Mortality and Longevity in Ireland

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To Mary Whelan (*née* Sweeney),

(22nd September 1939 – 15th August 2007: 67.90 years).

Rest in peace, Mum.

That the life-form as we have it
is inadequate in itself; but that
having discovered the compensatory devices

of Love and the creative and religious imaginations
we should gather in each generation
all the good we can from the past,

add our own best and,
advancing in our turn
outward into the dark,

leave to those behind us,
with Acts of Hope and Encouragement,
a growing total of Good (adequately recorded),

the Arts and the Sciences,
with their abstractions and techniques
– all of human endeavour –

in a flexible and elaborating
time-resisting fabric
of practical and moral beauty …

Thomas Kinsella
From 'Blood of the Innocent', *Marginal Economy* (2006)

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Foreword

Throughout the Covid-19 pandemic in Ireland, it became a habit for most people to check the daily reports from government sources and the media of the number of cases and the number of deaths. They will likely have considered the current trend in the figures, and may have made comparisons with earlier phases of the pandemic or the experience in other countries. While we were regularly reminded not to consider one day's data in isolation, people will have made their own judgements on the extent to which the situation was improving or deteriorating. There was plenty of data but inevitably it contained some imperfections, and so we tried to understand these and adjust for any that we were aware of. The pandemic has given everyone a better insight into these thought processes.

The Irish mortality data considered in this book of collected academic papers spans many years and predates the pandemic. These are all papers that Shane Whelan, a Fellow of the Society of Actuaries in Ireland, has authored or co-authored and they contain critical historical data and analysis. In the Prologue and Epilogue, Shane has thoughtfully put this analysis into context by also considering mortality trends and the growth in the global human population over a very long timeframe.

Actuaries use mortality projections for a variety of purposes, for example to price products and calculate capital requirements within insurance companies, to calculate funding requirements for pension schemes, to advise on compensation awards within the courts, and to advise on societal matters such as the future cost of healthcare or pensions. The mortality assumptions we use ultimately rely on an analysis of past data and assumptions about the future. Considerations may include known imperfections within the data, the extent to which the underlying population of lives is relevant for the particular projections required, as well as the trends over time and how these are likely to evolve. Sources of information, when considering Irish mortality, will typically include the mortality studies undertaken by the Demography Committee of the Society of Actuaries in Ireland as well as studies by the Institute and Faculty of Actuaries Continuous Mortality Investigation, the Irish Central Statistics Office and the World Health Organisation.

This book will provide actuaries, academics, professionals and policy makers with an insight into the detailed analysis and mathematical modelling that has been carried out on Irish population mortality over recent decades. I commend Shane for his contribution to this work, and his ambition in bringing these papers together so that they can be accessed more widely. This is important research and the Society of Actuaries in Ireland is very pleased to have provided some financial support to help make this happen.

Newer data and analysis will inevitably supplant some of the learning points from these papers over time. I believe that the human brain is wired is to look for patterns, seek coherence and rationalise outcomes. In thinking about future trends we may also be reluctant to let go of established norms, such as mortality differences by gender or country. While the key to making good decisions is good data, I therefore conclude on a word of caution, quoting Daniel Kahneman, "The illusion that we understand the past fosters overconfidence in our ability to predict the future."

Sheelagh Malin
President of the Society of Actuaries in Ireland
Dublin, February 2022

Preface and Acknowledgements

This book is based around a selection of academic papers that I authored or co-authored with PhD students over the last decade and a half. Almost all concern modelling trends in mortality in Ireland and forecasting their future trajectory. These papers would have been submitted to an Irish actuarial journal if one existed. Instead, they have been published in many different journals and the coherent story that collectively they tell is fragmented. This volume is an attempt to make the research readily accessible to the intended readership of actuaries, demographers, and interested others.

The findings of the research have been influential. The first paper, published in 2008 in the *Journal of the Statistical and Social Inquiry Society of Ireland*, outlines a new approach to mortality projections adopted by the Central Statistics Office (CSO) and contrasts it with other projection methodologies. The year previous I was invited to join the CSO Expert Group on Population and Labour Force Projections and the research summarised in this paper informed the new approach. The Demographic Committee of the Society of Actuaries in Ireland also endorsed the new approach in 2008 stating that "the Committee recommends that the rates of mortality improvements assumed for the purpose of the Society's ASPs [Actuarial Standards of Practice] should be the same as the rates of improvement assumed by the CSO in its recently published mortality projections."[1] Accordingly, it was also quickly adopted in actuarial applications and remains favoured to this day.[2]

However, to my mind, there remained three issues with the mortality projections that needed to be addressed with further research. It is only now, more than a decade later, that I am satisfied that all three issues have been satisfactorily resolved. Chapters 2–5 reproduce papers that deal with these issues.

First, should a cohort effect be incorporated into the mortality projection model? The official national projections and many actuarial projections models in the UK already did make such allowance, but the extension was seldom used elsewhere. The paper reproduced in Chapter 1 is ambivalent on the issue: it suggests that there is some evidence of a cohort effect in historic Irish mortality data but does not model it explicitly in the future. I elaborated on this issue in 2009 in an editorial to a special supplement on mortality modelling in the *British Actuarial Journal*. An edited extract from this is given in Chapter 2.

Second, previous estimates of mortality rates at older ages in Ireland (from about age 80 years onwards) are not fully credible and, therefore, nor were the estimated trends in mortality at these ages. I published a pair of papers in the *Annals of Actuarial Science* dealing with the data issues and proposed a parsimonious model that graduates the crude rates at advanced ages. This gives the best estimate of actual mortality rates at the later ages in Ireland and their trends over the last half-century. This approach was incorporated into subsequent official mortality projections in 2013 and 2018[3] but, unfortunately, it has not yet been adopted to graduate Irish Life Tables. These papers form Chapter 3 and Chapter 4.

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[1] Society of Actuaries in Ireland (2008), *Review of rates of mortality improvement*. Demography Committee Report (Chair: Armstrong, J.), 14th October 2008. Quote is from p.14.

[2] Demography Committee of the Society of Actuaries in Ireland (2020), *Review of Best Estimate Mortality Projection Methods*. September 2020.

[3] For a full treatment see Hall, M. (2013), *Analysis of Irish Mortality for Actuarial Applications*. Unpublished PhD Thesis, UCD School of Mathematical Sciences, and Naqvi, R. (2020), *Modelling the Pattern of Mortality Decline in Ireland and the United Kingdom*. Unpublished PhD Thesis, UCD School of Mathematics and Statistics.

Third, the mortality projection model is deterministic not stochastic, so no measure of uncertainty around the best estimate projection is given. Such prediction intervals would clearly help, especially in actuarial applications where more prudent estimates are required in certain circumstances. Rabia Naqvi and I consider the most recent official projections made by the CSO in 2018 and give measures of uncertainty about them using several approaches in a paper published by the Statistical and Social Inquiry Society of Ireland in 2019. The paper also gives estimates of cohort life expectancies. This paper is reproduced as Chapter 5.

Chapter 6, though short, outlines a simple and elegant way to turn the prediction bounds of a stochastic model into scenarios within the deterministic model used by the CSO. This allows us to create high and low estimates of future mortality at any explicit probability level. The chapter illustrates how this can be done with the UN stochastic model for forecasting life expectancies in Ireland. We then use the results to estimate prediction intervals around the projected cohort life expectancy for a new-born in Ireland during the 21st century. Chapter 6 reproduces the original paper submitted to the *Irish Medical Journal*. It is of particular interest to actuaries in this original form but, for a medical audience, it was necessary to shorten the paper for publication, with an emphasis on the results rather than the methodology. The paper co-authored with Rabia Naqvi was published in June 2020.

The final two chapters consider applications of the model to estimate compensation for future loss due to wrongful injury. It shows how to put a monetary value on human life. In particular, we consider injury to the new-born due to negligence in the delivery of maternity services. I began my actuarial career doing court work under the tutelage of Brian Reddin and have since maintained an interest in this specialism of actuarial practice. Chapter 7 sets out the principles behind the calculation of damages and is followed by a short Chapter 8 which highlights that claims currently settling for wrongful injury at birth are now greater than the day-to-day cost of running maternity services in Ireland. Quite apart from our parental duty, it is now cost effective to spend more to ensure the safe delivery of the next generation. Chapter 7 is co-authored with Maeve Hally and was originally published in the *Economic and Social Review* in autumn 2020.

Chapter 8 reproduces a paper published in the *Irish Medical Journal* in early 2021, co-authored with Maeve and Caoimhe Gaughan.

I have heroically resisted the temptation to edit and change the original papers other than in four minor respects. This leads to some repetition, particularly when setting the context at the beginning of each chapter, but this is no bad thing for a book designed to be read one chapter at a time over several sittings. All the original papers were published in colour, so some graphs and diagrams had to be redone to render their information accessible in the format of this volume. I also took the opportunity of moving appendices in the original papers to insert them at a more appropriate place in the text. Such original appendices now appear in boxes. The chapter text is based on either the original paper submitted or a near-final version of the actual published paper. New material is added in the form of the Prologue and the Epilogue. The table overleaf maps the chapters in this volume to the published papers.

I am indebted to my actuarial colleagues, Dr Mary Hall, Dr Rabia Naqvi, and (soon-to-be Dr) Maeve Hally, for friendly and productive collaborations over many years, some of which underlie this volume. I thank the editors of the journals for permission to use work originally published, and to the referees and others who made helpful suggestions for improvements. I thank the CSO Expert Group on Population and Labour Force Projections for many interesting discussions over the years. The Society of Actuaries in Ireland deserves special mention for providing many opportunities to present the on-going research work and, of course, for generously supporting the publication of its key findings in this volume. I thank Dr Kieran Rankin and Niamh Brennan of Dublin University Press for their professionalism and friendly efficiency.

Lines from Thomas Kinsella's "Blood of the Innocent" and "Songs of Understanding" are reproduced here with the kind permission of Carcanet Press (first published in *Marginal Economy*, 2006, republished by Carcanet Press in *Late Poems*, 2013).

Finally, I thank my wife, Pauline Mellon, and our children, Sorcha, Cathal, and Aisling for tolerating a distracted husband and father over many evenings over many years. These are some of those night thoughts that preoccupied me.

Mapping Chapters to Previous Published Papers

Chapter 1 'Projecting Population Mortality for Ireland', *Journal of the Statistical and Social Inquiry Society of Ireland*, Vol. XXXVII, (2007-08), 135-163.

Chapter 2 Edited Extract from 'Live Long and Prosper', *British Actuarial Journal*, 15, Supplement, (2009), 3-15.

Chapter 3 'Mortality in Ireland at Advanced Ages: Part 1: Crude Rates, 1950-2006', *Annals of Actuarial Science*, Vol. 4, Part 1, (2009), 33-66.

Chapter 4 'Mortality in Ireland at Advanced Ages: Part 2: Graduated Rates, 1950-2006', *Annals of Actuarial Science*, Vol. 4, Part 1, (2009), 67-104.

Chapter 5 'Future Life Expectancies in Ireland', [With Rabia Naqvi] *Journal of the Statistical and Social Inquiry Society of Ireland,* Vol. XLIX, (2019-20), 13-42.

Chapter 6 'The Life Expectancy of a Child Born in Ireland in the Twenty-First Century', [With Rabia Naqvi] *Irish Medical Journal*, Vol. 113, No. 6 (June 2020), P96.

Chapter 7 'Compensation for Wrongful Injury in Ireland: Principles, Practice and Cost to the State', [With Maeve Hally] *Economic and Social Review*, Vol. 52, No. 3, Autumn 2020, 425-460.

Chapter 8 'The True Cost to the State of Maternity Services in Ireland', [With Maeve Hally and Caoimhe Gaughan] *Irish Medical Journal*, Vol. 114, No. 1 (January 2021), P241.

Prologue

Close by Shanganagh Cemetery in Shankill, Co. Dublin, is perhaps the most suitable commemoration of the dead — a child's playground. The shouts and laughter of children absorbed in the adventures of living pass over the quieted graves with "all toil locked fast inside a dream with iron gates." Life has been successfully renewed once again. Such is immortality — the immortality of our race — not that foolish dream of individual immortality ridiculed by Jonathan Swift.[1]

The extinction of mankind was almost inevitable about 75,000 years ago. Evidence, principally from the limited genetic diversification displayed by modern humans throughout the world, points to a perilously small population of humans — perhaps as low as 3,000 in number.[2] The genetic diversity of humans, and certain associated bacterial infections we host, increases as one approaches east sub-Saharan Africa, consistent with the archaeological evidence that has long identified this region as the cradle of *Homo sapiens* as well as the broader class hominins. Nature's

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[1] Swift, J. (1726), *Travels into several remote nations of the world, in four parts. By Lemuel Gulliver, first a surgeon, and then a captain of several ships*. Reference is to Part III, when he considers the plight of the struldbrugs during his visit to Luggnagg.

[2] For an overview of the several competing theories of human evolution see, for instance, Scarre, C. (ed) (2005), *The human past: World prehistory and the development of human societies*. Thames and Hudson, London. The version presented, although of course not uncontentious, is probably the most generally accepted theory amongst anthropologists at present and the most consistent with the emerging evidence from genetics to date.

investment of several millions of years of evolutionary time in this one species was being put to the test, but the result was of no significance: extinction is a commonplace event in nature's trial and error experimentation. The outcome mattered only to the individuals of the species themselves, and that was a small number.

The primary objective of continuing our race requires, on average over the long term, that each of us has at least one offspring, and rears that offspring to reproduce themselves. Reproducing any less than that mathematically implies that the species will die out eventually, no matter what the starting population. The human population in the world has multiplied from the 3,000 persons back about 75,000 years ago to the current estimate of 7.9 billion. This period includes the explosive growth since 1850 in most places outside of Ireland.

So that small band of humans, and all their descendants to date, did meet the primary objective, but only barely. They reproduced and reared just enough, averaging 1.0049 per person, a tiny fraction over the required 1.[3] This is roughly equivalent to just 5 in a 1,000 reproducing and rearing to reproduce themselves one more than the minimum over the period.

It is not known what caused the population numbers to fall to endangered levels about 75,000 years ago, but clearly the inimical environment was gaining the upper hand. Some point to climate change with the return of glacial conditions, while others suggest a dramatic ecological event such as the mega-eruption of the Toba volcano in Indonesia. What is known, from mathematical modelling, is that dramatic declines in population and local extinctions were not uncommon events over our species history (Gurven et al. (2019)).

In Ireland we have a chronicle of such catastrophic events. The Commissioners of the Census in Ireland in 1851 put the Great Famine in an historic context. In the first section of the Table of Deaths volume

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[3] The mathematics is straightforward. Assume that age 25 years is effective reproductive age then over the time period of 75,000 years there was 75,000/25 = 3,000 generations to reproduce and over those 3,000 generations the initial 3,000 persons grew to 7.9 billion. So $\left(\frac{7{,}900{,}000{,}000}{3{,}000}\right) = f^{3000}$, which gives the answer. The number is surprisingly low because the period of time is so long.

they attempt to list all the famines, pestilences, and other calamities that befell those living in Ireland since the first inhabitants. The list covers more than 200 pages with sources from history, prehistory, and myth. They conclude:

> From an examination of this epitome of the most remarkable epidemic pestilences, as well as of the famines, epizootics, cosmical phenomena, and other circumstances, influencing, or supposed to influence mortality, we perceive that so far as the annals and records of the country afford information, Ireland has from the earliest period of its colonization to the present time been subjected to a series of dire calamities, affecting human life …
>
> *Census of Ireland* 1851, Part V; Table of Deaths, Volume I, p.2.

Ireland is, of course, not unique with such a dreadful catalogue.[4] Such calamities occurred with fearsome regularity wherever humans settled. The nineteenth century actuary Cornelius Walford compiled an exhaustive listing of all the known famines of the world (Walford (1879)), and, separately, of all known plagues and pestilences (Walford (1884)). The list continues with the Covid-19 pandemic just a recent minor addition.

This volume tells a story altogether different from the mortality of man up to 1850. It traces the course of mortality in Ireland after the Great Famine of 1845-49. Our analysis starts when official registration of deaths began in Ireland in 1864, and the key elements are the number of deaths by age and the regular population counts in censuses, formerly decennial and now quinquennial. These figures allow us to calculate mortality rates by age and by calendar year with a degree of accuracy impossible before then. Several of the early chapters deal with issues that need to be addressed before the story can be reliably told. The people of Ireland

[4] While we cannot say that Ireland was any better or worse a place to settle than any other, we can say that the Great Irish Famine ranks as one of the worse of the famines in the world since 1700. The Great Irish Famine killed about 1 million or one-eighth of the population. In terms of the absolute number of deaths this puts it in the league of a handful of famines in modern times — Ethiopia in mid-1980s, Biafra 1968-70, the Great Leap Forward famine in China 1959-62, Great Bengali Famine 1943-44, the Ukraine Famine 1932-33, the Soviet Famine 1918-22, and the Great Finnish Famine 1866-68. In relative terms — that is, the number of deaths as a proportion of the total population — it ranks in the top three with the Soviet Famine of 1918-22 and a previous Irish famine in 1740-1. See, Ó Gráda, C. (1999), *Black '47 and beyond*, Princeton University Press, Princeton, p.5 and Ó Gráda, C. (1989), *The Great Irish Famine*, Gill and MacMillan, Dublin, p.1.

were not that precise in stating their ages when requested for official records: there was a tendency to round their age, or exaggerate it, or both. Accordingly, we need to make a number of adjustments to the official records to tell the story straight. The larger part of this volume is devoted to getting the numbers to tell the story as truthfully as they can.

