des Opérations Financières,* first published in 1908 by the French actuary Alfred Barriol, which went into several editions over the following

decades. The second edition of Barriol's book was reviewed in the *Journal of the Institute of Actuaries* in 1914 where attention was drawn to the method of pricing options: "In connection with speculative Stock Exchange transactions, M. Barriol includes an investigation—based on the assumption that the distribution of prices would be in accordance with the normal law of error—of the theoretical relation of the *prime to the écart,* that is, as we understand it, of the price of a call-option to the difference between the call-quotation and the ordinary quotation for next settlement. "... But apart from doubts whether in such a case theory really exercises any influence over practice, and whether variations in prices could be regarded as following even approximately the law of error, it would seem to be difficult, in applying the formula practically, to determine the modulus of the particular curve to be employed for a specified security. At times of active speculation—when options are most in demand—the average deviation in the price of the security for the period covered by the option might be a very unreliable measure of the range of fluctuation."[3]

The above quote makes the point of this article. Actuaries value utility and this demands that the model must faithfully capture the phenomenon modeled. In the above instance, the model failure to capture the observed heteroscedasticity (i.e., changing variance) of the price change series presented a crucial difficulty in "applying the formula practically."

The same critique is made by actuaries over most of the key insights of financial economics since 1900. Phelim Boyle, the academic actuary and financial engineer, gave a lecture recently to the *Society of Actuaries in Ireland* where he highlighted the seven ideas in financial economics. Below I set out the "magnificent seven" ideas he identified and write a brief note on the response of each from the actuarial community.

Boyle's Magnificent Seven Key Ideas in Finance

1. The No-Arbitrage Principle: two identical cashflows must have the same price.

This fundamental principle is the common ground between financial economics and actuarial science. It is so ubiquitous that it has many equivalent formulations—consistency in actuarial valuations, Law of One Price, etc.

2. Capital Structure Irrelevance: the ideal capital structure of a firm (the optimum debt/equity mix) does not exist (under certain assumptions).

This insight is often viewed as being at the heart of the current debate in investing pension funds, so representing a demarcation line between actuarial science and financial economics. Yet many—actuaries and financial economists—agree that the Modigliani-Miller stylized proof that capital structure is 'irrelevant' is, in actual practice, a way of helping to identify those factors that do make it relevant.

3. Mean-Variance Portfolio Selection: if investment reward is taken to be the expected return and investment risk to be the standard deviation of returns, and we know these values together with the correlation between the returns, then portfolio selection is a relatively straightforward optimization problem.

Karl Borch, the academic actuary, made clear that using just the first two moments to define a preference ordering on probability distributions can lead to inconsistencies and mean-variance analysis can only be applied without restriction when asset returns follow the normal distribution (but they do not). These severe limitations allow Borch (1974) to conclude that "... I shall continue to use mean-variance analysis in teaching, but I shall warn students that such analysis must not be taken seriously and applied in practice" (p. 430). Other actuaries have pointed out that the requirement of estimating the expected returns, standard deviations and correlations is so problematic that Markowitz's approach may be no better in practice than naïve ways of constructing a portfolio (see, for instance, Windcliff & Boyle (2004), for a recent development of this argument).

4. The Capital Asset Pricing Model (CAPM): a theory that accounts for an individual asset's ex-ante excess return (over the risk-free rate) as related linearly to the expected excess returns from all risky assets, i.e.,

$$E[R_i]-r_f=\beta_i\left(E[R_m]-r_f\right)$$

The same critique as used against Mean-Variance Portfolio Selection is often advanced: this theory cannot be applied to construct portfolios as the parameters cannot be reliably estimated. Extensive tests have been performed on the CAPM model, estimating the unknown parameters from observed returns, and the general conclusion is that the single beta factor does not adequately explain the cross-sectional variation in stock returns. In fact, valuation measures commonly employed by fund managers such as the price-to-earnings ratio or book-to-market value have been shown to have more predictive ability than a firm's beta (see, for instance, Hawawini & Keim (2000)).

5. Equilibrium: the notion that the expected return, and therefore the expected price evolution, of a risky asset is a function of its relationship with all other risky assets as captured by the Capital Asset Pricing Model or other market pricing models.

The assumptions underlying the equilibrium—that all investors have same expectations and same attitude to risk—are so demanding that they prelude trading and therefore markets.[4]

6. The Black-Scholes-Merton Option Pricing Formula.

The objections to applying this are, again, that the geometric Brownian motion model (or more general diffusion models) of asset prices that underlies the formula does not adequately reflect reality. Tom Collins (1982) produced a detailed assessment of whether the Black-Scholes-Merton approach could be made to work reliably.[5] Collins concluded that that such a hedging strategy "compares unfavorably with the conventional strategy" of reserving and that a "disturbing reason for the poor performance of the immunization [or hedging] strategy was that from time to time (e.g., early in 1975) the unit price was subject to sudden large fluctuations which were inconsistent with the continuous model assumed in deriving it."