The story revealed is remarkable. First, it tells of a breaking of the cycle of periodic mortality catastrophes. Second, it tells of more children surviving to adulthood, until the commonplace task in our evolutionary history of parents burying offspring becomes a rare event. And third, it tells of the extension of lifespans in Ireland from middle age to increasingly older ages. The Irish experience is part of a global phenomenon, where "the bulk of this mortality reduction has occurred since 1900 and has been experienced by only 4 of the roughly 8,000 human generations that have ever lived" (Burger et al. (2012)). The reduction is staggering, in Ireland and across the world.

The remainder of this Prologue highlights the magnitude of the fall in mortality and increase in longevity in Ireland and puts it in the context of the mortality our ancestors endured.

Prehistory

Let us begin about 75,000 years ago when our species' existence was threatened by the perilously low number of our ancestors. Let us also agree to measure a human generation, that is the average time between birth and reproduction, as 25 years. Hence 75,000 years ago equates to 3,000 generations.

At the start of the story, the small band of *Homo sapiens* lived in Africa. They were better tooled, had more systematic and elaborate burial rituals, and apparently a more developed sense of the decorative arts than any other species, including other hominin species (e.g., Neanderthals, *Homo ergaster*, *Homo erectus*, *Homo Heidelbergensis*). In short, the creative imagination appears to have been more vigorous in this hominin.

Fast forward 1,000 generations to about 50,000 years ago when there is evidence of more organized behaviour suddenly beginning. At about the same time, some geneticists contend that there may have been a minor but momentous mutation in humans that led to the development of a more sophisticated language. The development of a nuanced language facilitated a second and more accelerated evolution of our race. Individuals could now share previously private thoughts and, from this time, a pooled race consciousness develops that helps preserve and pass the collective's wisdom down the generations.

Some modern humans began to leave Africa soon after language was acquired around 2,000 generations ago. No doubt they were driven out by the constant pressures on resources from the thriving population growth caused by the crucial advantage that language confers. They migrated in the hope of a better life. They left accompanied on their adventures by lice and bacteria that tell their own genetic tales of human travels.[5] There is evidence of their presence across most of the world by about 40,000 years ago — including Europe, Asia, Australia, New Guinea. Emigration to the Americas, via the inhospitable Bering Strait, was later, at about 17,000 years ago. Humans arrive in Ireland about 12,500 years ago, and there is evidence of continual occupation from about 9,000 years ago. Regions that were previously occupied by other hominins, such as the Neanderthals in Europe outside of Ireland, were 'replaced' (as it is euphemistically put), although there was limited interbreeding in some areas (Marth et al. (2002)). Eventually, the last major archipelago, New Zealand, was populated by humans about 1,300 years ago. Wherever they walked, they walked with the accumulating knowledge, wisdom, and folly of the species.

Remember that humans also journeyed with the imperative to future generations to reproduce and rear to effective reproductive age at least one other on average. This is a time-consuming task. It takes two — one of either sex — to produce another and that other is a unique genetic mix of both parents. However, the fertility of each sex is quite different. The human female is generally fertile between the ages of 15 and 45 years, a 30-year period. The male typically ties his more abundant reproductive capacity to that of his mate and invests his energies alongside hers to rear

[5] See, for instance, Linz et al. (2007) and Reed et al. (2004). As Alan Rogers once quipped "the record of our past is written in our parasites." (*Of lice and men: Parasite genes reveal modern and archaic humans made contact*, University of Utah, 5th October 2004).

the children over their long period of dependency.[6] So, thinking in terms of couples, each couple must on average rear a minimum of two children to reproductive age just to maintain our species. This was barely accomplished over the 2,500 generations from 75,000 years ago until 12,500 years ago. Estimates put the world human population 12,500 years ago at about 6 million (Bacci (2017)), which means that the original band of 3,000 or about 1,500 couples had reproduced and reared some 2.006 offspring per couple on average over the 2,500 generations.

Human Habitation in Ireland

When it comes to a long time ago — say 10,000 years or more — we have so little direct evidence that a single new discovery could dramatically alter the scientific tale of the prehistory of our race. Textbooks (for example, Waddell (1998) or Herity and Eoghan (1977)) that confidently state man first inhabited Ireland from about 9,000 years ago (according to radiocarbon dating of camps found at Mount Sandel, Co. Derry, and Lough Boora, Co. Offaly) need to be revised with the more recent dating to 12,500 years ago of a human-worked bear bone in a cave in Co. Clare.[7] This puts the first evidence of human traces in Ireland at the ending of an Ice Age that lasted from about 115,000 years ago to 12,000 years ago — the Midlandian glaciation. Most of Ireland was then covered with an ice sheet and the rest with a bleak tundra. The ice sheet lowered the sea level, so Ireland was not then an island but joined to Britain and thereby to continental Europe. The first humans probably came to Ireland over this land bridge, following the migration of large mammals.

Perhaps this first group left before the land bridge became submerged from about 12,000 to 9,000 years ago as a result of a rising sea-level from more northerly melting ice sheets. Or perhaps the harsh conditions caused a local extinction event. It seems unlikely that this group or their

[6] There are a few exceptions to this typical pattern of reproduction in our species, which maximises its genetic diversity. Research suggests that that the thirteenth century Mongol leader Genghis Khan had a sizeable impact on the gene pool in Asia. Niall of the Nine Hostages, the fourth century Irish king, is suggested as a person with a disproportionate impact on the gene pool in Ireland (Moore et al. (2006)).

[7] *Irish Archaeology*, March 2016, *New discovery pushes back date of human existence in Ireland by 2,500 years.*

descendants witnessed the warming of Ireland and the age of afforestation that followed — with early coverage of grasses, herbs, juniper and willow succeeded by birch and hazel which in turn gave way to Scots pine, elm and oak.

In any event, Ireland was a more habitable place by 10,000 years ago. The peninsula of Ireland was cut off to migrating animals early after the Ice Age, which accounts for Ireland's comparatively limited fauna. There were just 14 mammals to hunt or trap in Ireland at this time — the badger, brown bear, wild boar, wild cat, fox, hare, otter, pig, pinemartin, pygmy shrew, red squirrel, stoat, wolf and woodmouse.[8] We picture another migration to Ireland by coastal followers, with a diet primarily of fish, when the Irish coast would have been temptingly visible. Increasing archaeological finds dating from about 9,000 years ago make it reasonable to surmise that Ireland was continuously inhabited from this time.

Seven thousand years BC, or 9,000 years before the present, puts too great a distance between us today and the first Irish settlers. It is better to say that the Irish people go back at least 360 generations, maintaining the convention that a generation is 25 years (the average age of a mother giving birth). 360 generations is a blink of an eye in evolutionary terms. These people differed from us primarily in the challenges they faced and the tools at their disposal. Suitably attired, the first inhabitants could pass along Grafton Street with less attention than most, although they might be mistaken for Italian with their black hair and brown eyes (Cassidy et al. (2015)).

The mythical account of the settlement of Ireland, *Leabhar Gabhála na hÉireann* (The Book of Invasions), recounts six invasions of Ireland. There was certainly a number of separate groups coming over time, so that it was not until the Bronze Age (about 4,000 years ago) that the inhabitants of Ireland are genetically recognisable as similar to the modern Irish population.

Life and death for the earliest inhabitants of Ireland changed little over the first 120 generations (the three millennia: 7000 BC – 4000 BC).

[8] It is believed that the red deer was extinct in Ireland by this stage, along with the reindeer, mammoth, and lemming.

However, so little is known of these Mesolithic hunter-gatherers that we must leave it to our imagination to fill in the details of their daily lives.

Table P1: Eras of Mankind in Ireland

Era	Dates	Generations Ago	% of Total Inhabited Time
Early Mesolithic	7000-5500 BC	360-300	17
Later Mesolithic	5500-4000 BC	300-240	17
Neolithic	4000-2400 BC	240-176	18
Copper Age	2400-2200 BC	176-168	2
Bronze Age	2200-600 BC	168-104	18
Iron Age	600 BC-400 AD	104-64	11
Early Middle Ages	400-1000 AD	64-40	7
Late Middle Ages	1000-1550 AD	40-18	6
Pre-Famine	1550-1850 AD	18-6	3
Post-Famine	1850-Present AD	6-0	2

The lifestyle of the Irish person was transformed from about 4000 BC or 240 generations ago. The cultivation of crops (wheat and barley) and domestication of animals (sheep, goats, and later pigs and cows) dates from about 9000 BC in the Near East and, perhaps independently, in Northern China. By 7000 BC the new farming communities lived uneasily alongside hunter-gatherers. For example, the wall enclosing the settlement village of Jericho in Palestine dating from about 7000 BC (reaching 4 metres high and 2 metres wide in places) was not enough, we know from biblical stories and archaeological evidence, to keep safe the Neolithic inhabitants and their farming supplies (Cipolla (1974)). It took until about 4000 BC for farmers to arrive in Ireland.

Maybe the description agricultural 'revolution' is inappropriate to describe such a slow diffusion over millennia across Europe, but it is appropriate to describe its arrival in Ireland. The early agriculturalists in Ireland immediately redefined our relationship to the land from passively accepting its bounty. The introduction of cattle, sheep, and goats more than doubled the number of large mammals in Ireland. They cleared forests for pasture and to sow hardy wheat and barley seeds. In fact, as

Waddell (1998) points out, unambiguous evidence for early farming in Ireland comes from about 3900 BC with marked local changes in the pollen records, with grass and, to a lesser extent, cereal pollen replacing tree pollen. The Céide Fields in Co. Mayo were intensively farmed from 3700 BC to 3200 BC, and, like the other farming communities in Ireland, seem to have been predominately pastoral. Of course, aside from their radical new lifestyle, the farmers also brought some other technologies (e.g., pottery) and a new culture. Part of that culture was honouring the dead with stone monuments. The majority of the excess of 1,500 recorded megalithic tombs in Ireland were constructed between 4000 BC and 2000 BC (Waddell (1998), p.57).

The pattern of human habitation in Ireland changed from semi-nomadic and scattered to settled and denser populations. The control the early agriculturalist could exercise over their food supply, though primitive by modern farming methods, meant that the island's produce was expanded, removing the key constraint to growth. So, like every other place that experienced the agrarian revolution, the relatively stable population in Ireland became a growing population.

The practical maximum offspring per couple is obviously dependent on the time between maternities. As noted earlier, women are fertile for about thirty years. When humans lived as hunter-gatherers, the gap between births was probably around 3 years due to lower female fertility when breastfeeding (Konner et al. (1985). This gap gives a reasonable maximum of 10 offspring per couple. When mankind settled down to farm, the reasonable minimum period between maternities reduced as children were weaned earlier, maybe at about 1.5 years. This smaller gap results in a reasonable maximum number of offspring per female of roughly 20, so double that of hunter-gatherer times.

It is straightforward to estimate crude upper and lower limits for growth in the population of Ireland from earliest times until today. Let us say, to get an upper bound on population growth, that 9,000 years ago there was just one breeding pair in Ireland and now there are 6.83 million (that is, 4.94 million in the Republic of Ireland and 1.89 million in Northern Ireland). So, ignoring migrations, this implies each generation on average reproduced and reared no more than an average of 1.0427 offspring per person. For the lower bound, note that if the population of

Ireland grew by the average 1.0049 per person estimated earlier for the growth in human population over the last 75,000 years, then the current 6.83 million inhabitants of Ireland must have started 9,000 years ago with over 1.1 million persons (again, ignoring migrations). The original number of inhabitants of this island was considerably lower than one million, thus implying that the growth rate must be higher than 1.0049 per person. We can conclude from the arithmetic that there was an acceleration in population growth as a consequence of the agrarian revolution in Ireland, from the long-term average of 1.0049 per person, but the average growth rate did not exceed 1.0427 per person.

The coming of the farming lifestyle increased the number of lives but not the length of each life. The graph in Figure P1 (Plate 1) shows male life expectancies at each age for hunter-gatherers, for males in Ireland immediately before the Great Famine, during the great Famine, and for three periods after the famine — 1871, 1926 and 2016.

The graph shows that life expectancies for hunter-gatherers was close to that in early nineteenth century Ireland, especially at adult ages. Life expectancies in famine times were lowest at all ages, barely exceeding 30 years at the peak. In contrast, Figure P1 (Plate 1) also shows a trend of increasing life expectancies across all ages after the Famine. In 2016, male life expectancy at birth in Ireland was 80 years compared to 57 years in 1871 and 38 years in pre-Famine Ireland. Also, note that the sharp rise in life expectancies from age 0 to age 5 is no longer a feature of modern life tables, due to the pronounced fall in the mortality of infants.

It was once widely held that the agrarian revolution was accompanied by mortality improvements on foot of a more secure food supply lowering the risk of starvation (the so-called 'classic' theory). This is now challenged by those who suggest that the higher population densities in farming communities increased the risk of infectious diseases, made more virulent given their less nutritionally rich diet and the occasional failure of crops. Under this alternative theory, the first agriculturalists were exposed to the same diseases and infections as hunter-gatherers but the reservoir of disease and infections increased over time with closer contact between humans, domesticated animals, and the faeces of both. This is particularly true for certain infections (often with brief infectious stages) that reproduce through transmission from one host to the next and require a large host community to survive. These communicable diseases include some of the worse killers until modern times: measles, cholera, smallpox. So the higher density of farming communities and their interconnections could keep alive and collect more infectious diseases over time, leading to an increasing endemic mortality burden, especially amongst the very young and the very old.

Figure P1 (Plate 1) is not decisive on the issue between the classic and alternative theories. It shows that life expectancies were similar for hunter-gatherers and farmers outside those times of famine or plague.

Life expectancy at birth has doubled in Ireland in the space of two centuries. Equally remarkable is that we are rid of those awful periods of mortality crises from famine or pestilence. Close scrutiny of Figure P1 (Plate 1) at about age 50 reveals another remarkable feature. Life expectancy at this age was about 20 years for hunter-gatherers and for the Irish in the nineteenth century in good times. So, to generalise, a 50-year-old could expect to live to the age of threescore and ten for almost all of human history. However, the life expectancy of a 50-year-old now extends to over 30 years and is still increasing, in Ireland and over much of the developed world. This is an emerging pattern, only becoming discernible in the last half-century. Neither individuals nor societies have yet adapted to this growing extension of our lifespans.

Explanations of the underlying causes extending the longevity of our species are frustratingly vague. Our understanding, such as it is, is summarised in surveys such as Oliver Lancaster's opus *Expectations of life: A study in the demography, statistics and history of world population* (1990) or James Riley's more accessible *Rising life expectancy: A global history* (2001). They tell a complicated tale, with the causes often differing substantially from region to region but all leading to a similar effect. It was the work of a great many, in a great many ways. The tale of mankind's changing mortality cannot be divorced from the broader narrative of the survival of our species through the ages, told in, say, Massimo Livi Bacci's updated classic, *A concise history of world population* (2017), or his more local *The population of Europe* (2000).

Another way to view how human longevity has changed over the millennia is to consider mortality rates at each age. We do this in Figure P2 (Plate 2) which expresses mortality rates at each age in previous times as a multiple of mortality rates in Ireland in 2016.

The graph shows that mortality rates have fallen dramatically at all ages before adulthood. Mortality rates of the young were several hundred times higher in earlier times. It is not possible to discern from Figure P2 (Plate 2) how much higher mortality rates were at adult ages due to the linear scale on the y-axis. We redo Figure P2 (Plate 2) by changing the y-axis to a log-scale in Figure P3 (Plate 3).

Figure P3 (Plate 3) highlights that mortality rates at all ages were a multiple higher in the past, even at adult ages. It is fair to say that mortality rates up to age 50 years were at least 10 times higher before the twentieth century in Ireland compared to today. At younger ages, the mortality multiple was over 100 times higher compared to modern times.

A significant part of our inheritance from previous generations is our extended life expectancy. Yet this towering achievement figures, if at all, only as a footnote in the vast written histories of the accomplishments of our ancestors. The major themes explored by historians remain man's struggle for power over his fellow man, often recounted in extraordinary detail. The story told in statistics here is the outcome of the background struggle of our species to survive. The protagonist of our story is each individual. Many have taken on the direct task of rearing children to reproductive age, many have devoted energies to making life better or longer for more, and many have done both. The statistics are triumphant in telling of lowering infant mortality and of increasingly longer life for the higher numbers making it to adulthood. The vital story of the individual of our species is now having a different ending. Yet too few in Ireland or elsewhere know the rewritten ending. This volume answers the key question: how long can those alive in Ireland today, and their children, expect to live?

Chapter 1

Projecting Population Mortality for Ireland

Abstract

Mortality data for Ireland is analysed, for recent and long-run trends, and several methods of projecting mortality rates are outlined and the results compared. Interpretation of the results suggests that it is not unreasonable to forecast that males born in calendar year 2006 have a life expectancy of 91 years (females 93 years). On the same basis, males aged 65 years in calendar year 2006 can be expected to live another 20 years on average (females 23 years). The uncertainty surrounding the forecasts is outlined.

Introduction

Increasing human longevity in more advanced nations is one of the greatest social achievements over the last one-hundred and fifty years. In Ireland, life expectancy began to increase markedly from the last years of the nineteenth century. In 1900-1902, life expectancy at birth was 49.3 years for males and 49.6 years for females. The latest official Irish Life Table, reflecting the experience in the years 2001-2003, shows life expectancies have increased to 75.1 years for Irish males and 80.3 years for females — a rate of increase averaging 0.26 years for males and 0.30 years for females with the passage of each calendar year over the twentieth century.

It is of interest to ponder how life expectancies may change over the course of the twenty-first century. Aside from personal planning, a good estimate of longevity would aid the State in, say, designing and financing of a pension and healthcare system to better achieve sustainability and inter-generational equity. This paper reviews the different approaches to projecting mortality rates and applies a couple, using several different sets of assumptions, to help form a judgement on the course of mortality in Ireland over the 21st century. Specifically, we attempt to answer the question: how long will a child born in Ireland in 2006 live on average?

Mortality patterns have been changing in the developed world at a remarkable pace over the recent past. Mortality improvements have tended to accelerate at many ages and most especially at the older ages. The pattern is no different in Ireland. In particular, Ireland is now experiencing an average rate of mortality improvement higher than at any recorded period in the past. The actuarial profession, the profession that prices and reserves for mortality risks and generally advises on the prudent management of life offices and pension funds, has recently ceased publishing mortality tables with forecasts of mortality improvements because of the dramatic changes of late and the consequent very significant uncertainty inherent in any single projection. It is of interest to explore the possible long-term effects of current emerging patterns, even if the resultant projections must inevitably be surrounded by considerable uncertainty.

The structure of the chapter is as follows. First, we overview the different methods of projecting mortality rates. Second, we analyse mortality trends in Ireland over both long and short periods of time. Third, we project mortality rates in Ireland and estimate future life expectancies, including for children born in 2006, by different methods and on different bases. This section outlines the new approach to mortality forecasting adopted in the forthcoming official projection (CSO (2008)) and Box II outlines the new approach in detail. Finally, we conclude that children born in 2006 can reasonably be expected to live to their early nineties for males and mid-nineties for females. We begin with Box I critically reviewing the underlying data from which the Irish mortality experience is inferred.

Box I: Data from which Irish Population Mortality is Estimated

The continuous registration of deaths in Ireland[1] began in 1864, with each record of death including the sex, age, cause, and location of death. To estimate mortality rates, one requires the number in the population corresponding to the number of deaths — in this case the population in Ireland sub-divided by sex, age, and location. Censuses have been conducted in Ireland since 1821 but have been reasonably reliable only since 1841. Accordingly, the mortality experience in Ireland can be estimated from official sources from 1864.

Formal life tables, showing how mortality varies by age and sex have been prepared from the experience in calendar years 1925-7 (Irish Life Table 1) and since that time a total of fourteen have been prepared, the most recent relating to the period 2001-3 (Irish Life Table 14). Summary statistics of life expectancies in the twenty-six countries of Ireland prior to independence are reported in the *Report of the commission on emigration and other population problems 1948-1954* (Table 79, p.106).

In this chapter, which is more concerned with projecting future mortality, we restrict our analysis to mortality trends since 1926. Five caveats must be made on the mortality experience as it is recorded in Irish Life Tables 1-14.

First, birth registrations up to 1941 are judged to be under-reported by 3-10% prior to 1941 but essentially complete after 1956 (Coward (1982)). This entails that infant mortality may be overstated somewhat prior to 1941.

Second, people when asked their ages at the regular censuses had a marked tendency in earlier times to round their ages to an age ending with 0 or 5, particularly at the older ages. This is a well-documented and internationally observed tendency known as 'age heaping'. Figure 1.1 shows the person count by age in the censuses of 1926 and 2002, with age heaping evident in the former and not the latter. The method of graduation of the Irish life tables has been designed to remove much of the effects of this rounding.

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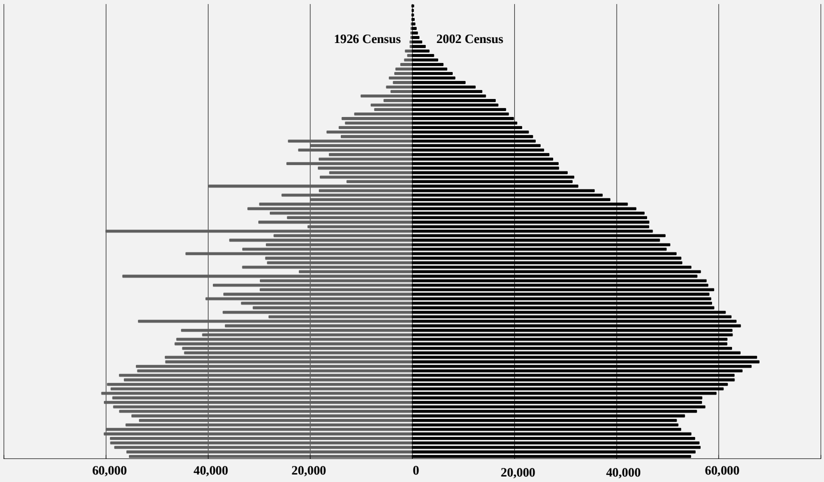
[1] Acts for the official registration of births, deaths, and Roman Catholic marriages came into operation on 1st January 1864, from which time continuous records have been maintained and published annually in the *Annual Reports of the Register-General.* Registration of Protestant marriages began somewhat earlier, from 1st April 1845.

***Figure 1.1: Persons in Ireland by Reported Age in Censuses of 1926 and 2002*[2]**

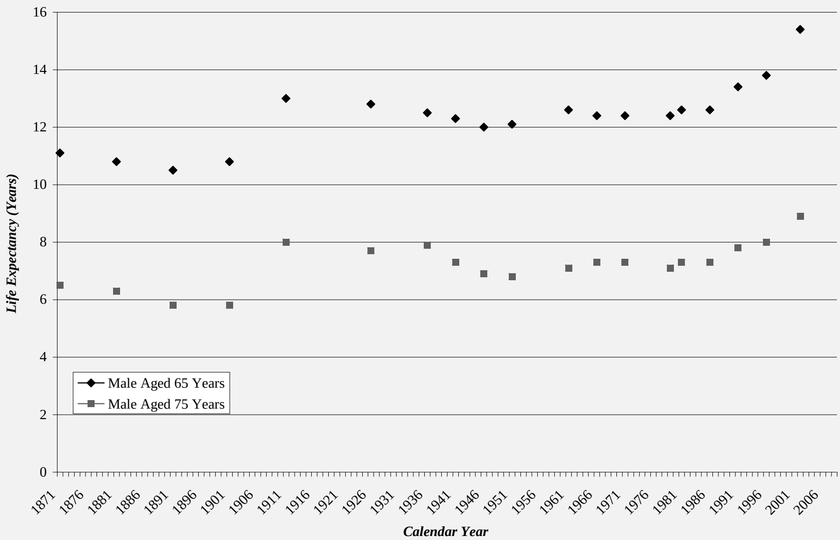


Third, again in the earlier years, there may have been a tendency for older people to not just round their age but to exaggerate it. Old age pensions were paid from 1909 to persons in the then United Kingdom over the age of 70 years, subject to means and other qualifying tests. While England registered births, deaths, and marriages since 1836, Ireland only began official records from 1864 as noted earlier. Thus there was no formal means to verify ages of anyone over 45 years old in 1909 (Wood (1908)) and, as could be anticipated, claims for pensions in Ireland exceeded that budgeted and, in fact amounted to "117 per-cent of the number of seventy and over, less paupers; and this assumes that not a single person of seventy and over in Ireland has an income of £31 per annum" (see O.T. Falk's discussion on Marr (1909), quote is from pp.270-1) In fact, the expenditure over-run of pensions in Ireland was one of the main reasons for Lloyd George's budget of 1909 that lead to the constitutional crisis (Ó Gráda (2002)). It could be expected that a person would report an age at subsequent censuses consistent with their declaration of age for pension. A male, say, aged 65 in 1909 claiming to be 70 years old could be 82 years old in 1926 and, according to Irish Life Table 1 Males, be alive in 1926 with probability 0.28. This individual would bias the estimated mortality rates downwards. We observe,

[2] Based on data sourced from the Central Statistics Office (CSO).

consistent with the exaggeration-for-pension hypothesis, that life expectancies for both Irish males and females at age 65 and 75 years show a suspicious jump between 1900-02 and 1910-12 of more than 20% and a decline thereafter. In fact, life expectancies for males aged 65 only rises above the 13.0 years estimated in 1910-12 in 1990-2 and life expectancies at age 75 take until 1995-7 to regain the level estimated in 1910-12.

Figure 1.2: Life Expectancies for Male Aged 65 and 75 Years, Ireland (26 Counties), Based on Experience Around the 3 years Centred in Calendar Year Shown[3]



The data for both deaths recorded and the exposed-to-risk population numbers in Ireland in respect of ages above 80 years, while improving with time, has been described even recently as "conditionally acceptable quality" and "data give probably a roughly correct description of the mortality trend though at a level artificially lowered by age overstatement" (Kannisto (1994)). The issue is that death certificates accurately report age at death but that age statements in census returns for the elderly tend to exaggerate the age — now for reasons other than the old age pension. Much of the problems can be overcome using the method of extinct generations (see, for instance, Humphrey (1970)),

[3] CSO (2004a), figures from Table 3.

which bases the analysis on death records only, but such an investigation is beyond the scope of this chapter (but see Chapter 4).

The reliability of records in regard to the cause of death are also questionable: in the 1920s over one-quarter of deaths were not certified by a medical practitioner (Brown (1930, p.101)) and even nowadays it is estimated that one-third of death certificates are likely to show incorrect cause of death (Roulson et al. (2005) quoted in O'Reilly (2006)). Only 26.5% of deaths in Ireland are referred to the coroner (O'Reilly (2006)).

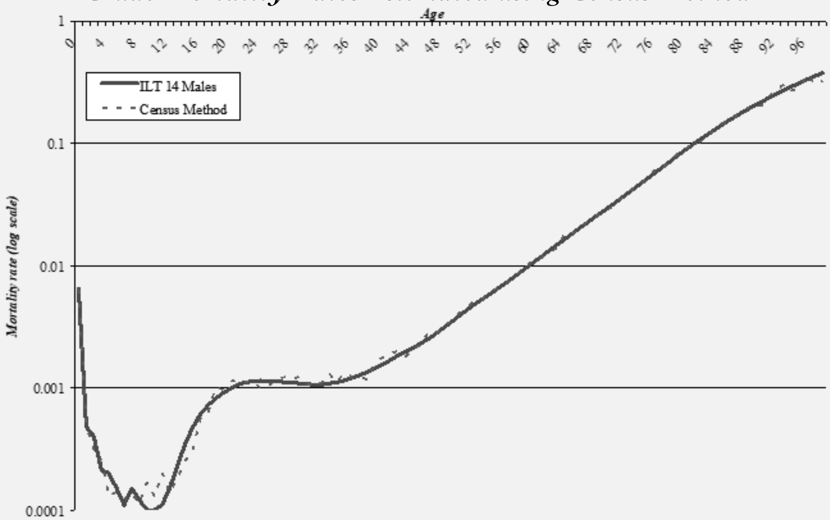
The method used to construct all 14 of the official Irish life tables is based on an old actuarial method, King's method (King (1909)), which was also employed to graduate life tables for England and Wales between 1901 and 1930-32 (ELT 7 to 10). The method involves smoothing the series of deaths by age and population by age to reduce the effects of age heaping, before estimating mortality rates and is typically applied to data grouped in quinquennial age groups. Osculatory interpolation is then used to estimate mortality rates at intervening ages. At the extremes of age — under 6 years and over 87 years — ad-hoc methods are employed. For instance, mortality rates above age 87 years are obtained by fitting a quadratic or Makeham curve (see Geary (1929), CSO (1986)).

Brown (1930) reviewed the construction of the first Irish life table. He judged that "Messrs Hooper and Geary in particular are to be congratulated on their enterprise to elucidate the obscurities of Irish population statistics" (p.103) but that the data problems mean that the "Saorstat Life Table cannot be unreservedly accepted as a reliable index of actual conditions" (pp.102-3). In particular, the exceptionally light mortality for both sexes at advanced ages might be partially because "as the pension age approaches the temptation to misstatement of age has still proved irresistible to a considerable section of the community" (p.102).

Investigation shows that King's method provides a reasonably smooth curve that closely fits the underlying crude specific age mortality rates. In fact, King's method with osculatory interpolation can be viewed to be a forerunner of modern spline graduation. However, the ad-hoc graduation method applied at the older ages, and the census method approach to estimating crude rates at these ages is no longer satisfactory. It is now well-established that the shape of mortality curve at advanced ages does not follow a Gompertz or Makeham curve but is better modelled using a logistic curve (Thatcher et al. (1998)). A better estimate is required for mortality at these advanced ages if only because more of the population can be expected to survive to these later ages.

We conclude that Irish life tables since 1926 give a best estimate of the mortality experienced based on the available data. Data quality had been improving over the years. There is an issue with estimating mortality at the highest ages due to age misstatements at censuses and, ideally, mortality rates above age 85 years or so could be better estimated, in line with international best practice, using the method of extinct generations and graduated using a logistic curve.

Figure 1.3: Irish Life Table 14 Males (2000-2002) Graphed Against Crude Mortality Rates Estimated using Census Method[4]



Methods of Projecting Mortality

There are several different approaches to projecting mortality rates. First, one can model the aging process and apply the model to forecast future changes. However, a satisfactory model or 'law of mortality' has

[4] CSO (2004a) for graduated age specific mortality rates of males; crude mortality rates calculated by author based on the average deaths in three years 2000-03 and the number of males at that age enumerated in the Census of 2001 (using census method).

proved elusive, despite notable attempts by, *inter alia*, Gompertz (1825, 1860), Makeham (1860), Perks (1932), and Beard (1971) (see, for instance, Olshansky and Carnes (1997) or Forfar (2004) for a review). In fact, attempts to derive a simple mathematical formula that governs mortality over the whole of life are now largely abandoned. Accordingly, this ideal approach is not practical.

The study of the aging process and how it evolves identifies two distinct mechanisms of mortality change. The first is secular change such as better nutrition, better housing, and innovations in diagnosis and treatment of life-threatening conditions. This must be modelled as a calendar year effect. The second mechanism, somewhat more speculative, is that the aging process is essentially the accumulation of damage to the body. This theory of aging predicts that the conditions an individual lives through are recorded on their body so, for instance, early-life conditions can affect late-life mortality patterns. This mechanism suggests that year of birth should also be incorporated into projections to proxy these 'cohort' effects.

Perhaps another obvious approach to mortality forecasting is a two-step method where, first, a forecast is made of those factors that are known to significantly impact mortality (such as marital status, smoking habits and wealth) and, second, the effect on mortality rates in such changing circumstances is estimated, in addition to some underlying secular improvement. However, again, this is not feasible in practice as explanatory variables prove just as difficult to forecast as the mortality rates themselves. In any event, the link between factors and mortality differentials tends not to be robust and the classification of the population by relevant factor is generally not available. Again, this theoretical approach is not used in practice.

We must settle on a more atheoretical approach. Typically, mortality projections identify historical trends or other patterns and extrapolate those trends or patterns to a greater or lesser degree. This broad approach can be effected in several ways. First, the observed rates of change of mortality rates over some period in the past, generally broken down by sex and age, are simply extrapolated into the future. This has been the approach used to date in Irish official forecasts (CSO (2004b)). Second, one can employ a 'targeting method', by assuming a target mortality rate of improvement from some future year and interpolating between current rates of mortality improvement and the targeted future rate. This method is used by the [UK] Government Actuary's Department (GAD) in making forecasts overall and for each separate region of Northern Ireland, Scotland, and England and Wales (GAD (2006a, 2006b)). Third, one can use parametric methods by fitting a mathematical curve that reasonably describes the mortality rates in the past, identify trends in the best fitting parameters over time, and then project the parameters and hence future mortality. Fourth, one can decompose historical mortality rates by underlying cause of death and make projections separately for each cause, typically using the main grouping of the International Classification of Diseases. For a more detailed overview of approaches applied in practice see, for instance, Wong-Fupuy and Haberman (2004) and, for approaches used in official national projections around the world see [UK] Government Actuary's Department (2001, Appendix H).

The out-turn from the different variants of atheoretical projections tend to be similar to each other. Each identifies the primary pattern in the past has been the near log-linear decline of age specific mortality rates with time (i.e., the annual rate of decline at each age tends to remain broadly constant with time). Another striking pattern is that the annual rate of decline of mortality diminishes with increasing age. These two patterns are evident in Ireland, as outlined in the next section.

All the above atheoretical approaches have another thing in common when applied in practice: the forecasts are wrong. Specifically, there has been a bias to underestimate mortality improvements. Official forecasts in many countries have a tendency to presume a reduced rate of improvement to that observed in the past, while the actual outcome has been closer to a level rate of improvement. The underlying reasons for mortality improvements in the past — better diet, vaccines and antibiotics to combat infectious diseases — are seen as 'once-off' and forecasters are reluctant to predict such major lifestyle changes or innovations in the future (Shaw (2007)). In particular, those who base their forecasts on separately modelling mortality by each main cause of death tend to overestimate future mortality. Projecting mortality rates by underlying cause of death separately produces an aggregate mortality improvement rate at each age that tends to decline with increasing forecast time as, quite simply, the weights attached to each underlying cause of death changes with time to emphasize more those with a slower rate of improvement.