7. Portfolio Selection in Continuous Time Presumably the same critique as applied to standard option pricing theory can be used here but, despite the obvious need to cope with dynamic investment conditions in actuarial applications, there is surprisingly little treatment of this topic in the actuarial literature.

History and Philosophy of Science

The actuaries' criticisms above are valid insofar as reality is considerably more complex than captured in the models. But this is always so. Roehner (2002) categorizes scientific models into four orders of complexity, depending on the phenomena modeled. He shows that there has never been an example of a satisfactory scientific model for phenomena as complex as those at the centre of actuarial science.

TABLE 1 ORDERS OF COMPLEXITY IN MODELING

Level 1 - Two body problem Examples of satisfactory models include gravity, light through prism, two heat reservoirs, liver-pancreas, proton-electron.
Level 2 - N-identical body with local interaction Examples of satisfactory models include Maxwell-Boltzmann's thermodynamics and the Ising model of ferromagnetism.
Level 3 - N-identical body with long-range interaction.
Level 4 - N-non-identical body with multi-interactions. Modeling markets. Modeling economic systems generally. General actuarial Modeling.

Adapted from Tables 1.1(a) and (b) in Roehner, B.M., *Patterns of Speculation: A Study in Observational Econophysics,* Cambridge University Press 2002.

Feyerabend, the philosopher of science, reinforces the above observations when he remarked: "To sum up this brief and very incomplete list: wherever we look, whenever we have a little patience and select our evidence in an unprejudiced manner, we find that theories fail adequately to reproduce certain *quantitative results*, and that they are *qualitatively incompetent* to a surprising degree. Science gives us theories of great beauty and sophistication. Modern science has developed mathematical structures which exceed anything that has existed so far in coherence, generality and empirical success. But in order to achieve this miracle all the existing troubles had to be pushed into the *relation* between theory and fact, and had to be concealed, by *ad hoc* hypotheses, *ad hoc*

Insight

approximations and other procedures." Feyerabend (1993), *Against Method*. Verso, London. (Chapter 5).

Conclusion

Theoretical models in actuarial science will never capture the detail of reality. It is unreasonable to ask the theoretician to come up with applicable models—it is the practitioner that must fashion tools. The relevant question for the practitioner is: is the insight useful? And there is a growing body of financial engineers that suggest that financial economics does produce useful insights.

Actuaries have been too dismissive of not just financial economists, but of heroes of actuarial science. Redington aside, the profession has made no or little fuss over the pioneering insights made by actuaries in the past—we are leaving it to historically minded financial economists to unearth and celebrate their achievements. We can speculate that even if an actuary did come up with the most profound theoretical insight, it would have been dismissed by the profession as practically useless. Depressingly, Hein Zimmermann and Wolfgang Hafner argue that this is not speculation. Zimmermann & Hafner (2006) make the case that the forgotten academic actuary Vinzenz Bronzin in his 80-page booklet of 1908, *Theorie der Prämiengeschäfte*, anticipated every modern idea in option pricing—the put-call parity, no-arbitrage arguments, perfect-hedging pricing conditions, risk neutral pricing, and "his equation is closer to the Black-Scholes formula than anything published before Black, Scholes, and Merton. He moreover develops a simplified procedure to find analytical solutions for [European] option prices. …"

The so-called controversy between some actuaries and some financial economists, given our common interests and commitment to the scientific method, is just another instance of Caliban's rage at seeing his own face in a mirror. ■

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For a list of key references, please visit http://misc.soa.org/key-references.htm

Footnotes

[1]See, for instance, Whelan (2002), *Actuaries' Contributions to Financial Economics,* for a brief overview.

[2]See, for instance, Boyle, P. & Boyle, F. (2001) *Derivatives: the Tools that Changed Finance,* Risk Books or Bernstein, P. (1992), *Capital Ideas: The Improbable Origins of Modern Wall Street,* The Free Press.

[3]Anonymous Book Review in Journal of the Institute of Actuaries, XLVIII (1914), 311-312.

[4]As an aside, some contend that the actuary Karl Borch anticipated this insight in Borch (1962), *Equilibrium in Reinsurance Markets,* Econometricia, 30, 424-444.

[5]Or, more accurately, the independent discovery of the basic idea by Colm Fagan presented in a paper to the *Society of Actuaries in Ireland* in 1977. Interestingly, Fagan (1977) sees his ideas as a generalization of the theory of immunization as developed in Redington (1952)—both being dynamic investment strategies designed to keep the market value of the assets and liabilities equal at all times by imposing certain constraints on the assets. Viewed in this way, Redington (1952) could mark the discovery of dynamic hedging strategies.


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