The inherent bias of past forecasters to predict slowing rates of improvement in the future, quite at odds with the historical record, is itself a stylized fact that requires explanation. We speculate that it could be due in part to a perception that an overestimation of future mortality rates is more prudent or conservative than an underestimation. It could be due in part to herding behaviour — where forecasters tend to anchor their forecasts close to others and those of past forecasters — which is often observed when the forecasting exercise is especially difficult.[5] Or perhaps it is due to a fundamental misunderstanding of the underlying process giving rise to mortality improvements. Mortality improvements are a manifestation of man's innovation in altering his environment to better suit his needs. As such, the process of mortality improvement can be seen to be akin to technological progress or economic productivity in that they share the common driving force of man's ingenuity. No one is seriously forecasting technological progress or economic productivity to slow. Quite the opposite, in fact, as the ingenuity of our race can be expected to be greater than at any time in the past, given that there is now more people alive, better educated, better resourced, and better incentivised to contribute to progress. On a simple analysis of the dramatic increase of measurable inputs, it would seem perverse to predict a decrease in the measurable outputs of ingenuity. The remarkably stable log-linear trend of mortality decline at each age, despite the complexity of the underlying process giving rise to it, demands, as Wilmoth (1998) observed, that "in this situation, the burden of proof lies with those who predict sharp deviations from past trends" (quote from p.397).

The caution of forecasters in projecting mortality improvements can be illustrated for Ireland by assumptions made in population projections in papers read to the Statistical and Social Inquiry Society of Ireland. True, future population numbers are considerably less sensitive to mortality than migration or fertility assumptions, but, nevertheless, it is as easy to adopt best estimate assumptions in this regard as any other.

[5] This theory would also account for the high significance attached to official projections, despite the inherent uncertainty and therefore scope for alternative views.

Each forecaster used a low or even zero rate of improvement and tended to be influenced by relatively short-term trends. Geary (1935), in forecasting the population of Ireland out to the year 2016 (an 81 year forecast horizon), assumed no change from the mortality rates of Irish Life Table 1, but acknowledged that "this is an assumption which is fortunately not likely to be realised" (p.28) and observed "there is little doubt that a figure of 70 [for life expectancy at birth] may be achieved during the next half century" (p.29). Geary (1941) revisited that analysis in the light of the 1936 Census, updating the base mortality assumed to Irish Life Table 2, and in two of the three new projections assumed no mortality improvement. In the third, he projected mortality improvements over the next 30 years, thereafter no improvements. The mortality improvements for both sexes were only made for ages up to 40 years for males and 66 years for females, with improvements assumed to be in line with (the higher) mortality improvements experienced by females over the decade to 1936. Knaggs and Keane (1971), in their population projections over 25 years to the year 1996, assumed that male mortality will improve marginally up to age 15 years, following the trend evident over the period 1961-66, with no improvements at higher ages. For females, mortality rates were assumed to decline in line with the age-specific rate of decline observed over the period 1961-66 up to age 80 years, with no improvements at later ages. Keating (1977), in making population projections out to 1986, assumed no improvements in mortality from the 1971 rates, on the basis that recent short-term trends (over 5 years or so) both in Ireland and internationally showed little change.

Mortality Trends in Ireland

The continuous registration of deaths in Ireland began in 1864, with each record of death including the sex, age, cause, and location of death. Regular censuses of the population in Ireland dating from before 1864, also sub-divided by sex, age, and location, allows us to construct the mortality experience in Ireland from official sources from that date. Summary statistics of life expectancies in the twenty-six countries of Ireland prior to independence are reported in the *Report of the commission on emigration and other population problems 1948-1954* (Table 79, p.106) and formal life tables, showing how mortality varies by age and sex have been prepared from the experience over calendar years 1925-7 (Irish Life Table 1) and since that time a total of fourteen have been prepared.

We shall restrict our analysis to mortality trends since 1926, and base it primarily on the mortality experience as it is recorded in Irish Life Tables 1-14. Box I considers the reliability of this source in detail. In summary, the Irish life tables give a best estimate of the mortality experienced based on the available data. Data quality has been improving over the years but there remains an issue with estimating mortality at the highest ages due to age misstatements at censuses. Ideally, mortality rates above age 85 years or so could be better estimated, in line with international best practice, using the method of extinct generations (Humphrey (1970), Thatcher (1999)). The approximate nature of mortality rates above age 85 years or so should be borne in mind in the sequel.

In the next subsections, trends in Irish mortality rates are first explored over the long term, by both calendar year and year of birth, and then over the short-term.

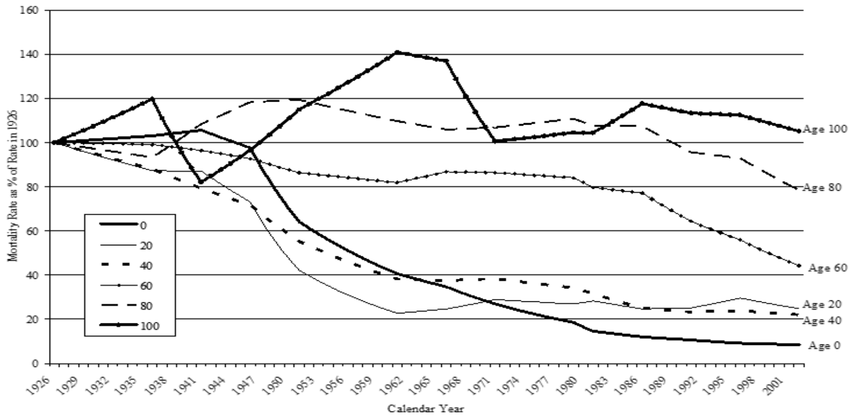
Long-term trends, by Calendar Year

The long-term trends in Irish population mortality are not dissimilar to trends observed in developed economies generally. Mortality rates for either sex have declined at every age except the very advanced, with the decline being most pronounced at the early ages. Figure 1.4 shows the decline in mortality from 1926 to 2002 for Irish males at ages 0, 20, 40, 60, 80 and 100 years, on both a linear and log scale.

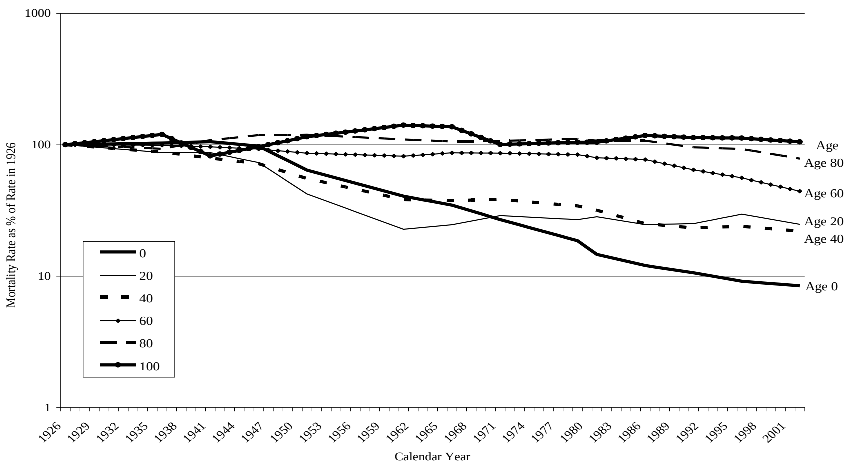
The extraordinary reductions in mortality rates at the younger ages over the 76-year period are displayed in Figure 1.4. The mortality rate at age 1 year for females showed the most improvement, falling to be just one-fiftieth of its rate by 2002 (for males the 2002 rate was just 2.6% of the rate in 1926). Also evident from Figure 1.4 is the pronounced age structure of mortality improvements, with the rate of improvement generally declining with increasing age so that, at very advanced ages, little or no improvement in observed.

Figure 1.4a and b: Mortality Rates of Irish Males, Selected Ages, as % of Rate in 1926

a. Percentage Scale



b. Log-Scale

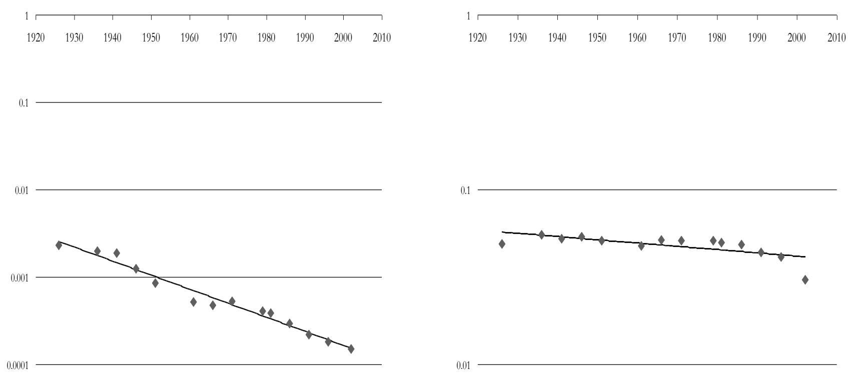


Note: Mortality rates at each age from ILT 1-14 Males, interpolated between census years by assuming that the age-specific mortality rates have same annualised rate of change.

It is not possible to see readily the timing of the improvements from Figure 1.4. Figure 1.5 plots the natural log of the mortality rate against calendar year at selected ages and shows the best fitting (log-linear) trend line. It is apparent that the trend-line captures much of the pattern in the secular improvement of mortality rates. There is nothing special about the ages selected below: a strong the log-linear pattern of mortality improvements emerges across all ages (with a better fit at the younger ages), across both sexes, over many different time periods, and across many different countries.

Figure 1.5a and b: Linear Regression of Log of Mortality Rate (q_x) Against Calendar for Irish Males at (a) age x=7 years and (b) age x=70 years

***a. Age 7 years* (R² is 0.97)** ***b. Age 70 Years* (R² is 0.46)**



A more natural, and wholly equivalent way, of expressing the log-linear relationship is to say that the age-specific mortality rate tends to decline by a fixed percentage with the passing of each calendar year. Figure 1.6 shows the annualised average rate of decline of mortality rates at each age for either sex over the 76 years ending 2002.

We see the declining rate of improvement with increasing age, so at ages close to 100 years, no improvement is recorded. While there are data issues associated with such high ages in Ireland (few deaths and overstatement of ages in census returns), this pattern is observed elsewhere (see Kannisto (1994)). Note that females show high rates of improvement in the late twenties and early thirties, while males show a dip in improvements in the late teens and early twenties. This is due to well-documented improvements in survival rates for females giving birth and the lifestyle-related 'testosterone spike' for males and, again, both patterns are observed internationally.

Figure 1.6: Average Annualised Decline in Irish Mortality Rates, by Age and Sex, 1926-2002

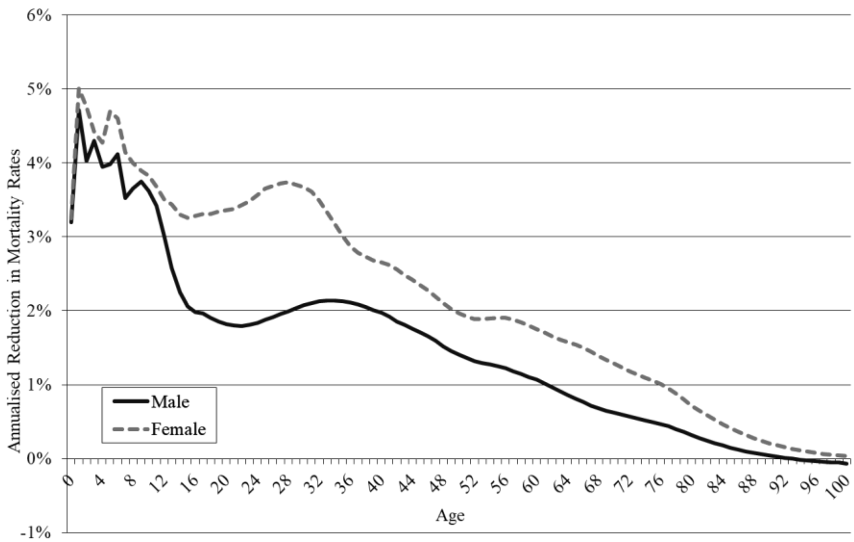


Table 1.1 further breaks down the annualised rate of decline by period to calendar year 2002.

Table 1.1: Annualised Percentage Rate of Decline in Mortality, Years Ending 2002, Various Ages, Each Sex

	10 Years		*20 Years*		*50 Years*		*76 Years*	
Age	*M*	*F*	*M*	*F*	*M*	*F*	*M*	*F*
0	2.0	1.9	2.5	2.8	3.9	3.9	3.2	3.2
10	4.2	1.5	4.9	2.8	3.7	3.4	3.6	3.8
20	0.5	-0.5	0.5	0.6	0.9	3.0	1.8	3.4
30	0.5	2.8	-0.1	1.8	1.6	3.7	2.1	3.7
40	0.6	0.7	1.6	1.4	1.8	2.6	2.0	2.7
50	1.9	2.0	2.7	2.2	1.7	2.0	1.4	2.0
60	3.4	2.8	2.9	2.8	1.3	1.9	1.1	1.8
70	3.4	3.0	2.4	2.4	1.0	1.7	0.6	1.3
80	1.9	1.7	1.6	1.9	0.8	1.4	0.3	0.7
90	1.1	1.5	0.8	1.1	0.5	0.7	0.0	0.2
100	0.7	1.5	0.1	0.7	0.2	0.3	-0.1	0.0

The table highlights another pattern of mortality with time that has been observed internationally (see, for instance, Willets et al. (2004)). Mortality improvements in earlier calendar years were concentrated at younger ages but, more recently, the higher improvements are at older ages. This pattern has been described as the 'aging of mortality improvements' (Wilmoth (1997)).

Long-term Trends, by Year of Birth

Theories of aging also suggest that year of birth, as a proxy for the conditions the individual lived through, might also be significant in forecasting mortality. Such a 'cohort' effect was, in fact, noticed by the UK Government Actuary's Department in 1995 (see Government Actuary's Department (1995)) when they pointed out the generation born in England and Wales between 1925 and 1945 have experienced more rapid improvements than generations born earlier and later. Since that time, the cohort effect has been investigated extensively in the actuarial literature (see, for instance, Willets (1999, 2004), Willets et al. (2004), Richards et al. (2006)) and actuarial projections have modelled it explicitly (see, for instance, GAD (2006a)). A cohort effect has also been detected in Japan, centred around the year of birth 1915. The generation in England and Wales borne between 1925 and 1945 is now 63-83 years old and lower mortality rates are being observed at these later ages.

We investigate whether a cohort effect is present in Irish mortality data. To this end, we broke down the annualised rate of improvement over each decade for decennial ages and set the results out in Table 1.2.

There appears to be a pattern along the diagonal of the table, consistent with a cohort effect. We notice that those born in 1931 would be 10 years of age in 1941, 20 years of age in 1951, 30 years of age in 1961, etc. The table shows that the cohorts born in 1931 and 1941 seem to have experienced a significantly lower mortality rate throughout their lives to date than preceding generations and, while generations following them also build on the decline in mortality, their rates of decrease are somewhat less spectacular. However, it is difficult to be conclusive from Table 1.2.

It is necessary to do a finer analysis. We used Irish Life tables when available. Otherwise, for each calendar year 1950 to 2001, we graduated mortality tables for Irish males between the ages of 12 years and 72 years, based on an average of three calendar years of deaths and population estimates in the centre year (kindly made available to us by the Central Statistics Office) using King's method and oscillatory interpolation (King (1909)). For earlier calendar years or ages outside the range, we interpolated between the closest known rates assuming a constant annual percentage rate of change in the age-specific rates. The results, for each five-year period and quinquennial age, are shown in Table 1.3a.

Table 1.2: Annualised Rate of Improvement Over Each Decade, 1941-2001, by Decennial Age, Irish Males

Age	1941	1951	1961	1971	1981	1991	2001
0	-0.39	**4.87**	**4.47**	**4.02**	**5.93**	**3.16**	2.13
10	**2.59**	**4.91**	**5.72**	0.40	**2.59**	**6.10**	3.14
20	0.70	**6.98**	**6.01**	-2.44	0.20	1.20	-0.17
30	1.24	**5.17**	**5.87**	**1.34**	**1.22**	-0.51	0.14
40	1.66	3.54	**3.63**	-0.05	**1.91**	**3.02**	0.42
50	0.88	1.53	1.87	-0.31	**1.19**	**3.67**	1.81
60	0.32	1.11	0.52	-0.53	0.80	2.11	**3.30**
70	-0.10	0.28	0.66	-0.66	0.27	1.28	**3.02**
80	-1.16	-0.96	0.83	0.27	-0.07	1.16	1.68
90	0.56	-2.40	-0.78	1.23	0.18	0.46	0.95

Table 1.3a: Annualised Rate of Improvement Over Each Five-Year Period, 1931-2001, by Quinquennial Ages, Irish Males

Year	*1931*	*1936*	*1941*	*1946*	*1951*	*1956*	*1961*	*1966*	*1971*	*1976*	*1981*	*1986*	*1991*	*1996*	*2001*
Age															
0	-0.3	-0.3	-0.5	1.6	8.1	4.5	4.5	3.1	5.0	4.5	7.3	3.8	2.5	2.9	1.3
5	0.0	0.0	4.1	8.9	9.3	4.9	4.9	6.0	-3.2	3.0	2.3	6.7	6.9	1.9	10.3
10	2.2	2.2	3.0	2.1	7.6	5.7	5.7	1.8	-1.0	1.6	3.6	3.1	6.7	5.9	0.3
15	0.4	0.4	2.2	2.6	12.8	9.2	-0.6	-0.6	-3.5	2.3	-0.5	2.8	5.9	3.2	-8.8
20	1.4	1.4	0.0	3.4	10.5	10.1	1.6	-1.6	-3.2	0.4	0.0	2.7	-0.2	-1.4	-0.3
25	1.8	1.8	0.5	2.8	8.7	8.9	3.5	0.7	-1.0	-1.4	1.6	3.4	-2.5	-2.5	-0.1
30	1.9	1.9	0.6	3.9	6.4	8.0	3.7	4.0	-1.4	2.2	0.2	0.7	1.9	-8.0	1.7
35	1.6	1.6	1.0	3.8	5.4	6.8	2.0	3.1	4.6	-1.6	0.6	2.0	-4.2	0.9	2.7
40	1.3	1.3	2.1	2.0	5.0	5.7	1.5	0.3	-0.4	3.7	0.1	4.5	2.1	-3.1	1.4
45	0.2	0.2	2.0	1.1	4.2	5.2	1.1	1.8	-3.4	3.3	1.1	2.9	2.9	0.5	0.9
50	-0.4	-0.4	2.1	1.1	2.0	1.9	1.8	0.0	-0.6	0.8	1.6	3.6	3.0	0.8	2.6
55	0.0	0.0	1.3	0.8	1.2	2.5	-0.1	0.9	-0.4	0.5	-0.5	2.7	3.6	1.2	3.5
60	0.1	0.1	0.5	0.8	1.5	0.1	0.9	-1.2	0.1	0.5	1.1	0.6	3.9	3.2	2.9
65	-0.9	-0.9	1.2	-0.1	1.6	1.2	-0.9	-0.6	-0.6	1.0	1.1	0.2	3.2	1.7	4.0
70	-1.2	-1.2	0.9	-0.5	1.1	1.1	0.2	-1.6	0.2	0.3	0.2	0.5	2.0	1.4	4.4
75	0.0	0.0	-1.8	-1.3	1.1	1.0	1.0	-0.1	-0.6	-0.7	0.4	0.4	2.4	0.6	3.4
80	0.7	0.7	-3.0	-1.8	-0.3	0.9	0.9	0.7	-0.2	-0.5	0.3	0.0	2.1	1.0	2.3
85	0.5	0.5	-1.4	-1.6	-2.3	0.2	0.2	0.9	0.8	-0.1	0.2	-0.3	2.0	0.5	1.5
90	-0.2	-0.2	1.3	-1.7	-3.1	-0.8	-0.8	0.8	1.7	0.3	0.1	-0.6	1.9	0.5	-0.3
95	-1.0	-1.0	4.3	-2.3	-3.6	-1.4	-1.4	0.5	2.6	0.6	0.0	-1.0	1.3	-0.5	5.5
95	-1	-1	4.3	-2.3	-3.6	-1.4	-1.4	0.5	2.6	0.6	0	-1	1.3	-0.5	5.5

It is difficult to see a pattern in all the numbers. Table 1.3b is Table 1.3a adjusted by deleting any entries where the mortality improvement over the five-year period is below 3% per annum. A pattern is more apparent in Table 1.3b.

Table 1.3b: Rate of Improvement Over Each Five-Year Period Exceeding 3%, 1931-2001, by Quinquennial Ages, Irish Males

Year	*1931*	*1936*	*1941*	*1946*	*1951*	*1956*	*1961*	*1966*	*1971*	*1976*	*1981*	*1986*	*1991*	*1996*	*2001*
Age															
0					8.1	4.5	4.5	3.1	5.0	4.5	7.3	3.8			
5			4.1	8.9	9.3	4.9	4.9	6.0				6.7	6.9		10.3
10			3.0		7.6	5.7	5.7				3.6	3.1	6.7	5.9	
15					12.8	9.2							5.9	3.2	
20				3.4	10.5	10.1									
25					8.7	8.9	3.5					3.4			
30				3.9	6.4	8.0	3.7	4.0							
35				3.8	5.4	6.8		3.1	4.6						
40					5.0	5.7				3.7		4.5			
45					4.2	5.2				3.3					
50												3.6			
55													3.6		3.5
60													3.9	3.2	
65													3.2		4.0
70															4.4
75															3.4
80															
85															
90															
95			4.3												5.5

Table 1.3b shows the generation born in the early 1930s are showing a step-down in mortality rates compared to previous and subsequent generations. Forecasting this trend forward would predict a fall of 3-4% per annum for those aged 65-70 years in 2001, which would follow them as they age. This rate of improvement at such advanced ages is considerably higher than that observed in the past.

One final method of visualisation of the two-dimensional data by age and year of birth is given in the so-called 'heat-map'.

One would, of course, expect the same cohort pattern to be discernible in female mortality statistics and, as demonstrated in Tables 1.4a and b and the heat-maps in Figures 1.7 and 1.8 (Plates 4 and 5), this appears to be the case.

The cohort pattern for Irish males and females identified above is centred in a calendar year close to that identified in the UK. This is further confirmation of the general similarity of our mortality experiences (see Whelan (2006) for a comparative study, especially with Northern Ireland over much of the twentieth century).

Table 1.4a: Annualised Rate of Improvement Over Each Five-Year Period, 1931-2001, by Quinquennial Ages, Irish Females[6]

Year *Age*	*1931*	*1936*	*1941*	*1946*	*1951*	*1956*	*1961*	*1966*	*1971*	*1976*	*1981*	*1986*	*1991*	*1996*	*2001*
0	0.0	0.0	-0.2	0.9	8.5	4.6	4.6	3.2	4.5	4.5	6.3	4.4	3.5	1.0	2.5
5	1.4	1.4	4.8	4.8	14.1	5.1	5.1	5.9	2.9	1.7	1.8	8.9	-2.2	10.4	3.3
10	0.5	0.5	7.5	7.5	6.1	7.4	7.4	2.3	2.6	1.9	2.2	2.4	5.6	3.6	0.0
15	2.8	2.8	0.4	2.4	11.9	10.7	10.7	0.8	-5.8	5.3	2.5	4.1	-0.9	-1.7	1.4
20	1.9	1.9	-0.4	2.2	12.3	11.4	11.4	5.0	-4.6	2.9	1.8	5.8	-1.3	-1.2	0.0
25	1.6	1.6	1.6	2.2	10.3	11.3	11.3	2.7	-1.0	3.9	6.2	5.0	-6.3	0.0	2.8
30	1.3	1.3	2.8	2.2	9.3	7.9	7.9	4.4	4.0	2.9	3.3	2.6	0.0	-2.1	5.9
35	1.4	1.4	2.9	2.3	6.5	6.0	6.0	7.9	1.9	2.7	3.2	1.5	0.9	0.7	0.0
40	1.7	1.7	1.8	2.2	6.1	5.3	5.3	2.3	1.9	2.0	3.6	3.6	1.1	0.6	0.7
45	1.2	1.2	2.6	1.4	5.1	3.9	3.9	0.9	0.7	1.8	3.2	3.3	0.1	2.7	2.7
50	0.8	0.8	1.1	1.6	5.2	1.7	1.7	1.4	1.0	2.2	2.7	3.5	1.8	1.4	2.4
55	0.9	0.9	1.3	1.9	3.4	2.6	2.6	2.0	0.2	0.9	1.2	2.6	2.9	2.2	2.6
60	0.7	0.7	0.7	2.9	2.5	2.6	2.6	-0.6	0.1	1.4	1.6	1.6	3.5	2.5	3.1
65	-0.5	-0.5	1.6	1.3	3.1	2.0	2.0	2.1	-0.6	0.9	1.2	1.5	2.3	2.2	3.9
70	-0.9	-0.9	1.4	0.8	1.3	2.0	2.0	0.1	2.2	0.8	1.1	1.2	2.2	1.1	4.3
75	-0.1	-0.1	-1.3	-0.6	0.7	1.8	1.8	0.6	1.1	0.8	1.6	2.1	2.9	0.5	3.3
80	0.4	0.4	-2.5	-1.3	-0.1	1.2	1.2	0.8	0.6	0.7	1.6	1.7	2.8	0.4	2.5
85	0.1	0.1	-0.6	-1.5	-1.8	0.4	0.4	0.9	0.5	0.6	1.3	0.5	2.3	0.5	2.1
90	-0.6	-0.6	2.4	-1.7	-3.1	-0.3	-0.3	0.8	0.6	0.6	1.0	-0.6	1.9	0.6	2.0
95	-1.5	-1.5	5.9	-2.2	-4.0	-0.9	-0.9	0.7	0.8	0.5	0.9	-1.4	1.6	0.7	2.0

Table 1.4b: Rate of Improvement Over Each Five-Year Period Exceeding 3%, 1931-2001, by Quinquennial Ages, Irish Females

Year *Age*	*1931*	*1936*	*1941*	*1946*	*1951*	*1956*	*1961*	*1966*	*1971*	*1976*	*1981*	*1986*	*1991*	*1996*	*2001*
0					8.5	4.6	4.6	3.2	4.5	4.5	6.3	4.4	3.5		
5			4.8	4.8	14.1	5.1	5.1	5.9				8.9		10.4	3.3
10			7.5	7.5	6.1	7.4	7.4						5.6	3.6	
15					11.9	10.7	10.7			5.3		4.1			
20					12.3	11.4	11.4	5.0				5.8			
25					10.3	11.3	11.3			3.9	6.2	5.0			
30					9.3	7.9	7.9	4.4	4.0		3.3				5.9
35					6.5	6.0	6.0	7.9			3.2				
40					6.1	5.3	5.3				3.6	3.6			
45					5.1	3.9	3.9				3.2	3.3			
50					5.2							3.5			
55					3.4										
60													3.5		3.1
65					3.1										3.9
70															4.3
75															3.3
80															
85															
90															
95			5.9												

[6] Mortality rates at each age from ILT 1-14 Females and interpolated between census years by assuming that the age-specific mortality rates have same annualised rate of change.

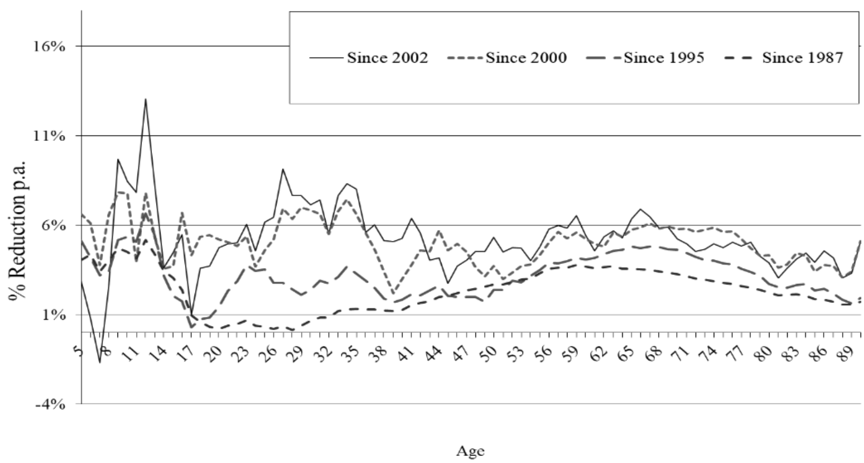
Short-term Trends in Irish Mortality Rates

The long-run trends in mortality rates explored earlier treated periods up to 2002 (Irish Life Table 14). That long-run analysis suggests that we can expect: (i) a continued log-linear pattern of decline of age specific rate; (ii) a continued pattern of 'the aging of mortality improvements', and, in particular, the cohort effect will work itself through the older ages.

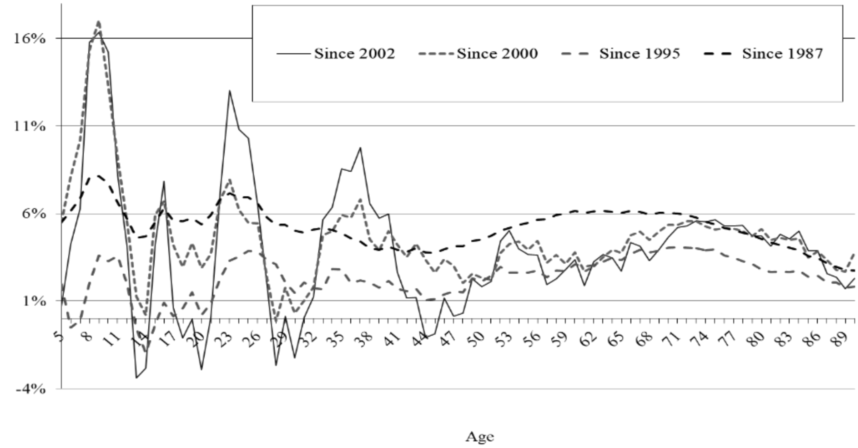
Figure 1.9 sets out smoothed mortality rates by age observed over the three, five, ten and seventeen years ending 2005, for both males and females. First, we note that observed rates of improvements are particularly high over each of the periods, and high across all ages. Annualised rates of improvement across ages and both sexes appear to be averaging 4% or so, which is a considerably higher average rate than observed previously (see Table 1.1). Second, Irish males show a markedly accelerating rate of improvement in recent years over all ages, with the smoothed average improvements over shorter periods higher than over longer periods. For females, no such accelerating pattern of improvement is evident across all ages.

Figure 1.9a and b: Smoothed Annualised Average Rate of Improvement, at Each Age, Over Periods Ending 2005

a. Irish Males



b. Irish Females

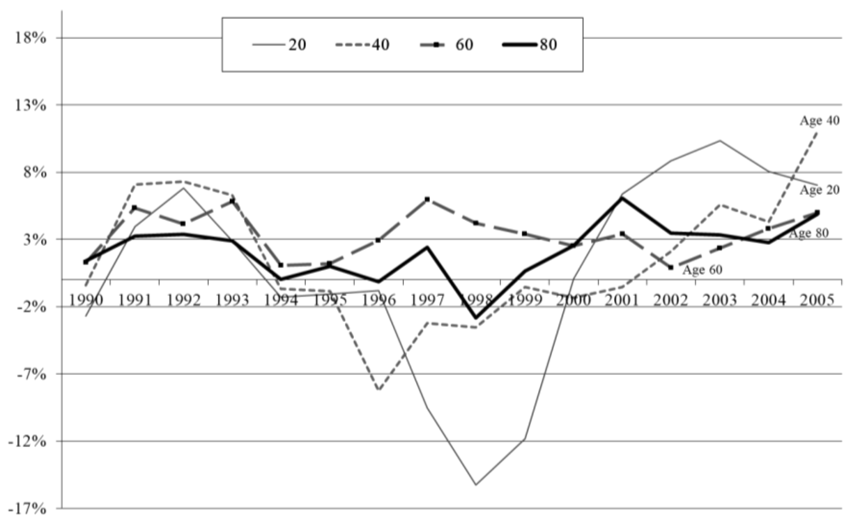


Note: Author's computations based on deaths and estimates of population by age kindly provided by the CSO. Mortality rates in each year calculated by census method with deaths averaged over three years and then averaged over 5 years of age centred at age shown.

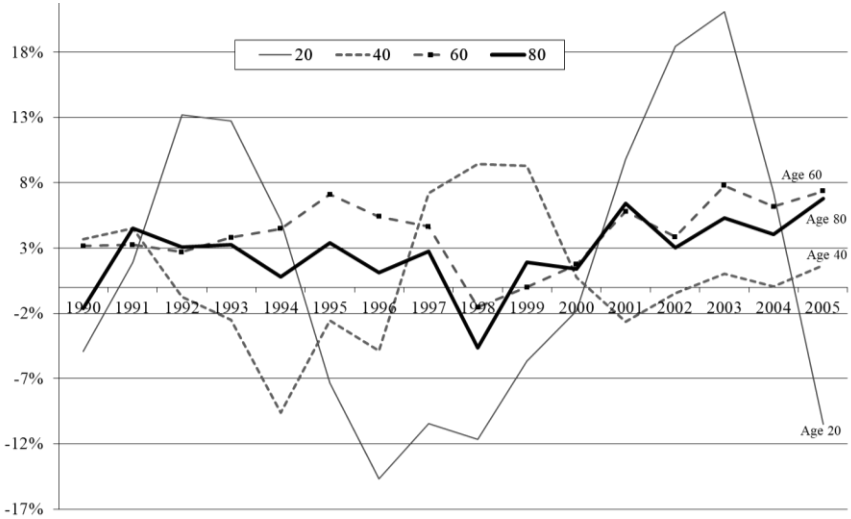
Figure 1.10 takes a closer look at the trend in mortality improvements with time at selected ages.

Figure 1.10a and b: Annualised Average Rate of Improvement Over 3 Years Ending 1990-2005, at Selected Ages

a. Irish Males



b. Irish Females



Note: Author's computations based on deaths and estimates of population by age kindly provided by the CSO. Mortality rates in each year calculated by census method with deaths averaged over three years.

The time series of short-term improvements, even when smoothed, remains quite a noisy series, especially for females, so patterns are not clear-cut. From age 40 years upwards there has been an accelerating rate of improvement since 2002. Rates of improvements in recent years tend to be higher at each age than observed since 1990, except at younger female ages.

The patterns above are not completely consistent with our earlier expectations. First, there has been a deviation from the log-linear pattern of decline by age, as rates in more recent years have declined more than the long-run average. Second, mortality rates across all ages, even the very advanced, have participated in the decline. Third, the above pattern is more marked for males than females. In particular, the expected cohort effect at late ages is not apparent as calendar year improvements, giving a decline across all ages, masks it.

Projecting Mortality Rates

Recent trends in mortality rates highlight the difficulties in making predictions about the future with any confidence. Yet an assessment is necessary, with an appreciation of the associated uncertainty.

We treat in detail two different projection methods: the Logarithmic Method and the Targeting Approach. We analyse the forecasts from each on several different bases. The two methods are popular amongst official forecasters. In fact, the Central Statistics Office applied the first method in its population and labour force forecasts (CSO (2004b)) and this was also the method applied in all papers presented to the Statistical and Social Inquiry Society of Ireland surveyed earlier. This method is known as the 'logarithmic method'.

Projection Method 1: Logarithmic Method

The logarithmic method extrapolates the log-linear trend in age-specific mortality rates observed in the past. As remarked earlier, this has been the most obvious pattern in past rates and remarkably stable over long periods of time.

Let us assume we have the mortality rate at age x in base calendar year 0. Then, by this method, the expected mortality rate at age x in calendar year t years from the base year is given by:

$$q_{x,t} = \alpha^t q_{x,0}$$

Of course, $(1-\alpha).100\%$ is the fixed annualised percentage decline in mortality rates expected in each future year. Taking natural logarithms, we get a linear relationship between the future mortality rate and time:

$$\ln q_{x,t} = t \ln \alpha + \ln q_{x,0}$$

Generally, the term $\ln \alpha$ and therefore the parameter α is determined by least-squares linear regression of the log of past age-specific mortality rates against calendar year (see Figure 1.5 earlier). However, an approximation to α, generally reasonable given the strength of the log-linear trend, is to estimate the annualised rate of improvement over some suitable period in the past, i.e., over n calendar years ending in the base year by

$$\hat{\alpha} \cong \left({}^{q_{x,-n}}\!\big/_{q_{x,0}} \right)^{-1/n}$$

This latter approximation has been employed by the CSO and the previous forecasts published in the *Journal of the Statistical and Social Inquiry Society of Ireland*, so we use it here to estimate α.

Materially, it is necessary is specify what period over the past is most appropriate to use to estimate α. This is not obvious and, as can be judged from its variability with time period (see Figure 1.11: Plate 6), it will significantly affect the projections. Typically, as noted earlier, forecasters used short periods of time despite then making long-term forecasts. CSO

(2004b) used a sixteen-year period, adjusting those rates to zero that were negative.

Both to illustrate the sensitivity of this approach to the past time period used to determine α and to give some measure of the uncertainty surrounding the forecasts based on forecasting a log-linear trend, we set out in Table 1.5 the projected period and cohort life expectancies at birth and at the current common retirement age of 65 years, based on extrapolating the log-linear trend observed to calendar year 2005 over, alternatively, one decade, two decades, five decades and since the first Irish Life Table in 1926.

Table 1.5: Projected Period and Cohort Life Expectancies (LE), Irish Males and Females, Log-Linear Trend Extrapolation

	Based on log-linear trend over *n* years ending 2005, where *n*=	**Period LE in 2021**		**Period LE in 2041**		**Cohort LE in 2006**	
		Age 0	**Age 65**	**Age 0**	**Age 65**	**Age 0**	**Age 65**
Males	10 Years	82.20	20.37	87.42	23.37	93.40	19.59
	20 Years	80.85	19.29	85.08	22.50	90.78	18.68
	50 Years	78.95	17.70	81.34	19.27	84.52	17.42
	Since 1926 (79 Years)	78.48	17.10	80.21	17.90	81.63	16.86
Females	10 Years	85.51	22.83	89.65	26.18	94.94	22.62
	20 Years	84.74	22.16	88.31	25.01	93.47	21.99
	50 Years	83.92	21.26	86.58	23.16	90.09	21.05
	Since 1926 (79 Years)	83.29	20.58	85.18	21.70	87.15	20.33

The table reports the projected values of two different types of life expectancy, period and cohort. The period life expectancy is derived from the life table constructed from the mortality experience in the indicated calendar year, i.e., it represents the expected number of years until death of an individual subject to the mortality rates of the life table constructed in that year. This is somewhat of a theoretical concept as the life table is a mixture of the mortality experience of different generations — that of a 70-year-old born 70 years ago and that of a 7-year-old born just 7 years ago. In order to compute cohort life expectancies, one must allow for future mortality improvements. Cohort life expectancy at age x is the expected number of years that a person aged x in the given calendar year will live, and factors in the expected change to mortality rates — so it factors in mortality improvements over the n years until the person is aged $x + n$. Accordingly, the cohort life expectancy is the pertinent life expectancy for personal planning. In order to compute the cohort life expectancy at age 0 it is necessary to project mortality improvements over more than a 100-year period.

Let us focus on the cohort life expectancy for a child born in 2006. First, we note, the longer the period in the past used to determine the projected trend, the lower the life expectancy computed for both males and females. This is a consequence of the accelerating trend in mortality improvements observed, which is more marked for males. Second, we note that cohort life expectancies for males differ by as much as a dozen years, depending on the past period used to determine the trend.

The principal problems in using the logarithmic method can be listed as:

1. The choice of past period is somewhat arbitrary but has a key impact on the results. A general rule is that the longer past period used, the lower the trend rate of improvements forecast. This is because of the accelerating trend of improvement seen in recent years, especially at the older ages.
2. It makes no attempt to forecast the 'cohort' pattern of mortality improvement evident in the past.
3. It produces a discontinuity in (the first derivative of) mortality rates with time, as current rates are projected to make a step change in the first year of the forecast to the long-term average rate.
4. If age-specific rates of improvement are estimated and projected for each age separately, then the method is likely to produce projected (period) mortality tables that do not progress smoothly with age and where, in fact, the monotonic increase in mortality rates at later adult ages is not always predicted. One way to overcome this problem is to smooth age-specific rates of improvement but has generally not been done to date.

Projection Method 2: Targeting Approach

A 'targeting approach' has been used by, inter alia, the UK Government Actuary's Department (GAD) in forecasting mortality in their national population projections and, in particular, in projecting the age and sex composition of the population of Northern Ireland to 2074 (GAD (2006b)). The targeting approach has three distinct components: (a) an estimate of short-term mortality trends by age and sex; (b) a judgement of the long-term rate of improvement at each age and sex from some target year in the future; and (c) interpolating in some manner between the observed short-term trend now and the long-term trend assumed from the target year.

The GAD takes the target year to be 25 years from the base year of the projections, anchor expectations about the long-term rate of improvement from the target year by the rates of improvement observed over the long-term past (i.e., the entire 20th century), and interpolate between the current short-term trend and the assumed long-term trend rates of improvement in a linear manner, but explicitly allowing for a cohort effect at later ages. The methodology is critically reviewed and compared with other projection methods in GAD (2001), including an evaluation of how the different methods performed in the past. Methods evaluated include the logarithmic, logit, Lee-Carter (modified and stochastic), as well as the various methodologies employed by the Continuous Mortality Investigation Bureau of the Faculty and Institute of Actuaries. The report concludes "…the clear conclusion was that there were no grounds for believing that an alternative methodology would be likely to outperform the present method" (p.109) and that a key strength of their methodology is its ability to incorporate the cohort effect. However, the targeting approach as employed by the GAD requires two key inputs from the user — the target date and the assumed rate of improvement in mortality rate from that target date — and the results are obviously very sensitive to these user-determined assumptions. In fact, in applying this method over recent years, the GAD has been revising upward the long-term trend assumed. Even the currently assumed 1.0% per annum rate of decline across all ages and both sexes from 2029 — a rate close to the average rate of improvement over 20th century — does not get the endorsement of their expert advisory panel who consider it too low (see Appendix III of GAD (2006a, p. 79)).

The targeting method does, though, overcome most of the drawbacks of the logarithmic method: it can readily be adapted to allow for cohort effects, its projections tie-in smoothly with current trends, and it can produce period mortality tables with the right shape in each future year.

In fact, the forthcoming population and labour force projections of Ireland (CSO (2008))[7] adopt a targeting method. The approach was similar in principle to the GAD approach but somewhat different in detail, and more importantly, in the parameter adopted for the long-term rate of improvement. So, like the GAD approach, mortality rates were forecast by estimating the current rate of improvement for each sex at each age and assuming that the current rate of improvement will decline over a twenty-five-year period to a long-term average improvement rate not dissimilar to that observed rates in the long-term past. It turned out with the Irish data that the current rate of decline of mortality for males averaged at 5% per annum across most ages, with surprisingly little variation. For females, the current rate of decline oscillated with age about an average rate of 3.5% per annum. It was judged reasonable in the estimates for the long-term future to apply the same rate of decline to male and female mortality rates and, on discussion, a long-term rate of 1.5% per annum was settled upon as not unreasonable for all ages up to age 90 years after calendar year 2031. The remarkably stable level of current improvements for each sex across most ages mean that future projections can be interpreted as either cohort projections or calendar year projections, with the rate of improvement smoothly decaying by calendar year or year of birth at the same rate. It was assumed, because of the paucity of data, that there would be no mortality improvements at ages of 100 years upwards. For each year between 2005 and 2031, the mortality declines for that year were calculated by linear interpolation. For each age between age 90 years and 100 years, the rate of mortality decline for that age was estimated by linear interpolation in each future calendar year. The approach, in effect, assumes that the secular and cohort effects begin to decay after age 90 and by age 100 no improvement in mortality rates occur. Box II sets out, step-by-step, how the target method was applied to project Irish population mortality from calendar year 2005, together with a brief rationale.

Table 1.6 below sets out the results of applying the targeting method and gives an indication of the sensitivity of the projected life expectancies to the assumed parameters.

[7] The author is a member of the expert advisory panel and produced, on the agreed basis, the mortality forecasts.

Table 1.6a and b: Projected Period and Cohort Life Expectancies (LE), Targeting Approach

a. Irish Males

Parameter	*Period LE in 2021*		*Period LE in 2041*		*Cohort LE in 2006*	
	Age 0	*Age 65*	*Age 0*	*Age 65*	*Age 0*	*Age 65*
Central Projection Basis	83.1	21.1	86.5	23.7	91.0	20.6
Initial Decline Up 1.0% p.a.	84.1	21.8	87.5	24.5	91.8	21.2
Initial Decline Down 1.0% p.a.	82.1	20.3	85.4	22.9	90.1	19.8
Long-term Decline Up 0.5% p.a.	83.4	21.3	87.6	24.6	93.0	20.8
Long-term Decline Down 0.5% p.a.	82.9	20.9	85.4	22.9	88.6	20.3
					88.7	

b. Irish Females

Parameter	*Period LE in 2021*		*Period LE in 2041*		*Cohort LE in 2006*	
	Age 0	*Age 65*	*Age 0*	*Age 65*	*Age 0*	*Age 65*
Central Projection Basis	85.5	22.9	88.2	25.1	92.5	22.7
Initial Decline Up 1.0% p.a.	86.3	23.4	89.1	25.6	93.1	23.2
Initial Decline Down 1.0% p.a.	84.6	22.1	87.4	24.3	91.9	22.1
Long-term Decline Up 0.5% p.a.	85.72	22.9	89.2	25.8	94.3	22.9
Long-term Decline Down 0.5% p.a.	85.2	22.5	87.3	24.2	90.4	22.4

On the central projection, the life expectancy of a female born in 2006 is 92.5 years, while it is 91 years for a male. A female aged 65 years in 2006 is projected to have a future life expectancy of 22.7 years, while it is 20.6 years for a male on the central projection basis.

The rule-of-thumb from the above table is that a change in the current rate of decline by 1%, will change projected period life expectancies by 1 year at age 0 and 0.7 years at age 65 (i.e., by 1.2% and 3% respectively). The change in cohort life expectancy in 2006 is about the same for 65-year-olds but less at age 0 at 0.8 of a year (0.9%). A change to the long-term rate of decline has a bigger influence the longer the projected period and, in particular, cohort life expectancies at younger ages are particularly sensitive to this assumption.

If we assume that the long-term rate of improvement is 3% per annum (rather than the 1.5% per annum in the central projection) then the figures in Table 1.6 suggests this change will increase the cohort life expectancy of a male born in 2006 to about 97 years (in fact, the actual answer is 95.9 years) and that of a female to 97.9 years (the actual answer is 96.7 years).

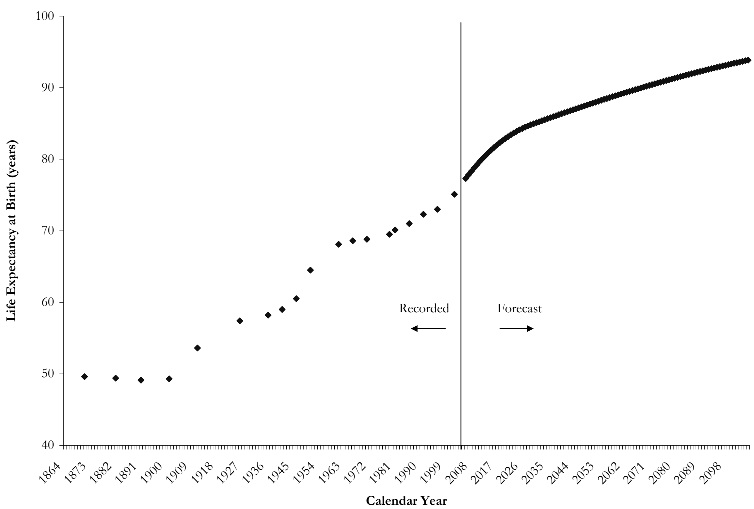
Conclusion

We have analysed the patterns, both long-term and short-term, in the mortality experience of the Irish population. We identified a cohort effect, where those born in the 1930s have been experiencing a step-down in mortality rates as they age. The trend in mortality improvements has steepened significantly in more recent years. In particular, improvements are now being observed at the more advanced ages.

We briefly overviewed different methods of projecting mortality rates and applied two of the more popular to project Irish population mortality, each on several different bases. There is considerable uncertainty in the projections, as illustrated in Tables 1.5 and 1.6. However, it is not unreasonable to conclude, in answer to the question posed in the Introduction, that children born in 2006 can reasonably expect to live to their nineties — the early nineties for males and mid-nineties for females.

Figure 1.12 sets out the recorded (period) life expectancy for a male born in Ireland from 1871. The forecasted life expectancy graphed is based on the central projection basis of the targeting approach, as adopted in Ireland's Central Statistics Office (2008), *Population and labour force projections 2011-2041*.

Figure 1.12: Recorded and Forecast (Period) Life Expectancy at Birth for Males in Ireland



We conclude by pointing out the growing need for better estimates of mortality rates at ages above 85 years, as current methods employed are not in accordance with best international practice.

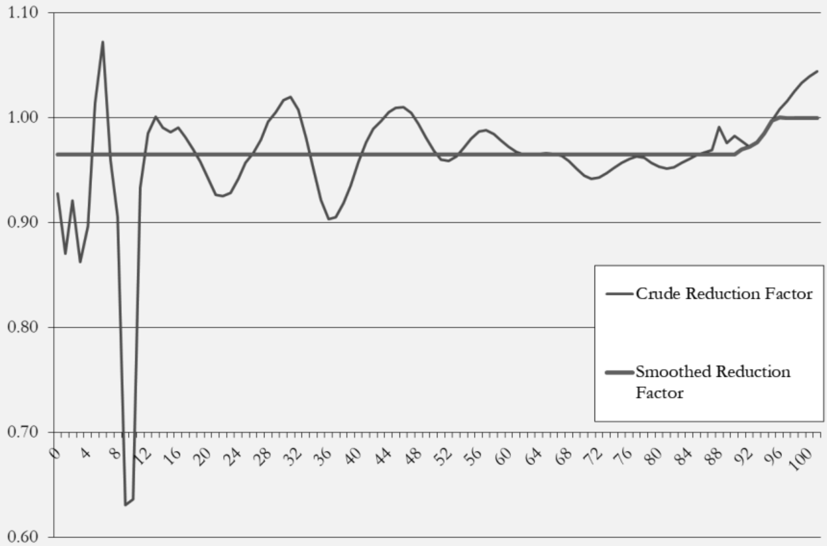
Box II: Steps in Forecasting Irish Population Mortality by Target Method

1. Recent graduated life tables are prepared for each sex, separated by a short number of years. We used Irish Life Tables 14 (corresponding to morality experience 2001-2003, so centred in 2002) and, applying the same method, a graduated table was prepared for the experience 2004-2006, so centred in 2005.
2. The annualised percentage fall in mortality at each age for each sex was calculated from the graduated rates. This gave the average rate of improvement per annum over the three-year period 2002 to 2005. These rates are then expressed as a reduction factor (RF), i.e., unity less the annual percentage rate of decline.
3. Figure 1.13 graphs these crude reduction factors by age. We notice that there are large fluctuations at the early ages but from age 11 to 90 or so the RFs tend to oscillate about 0.95 for males and, with somewhat greater amplitude, about 0.965 for females. In fact, the mean of the crude RF from age 0 to 90 is 0.949 for males and 0.957 for females and from age 11 to 90 the means are 0.950 and 0.967 respectively.
4. Most — almost all — projection methods in common use do not attempt to smooth the crude RF. This can lead to inconsistencies in the projections, with, say, age $x+1$ having lower mortality than age x at some point in the future or, in general, the projections producing a very oddly shaped mortality curve. To avoid this, we smoothed the RF factors across ages. In fact, we adopted a very strong smoothing approaching by essentially replacing the RFs up to age 90 with their average rate. The smoothed RF was 0.95 for males and 0.965 for females, for all ages up to age 90 years. The ultimate justification for the strong smoothing adopted is that life expectancies at ages 0 up to age 65 years showed no significant differences in each future year whether the unsmoothed RF factors or the strongly smoothed factors were used. From age 100, we assumed the RF was 1 (i.e., no reduction) for both sexes.
5. The smoothed RF factor was assumed to apply to mortality rates between calendar year 2004 and 2005.

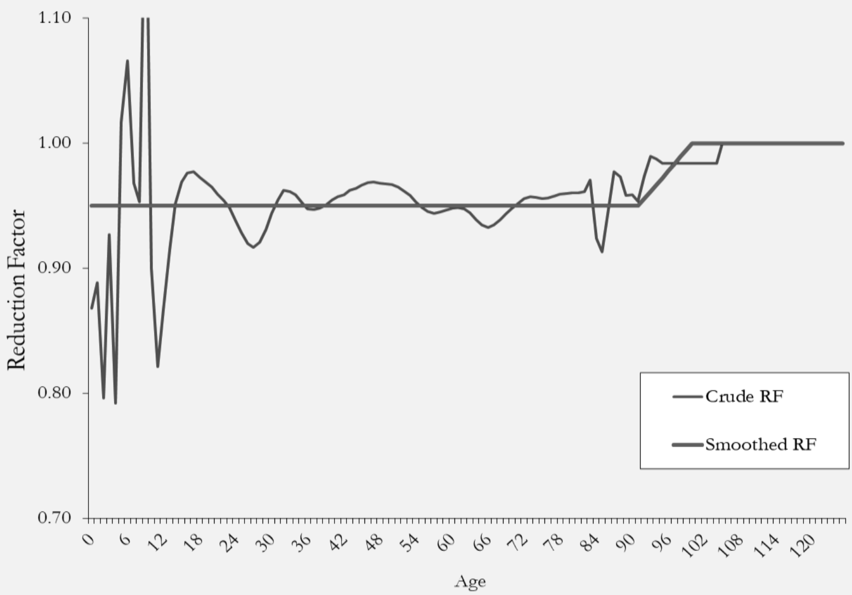
6. In 25 years from 2005, (that is, from calendar year 2031) the reduction factor in any one year is assumed to be 0.985 for both males and females. This is justified on the basis that it is close to the average mortality improvement over the long-term past and, as such, might be reasonably assumed in the long-term future. The average annual rate of improvement over the 76 years, 1926 to 2002, was 1.4% for males (estimated as a simple average of rates of improvement at each age from 0 to 100 over that period). For females, the correspondingly average annual rate was higher at 2.1%. A simple average over the period would suggest a long-term rate of decline of about 1.75% p.a. The selection of a rate is difficult and eventually a rate of 1.5% p.a. was settled on as it is close to the long-term trend and, when the other elements of the approach are included, produces estimates of period life expectancies in 2041 not very dissimilar to the last official projection.
7. In summary, the projection methodology is to assume that in any calendar year after 2031, the reduction in mortality over that year is 1.5% (i.e., apply the reduction factor of 0.985 to the previous year's rates). In 2004 to 2005 the smoothed RF is assumed to apply, so 0.95 for males and 0.965 for females. For all years between 2005 and 2031, the RF for that year is a simple linear interpolation between these two extremes. The base table that the resultant cumulative RFs are applied to is the graduated life table for each sex, centred at calendar year 2004.

Figure 1.13: Reduction Factors for Females and Males, Crude and Smoothed

Females



Males



Chapter 2

Should the Mortality Projection Model Incorporate a Cohort Effect?

Abstract

Should the Mortality Projection Model incorporate a cohort effect? Probably not back in 2008 and definitely not in 2022.

Introduction

Proponents of cohort modelling argue that, as each cohort's life experiences have been so different, population mortality is best projected on a cohort basis. Accordingly, the model must incorporate a year of birth parameter alongside parameters for age and calendar year. The hypothesis finds support in studies of the mortality of individuals suggesting that acute exposure to inimical conditions early in life can manifest later in life through elevated mortality (see Barker et al. (1989), Barker (1994), Barker (1998), Barker (2001), Finch and Crimmins (2004), Gluckman and Hanson (2006), Gluckman et al. (2009)).

The cohort effect in populations has been a major theme explored by actuaries in the UK over the last quarter-century (see, for example, Willets (2004), Willets et al. (2004), Richards et al. (2006), Richards et al. (2007)). In fact, such a cohort analysis and its rationale was first proposed as early as 1927 by the actuary Victor Derrick (Derrick (1927)). Population mortality projections in the UK and mortality projections for many actuarial purposes have incorporated the so-called 'cohort effect' for two decades now although, curiously, the cohort effect does not seem to be manifest in many countries outside of the British Isles and Japan.

Adding a parameter for year of birth as a proxy for cohort effects (even with the restriction of some level of smoothness from one birth year to the next), alongside the usual age and calendar year parameters, runs the risk of over-parameterising the mortality projection model. Over-parameterised models produce spuriously good in-sample fits, unstable projections, and confidence intervals associated with the projections that are too narrow. Accordingly, the introduction of another factor for each cohort is resisted by some who remain unconvinced that the evidence is sufficiently compelling to move from the parsimony of the traditional model with just age and calendar year parameters.

The Cohort Effect in the UK

Three important observations must be made about the cohort effect in the UK. First, we must acknowledge, as the original discoverer of the UK 'cohort effect' pattern did, that it was observed in the data during an exploratory analysis before it was hypothesised (Office of Population Censuses and Surveys (1995)). To use the data to suggest the hypothesis to test on the same data is not statistics but data-mining and, accordingly, the p-value of such tests must be adjusted significantly. In short, one cannot rely on the reported statistical significance of the effect.

Second, the so-called cohort effect is telling us little that we did not already know. Mortality improvements, in the UK and elsewhere in the developed world, were well-known to be concentrated at the early ages early in the twentieth century and, from about the 1970s mortality improvements began to be detected at later ages (a pattern described as the 'aging of mortality improvements' (Wilmoth (1997)). The new insight from the cohort effect is then to draw a line from young adult ages to late middle ages and suggest the uniqueness of the mortality experience of particular cohorts. But the main causes of death in young adults and early middle age are accidents and injuries, so it is difficult to make a plausible case that trends in mortality over these ages are linked to year-of-birth rather than age. For the 'golden generation' born around 1931, their twenties were in the austere and rationed 1950s which did not afford the same opportunities for such risk exposure — English Life Table 12

(Table C, p. 6) shows the relative fall in mortality across ages for both males and females was highest at age 25 years over the 1950s.

Third, there are many potential causes of the observed effect in the UK and some are best projected using the traditional calendar year approach. Murphy (2009), for instance, challenges the 'cohort effect' and points out that the pattern observed in UK mortality statistics is not incompatible with a period effect. Using stylised modelling, he attempts to identify the most likely candidate from the five main hypotheses that have been advanced to date, ranging from the simple suggestion of a change in smoking patterns within the population over time to the more nebulous hypothesis that it arises from medical and welfare advances. Though more work is needed, it appears that the observed 'cohort effect' might simply be accounted for by changing smoking patterns and that, rather than a 'golden generation', what we are observing could be simply a pattern in population mortality arising when one generation is succeeded by another generation comprising fewer smokers. In short, Murphy (2009) suggests that the observed 'cohort effect' is not reflecting individual mortality patterns (i.e., it does not reflect changing mortality of the typical person in the population) but the changing mix of two separate populations within the overall population that have a material mortality differential.

The Cohort Effect Outside the UK

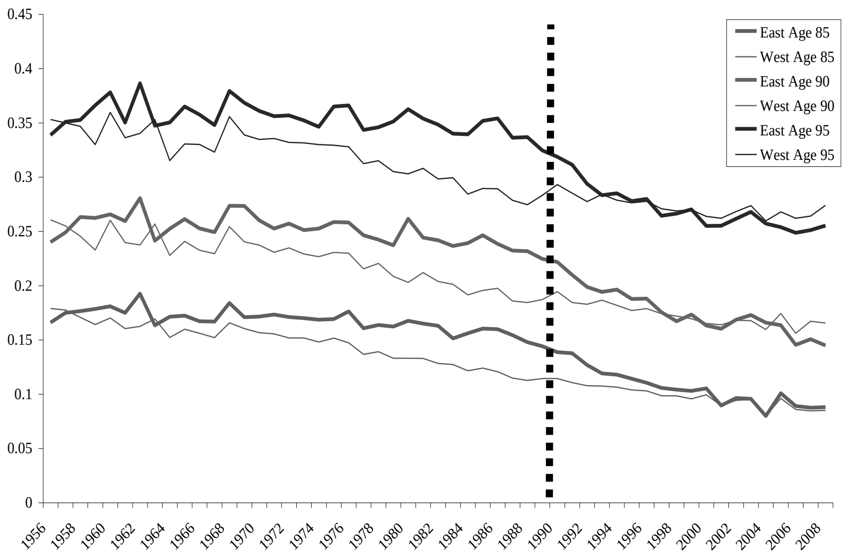
It is potentially a grave error to generalise from a unique set of circumstances to build models for future mortality that too closely capture past mortality patterns. Observations from mortality patterns over diverse environments suggest that the extra year-of-birth parameter has a low level of explanatory power in models of population mortality. Outside of the UK and Japan, the suspicious lack of a cohort effect in other developed economies must also be explained.

Two case studies might be cited to caution the introduction of the year-of-birth parameter alongside age: (i) the convergence between East and West German mortality rates following reunification; (ii) the difficulty in detecting lasting impact on mortality after the population experience an acute trauma such as in Ireland after the Great Famine. It is worthwhile to elaborate on these case studies.

Case Study 1: Mortality Rates after Reunification of Germany

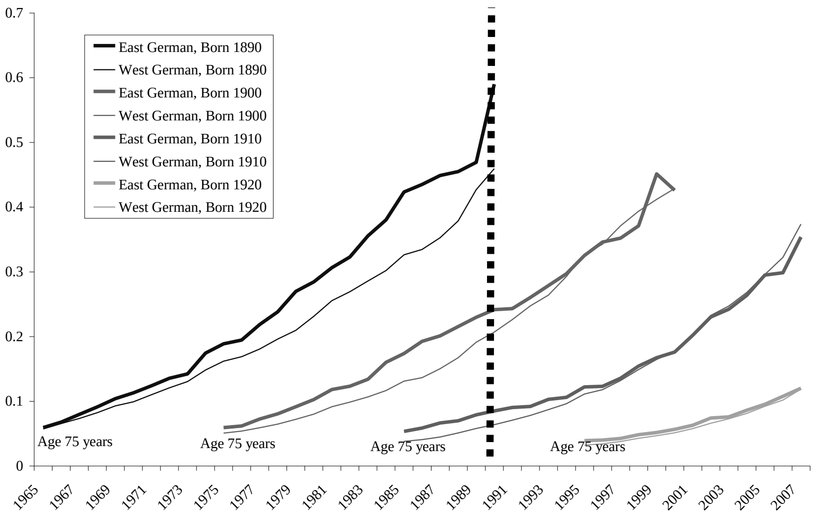
The key observation (originally made in Vaupel, Carey and Christensen (2003)) comes from the rapid convergence of mortality rates at advanced ages following the reunification of Germany in 1990. Figure 2 shows the convergence of mortality rates at selected advanced ages by calendar year and Figure 3 highlights the convergence of mortality rates between different birth cohorts after reunification. These figures make the case that mortality differentials need not persist even at advanced ages, even after a near-lifetime lived under quite different conditions with quite different mortality pertaining. Age and calendar year effects appear to dominate with apparently no lasting effect of past lifetime.

Figure 2.1: Convergence of Mortality Rates at Advanced Ages following German Reunification in 1990: Mortality Rates at Fixed Ages Tracked over Calendar Years[1]



[1] Mortality rates sourced from mortality.org.

Figure 2.2: Convergence of Mortality Rates at Advanced Ages following German Reunification in 1990: Mortality Rates Tracked by Cohort (Fixed Year of Birth)[2]



So, from the German experience and controlled studies in other species (e.g., Mair et al. (2003), Carey (2003)), one could form the hypothesis that a chronic exposure to a relatively poor environment appears to have no lasting effect (after a select period) once the subject is removed from that environment. The hypothesis stated in its most general form applies equally to, say, smoking, poor diet, or alcohol abuse (although the select period will vary for each).

We need further evidence to support the hypothesis of the transitory impact on mortality risk of deleterious conditions. To my mind, evidence is provided in the difficulty of detecting a lasting impact on population mortality after the population experiences an acute trauma such as a famine. Research reports that cohorts that were born in famine conditions, such as the Finnish Famine of 1866–68 or the Siege of Leningrad in 1941–43 or the Dutch Winter of Hunger in 1944, do not have appreciably higher mortality later in life (see Murphy (2009) for a

[2] Based on mortality rates sourced from www.mortality.org.

review). We can add to that list the Great Irish Famine of 1845–49 based on the following evidence.

Case Study 2: Mortality Rates after the Great Irish Famine

The Great Irish Famine was one of the longest and worst famines in modern times (Ó Gráda (2007), pp. 19–20 and Table 3), with an excess death rate of 12%, compared to, say, the 7% excess death rate of the Finnish Famine of 1866–68. During famine times, starvation is only a relatively minor cause of death, the major causes being dysentery, diarrhea, fevers and consumption. The whole population is exposed to some degree to inimical mortality influences during a famine time (for causes of death in Ireland during the Famine, see Mokyr and Ó Gráda (2002), Ó Gráda (2007)). Mortality rates during the famine years in Ireland were approximately double that during the pre-famine years, a relationship that holds reasonably stable across both age and sex for those over 4 years of age (Boyle and Ó Gráda (1986)). Accordingly, famine times provide a grim test to see if exposure to such deleterious conditions at young ages has a discernible lasting impact on mortality.

Ireland introduced compulsory registration of deaths from 1864 and had reasonable accurate censuses every decade or so. It is therefore possible to get a good estimate of mortality rates at around the census of 1871 and subsequent censuses. Table 1a and 1b show mortality rates in the area now identified as the Republic of Ireland. Rates in bold with shaded background down a diagonal highlight the mortality rates of those who were born during the Famine or were just one year of age at the start of the Famine. A study of the table reveals that the cohorts born just before or during the Famine had, in later life, mortality very similar to the cohorts before and after them. This evidence indicates that exposure to famine conditions early in life appears to leave no trace in later life mortality statistics at the population level.

Table 2.1a: Mortality Rates of Females in Ireland, Post Great Famine[3]

	Calendar Year					
Age	1871	1882	1891	1901	1911	1926
7	0.0051	0.0048	0.0042	0.0047	0.0037	0.0025
12	0.0033	0.0033	0.0035	0.0036	0.0029	0.0020
17	0.0048	0.0050	0.0054	0.0054	0.0045	0.0038
22	**0.0058**	0.0063	0.0065	0.0061	0.0054	0.0049
27	**0.0072**	0.0077	0.0081	0.0073	0.0064	0.0058
32	0.0083	**0.0089**	0.0092	0.0085	0.0073	0.0060
37	0.0088	**0.0094**	0.0094	0.0095	0.0081	0.0068
42	0.0096	0.0104	**0.0103**	0.0107	0.0093	0.0080
47	0.0106	0.0120	**0.0120**	0.0120	0.0107	0.0095
52	0.0136	0.0159	0.0162	**0.0157**	0.0145	0.0132
57	0.0187	0.0225	0.0233	**0.0220**	0.0213	0.0192
62	0.0291	0.0338	0.0354	0.0331	**0.0284**	0.0274
67	0.0467	0.0515	0.0543	0.0505	**0.0350**	0.0375
72	0.0699	0.0752	0.0799	0.0751	0.0492	0.0543
77	0.1006	0.1068	0.1143	0.1098	0.0742	**0.0806**
82	0.1382	0.1487	0.1568	0.1551	0.1124	**0.1157**
87	0.1842	0.2046	0.2089	0.2143	0.1703	0.1622

Table 2.1b: Mortality Rates of Males in Ireland, Post Great Famine

	Calendar Year					
Age	1871	1882	1891	1901	1911	1926
7	0.0051	0.0044	0.0037	0.0037	0.0032	0.0019
12	0.0029	0.0027	0.0026	0.0026	0.0022	0.0012
17	0.0050	0.0046	0.0044	0.0044	0.0037	0.0027
22	**0.0078**	0.0073	0.0073	0.0068	0.0055	0.0039
27	**0.0087**	0.0084	0.0088	0.0083	0.0067	0.0039
32	0.0091	**0.0091**	0.0096	0.0094	0.0075	0.0045
37	0.0093	**0.0095**	0.0100	0.0100	0.0080	0.0050
42	0.0105	0.0110	**0.0112**	0.0113	0.0093	0.0062
47	0.0126	0.0136	**0.0132**	0.0133	0.0115	0.0087
52	0.0160	0.0179	0.0172	**0.0171**	0.0155	0.0122
57	0.0205	0.0240	0.0232	**0.0226**	0.0218	0.0182
62	0.0301	0.0344	0.0339	0.0324	**0.0291**	0.0277
67	0.0467	0.0502	0.0508	0.0475	**0.0371**	0.0410
72	0.0697	0.0728	0.0752	0.0707	0.0524	0.0630
77	0.1012	0.1047	0.1104	0.1052	0.0782	**0.0988**
82	0.1409	0.1498	0.1552	0.1528	0.1178	**0.1402**
87	0.1906	0.2144	0.2116	0.2188	0.1784	0.1860

[3] The author's calculations based on official death and population counts in the area now constituting the Republic of Ireland. King's method was used to estimate mortality rates at each age (King (1909)) because of pronounced age rounding (see Chapter 3). The ages shown are generally 'pivotal' ages under the method but, where necessary, oscillatory interpolation was used.

Conclusion

The evidence available in 2008 suggested that introducing a parameter for each year of birth alongside age and calendar year in mortality projections models was not necessary. The resultant three parameter model would too prolific, too much opposed to Occam's dedicate. It could reasonably be held that mortality improvements in the past relate to age and calendar year alone. Therefore, there is no need — and much danger — in incorporating birth year into mortality projection models.

Recent mortality data in the UK in 2017 shows that the "golden cohort" in the UK that gave rise to the cohort hypothesis no longer experiences significantly higher rates of improvement than other generations. The official UK mortality projections no longer project by cohort for this group (ONS (2017a) and Chapter 5).

Chapter 3

Mortality in Ireland at Advanced Ages, 1950-2006: Crude Rates

Abstract

We examine the data and techniques underlying the estimation of mortality rates at older ages in Ireland since 1950. Previous attempts to elucidate the level and trends in mortality at advanced ages in Ireland have been frustrated by significant non-random biases arising from age exaggeration and age heaping, together a lack of correspondence, growing with increasing age, between the exposed-to-risk estimated from census data and the death count from registration data. Applying the method of extinct generations, we re-estimate crude mortality rates and report the somewhat unexpected result that mortality rates were lower, and did not increase as steeply with age, than those recorded in the official Irish Life tables. The re-estimated crude rates show, for both sexes, a very slight decrease in mortality rates between the 1950s and 1980s up to age 90 years, with no improvement discernible at older ages. Improvements at advanced ages in Ireland have lagged those in England and Wales and other developed countries over the same period. The next chapter graduates the crude rates and extends the method of extinct generations to estimate mortality rates of more recent, still surviving, generations.

Introduction

The general trend of mortality in Ireland over the last century follows the well-known pattern for developed nations. Mortality rates at all ages have fallen, with the greatest proportional improvement at the lower ages and being greater for females than males (Chapter 1, Whelan (2008)). In more recent years, mortality declines at older ages have been even greater than those observed at younger ages, a secular pattern described as the "aging of mortality improvements" (Wilmoth (1997)). This more recent development may be related to the co-called "cohort effect" evident in Irish, UK, Japanese and other national mortality statistics (Whelan (2008), Willets (2004), Willets et al. (2004)).

Falling mortality at younger ages has shifted the expected age at death upwards so now over three-fifths of females and over two-fifths of males in Ireland can expect to live beyond 80 years of age, according to Irish Life Table 14 (reflecting the mortality experience in 2001-2003). However, the data underlying mortality estimates for Ireland at ages of 80 years and over fall below international standards of "good quality" (which includes Scotland, England and Wales and most EU countries) and even "acceptable quality", being classed as "conditionally acceptable quality" (Kannisto (1994)). Irish data is believed to be biased in a manner that understates mortality at the more advanced ages: "these data [for Ireland] give probably a roughly correct description of the mortality trend though at a level artificially lowered by age overstatement" (Kannisto (1994), Section 2).

In this chapter we examine the data since 1950 and re-estimate mortality rates at advanced ages in Ireland. Using the method of extinct generations, we provide estimates of Irish crude mortality rates from 1950, for both males and females at ages of 75 years and upwards. We report the somewhat unexpected result that mortality rates were, in fact, lower than those recorded in the official Irish Life tables. The shape of the curve at advanced ages is also different to that recorded in the official tables, with the rate of increase in mortality rates decelerating more markedly with age.

The layout of the chapter is as follows. The section following this introduction summarises the level and trends in mortality at advanced ages in Ireland as reported in the official Irish Life tables, which are regarded as the best estimate to date. Quite at odds with the generally accelerating decline in rates observed in England and Wales since the 1930s, Irish rates show no trend improvement until the 1990s.

The next section after critiques the official Irish mortality record, systematically itemising each type of error than can occur and discussing its significance in the Irish context. We report and appraise the significance of (i) age heaping in census counts; (ii) age heaping in reported age at death, which is still discernible; (iii) age exaggeration in census returns, especially at age 100 years and over; (iv) errors in estimating the crude mortality rates due to a lack of correspondence between deaths and exposed-to-risk, which grow in significance with advancing age; (v) the importance of random error, heterogeneity in underlying rates, and stochastic variation in underlying mortality rates; and, materially for official Irish rates as they are estimated by curve fitting to grouped data, (vi) the inconsistency created by employing different models to graduate and extrapolate mortality rates at advanced ages from one life table to the next.

Then the next section considers an alternative method to estimate crude mortality rates at late ages, known as the method of extinct generations, an extended version of which has been employed to estimate crude mortality at advanced ages in the Kannisto-Thatcher Database on Old Age Mortality maintained by Max Planck Institute for Demographic Research and has been employed in the estimation of rates in England and Wales since English Life Table 15 (1990-1992). It is shown that the method of extinct generations is preferable to using census data for estimating the exposed-to-risk, not because it avoids the well-documented problems with age exaggeration at censuses (as previously maintained by, for instance, Thatcher (1987)), but because the method achieves a perfect correspondence between the exposed-to-risk and death count. As an aside, in Box II, we provide a better estimate of the number of centenarians in Ireland over the last half century and note the longest lived in Ireland.

The section after uses the method of extinct generations to estimate crude Irish mortality rates and their trends. The levels and trends revealed are compared with the recorded official rates and, in the following section, with international rates from the 1950s to the 1980s.

It is shown that the mortality rates lie below the official recorded rates, but their secular decline lags that in other developed countries. In particular, we report no discernible improvement in crude mortality rates above age 90 years. The last section outlines the conclusions from our investigations.

The companion paper and next chapter, *Mortality in Ireland at Advanced Ages, 1950-2006: Part 2: Graduated Rates,* graduates the crude rates using different curves, reporting that the curve giving the best fit is Kannisto's version of Perks's Law, evaluates various approaches to extend the method of extinct generations so mortality rates for non-extinct generations can be estimated, and concludes that a modest trend of improvement in male and female mortality at advanced ages is evident in Ireland over the last five decades but that the rate of improvement lags those evident in England and Wales.

Pattern of Irish Mortality at Advanced Ages from Irish Life Tables

Irish life tables have been published by the Central Statistics Office (and its forerunner), outlining the mortality experienced by males and females in Ireland around each census year. In total, fourteen censuses have been taken in Ireland between 1926 and 2002 and, accordingly, there are fourteen Irish Life Tables (ILTs) for each sex, numbered 1 to 14. Censuses have not been evenly spaced, but have generally been taken every five years, Summary mortality statistics prior to 1926 for the area now in the Republic of Ireland have been published in the *Report of the commission on emigration and other population problems, 1948-1954*.

The life expectancy at age 75 years computed from the official Irish Life tables is summarised in Figure 3.1 for Irish males and females. Life expectancies at age 75 years have been remarkably stable since the first Irish Life table in 1926, with a rapid improvement evident only in very recent times. To better understand trends in mortality rates at advanced ages over this period, Table 3.1 shows the recorded mortality rates for males at selected ages over time periods separated by approximately a decade.

Figure 3.1: Life Expectancies for Males and Females, Aged 75 Years, Ireland, Based on Experience Around the 3 years Centred in Calendar Year Shown[1]

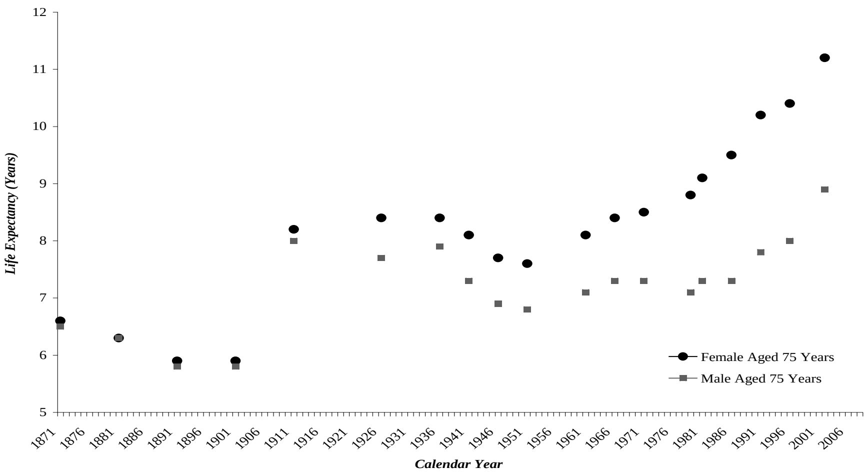


Table 3.1: 1000q_x from Irish Life Tables, Males, 1926- 2002. Ages 80 and Over[2]

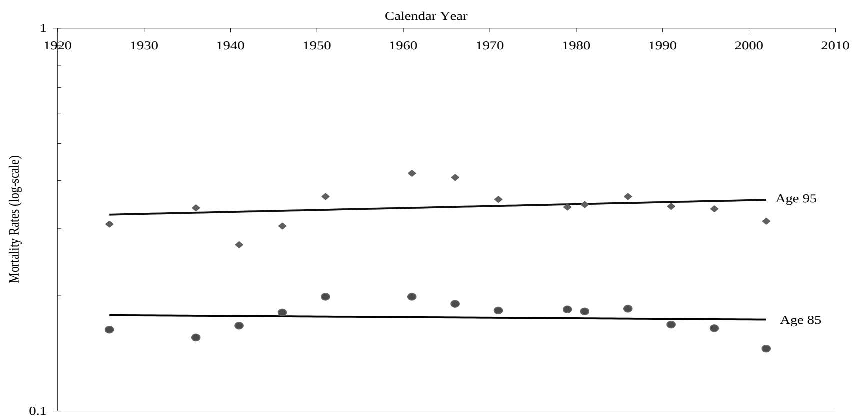
Source	Period	1000xMortality Rate at Age x years, where x=				
		80	*85*	*90*	*95*	*100*
ILT 14	2001-03	89	145	220	313	427
ILT 12	1990-92	109	168	246	342	460
ILT 10	1980-82	122	182	257	346	424
ILT 8	1970-72	122	183	262	357	408
ILT 6	1960-62	125	199	296	418	571
ILT 5	1950-52	136	199	274	363	467
ILT 3	1940-42	123	167	216	272	333
ILT 1	1925-27	114	163	227	308	406

[1] Central Statistics Office (CSO) (2004), figures from Table 3.
[2] From various Irish Life Tables, published in more recent years by the Central Statistics Office (CSO), and previously published in various reports of the Census of Population when compiled by the Department of Industry and Commerce.

Reported mortality levels for males in their nineties in 2002 are much the same as those reported in 1926. At all advanced ages, reported mortality increases until the 1960s, with the increases being greatest at the higher ages. However, since the 1960s, there is a trend of improvements evident across all advanced ages. The overall uneven trend in Irish mortality at advanced ages is at odds with those recorded in England and Wales, where mortality at ages up to about 95 years has been in a slow decline from 1930 but showing a rapid acceleration after 1950 (Humphrey (1970), Thatcher (1987), Thatcher et al. (1998), Gallop (2002), Gallop and Macdonald (2005)).

Figure 3.2 graphs how mortality rates for Irish males at ages 85 and 95 years have evolved since 1926, together with a log-linear trend line fitted by least squares. There appears to be a very weak secular trend — marginal decrease at aged 85 years and marginal increase at age 95 years.

Figure 3.2: Mortality Rate of Irish Males, Age 85 and Age 95, 1926-2002 [log-scale][3]



It is evident from the foregoing graphs and tables that reported mortality rates at older ages show neither pronounced nor regular improvement, in contrast to trends at earlier ages (see Chapter 1 or

[3] Data from Irish Life Tables 1-14, Males, published by the Central Statistics Office (CSO).

Whelan (2008)). However, as developed in the next section, this conclusion must be qualified by the observation that the underlying data and method used to estimate mortality is considerably less reliable at advanced ages than at younger ages. In short, the reported mortality rates for the population of Ireland at advanced ages, and the secular trends in these rates, require further analysis. In particular, we shall show that the disimprovement in reported mortality rates at later ages in Ireland in the early 1960s could be simply due to the Central Statistics Office's method of extrapolating rates at later ages rather than any underlying disimprovement in mortality.

Critique of Mortality Rates at Advanced Ages Reported in Irish Life Tables

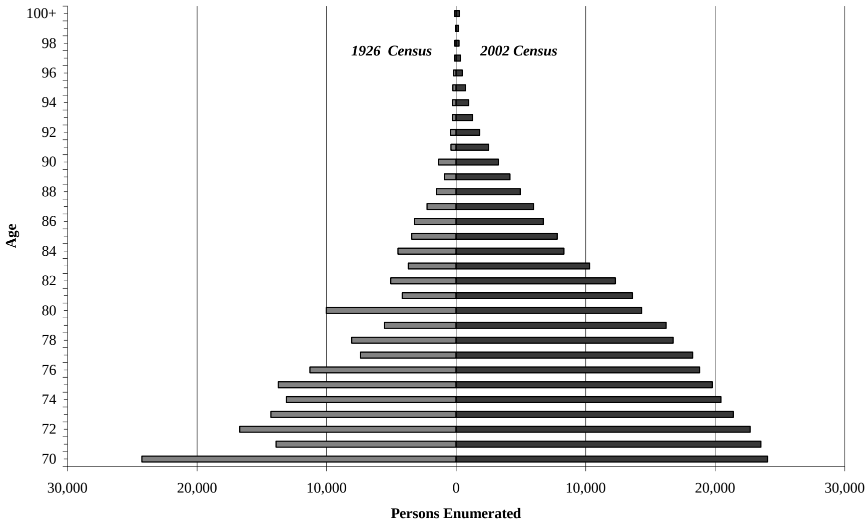
Errors can arise in estimating mortality rates in four generic ways: (1) data errors arising from inaccurate population or death records, (2) errors in estimating crude mortality rates when the death count and the exposed-to-risk do not perfectly match, (3) errors in statistically modelling the crude rates, and (4) model misspecification. We treat each of the four sources of error in turn below, discussing its significance for Irish mortality rates reported at older ages.

Data Errors in Population and Death Records

Errors in the data underlying the construction of crude mortality rates can take two distinct forms: (i) errors in age statement in census, and (ii) errors in age reported at death. Aside from mortality estimation, it is necessary to correct for these errors so as to, for instance, achieve a more accurate estimate of the age distribution of the population.

There is an internationally observed tendency, especially evident in earlier times, for people in declaring ages to round to a number ending with either 0 or 5 (or, to a lesser extent, to prefer to report an even rather than an odd number) (see, for instance, Myers (1940)). This tendency is known as 'age heaping'. Figure 3.3 shows the person count by age over age 70 years in the censuses of Ireland in 1926 and 2002. Age heaping is evident at a glance in the former — especially at ages 75, 80 and 90 years — but not the latter.

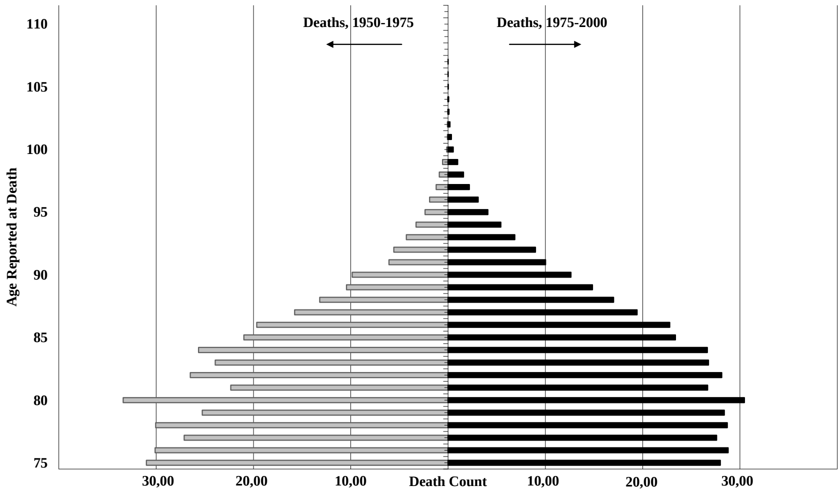
Figure 3.3: Age Declarations in Censuses of Ireland, 1926 and 2002, Males and Females Combined[4]



Age heaping is also evident in reported age at death. Figure 3.4 graphs the reported age at death in Ireland for all deaths at or over age 75 years reported in the periods from 1950 to 1975 and from 1975 to 2000 (inclusive). There is an unusually large number of recorded deaths at age 80 years in both periods, but also at ages 78, 82 and 84 years. Age heaping seems to be a more pronounced feature of earlier times but it still persists. Myers's Blended Index (Myers (1940)) was calculated from the age reported at death in each calendar year from 1950. This index should take a value of zero if there was no age heaping but, in fact, took a value of about 10% in the 1950s, falling to 3% in the late 1990s and early 2000s, for both males and females.

[4] Based on data kindly provided by the Central Statistics Office (CSO).

***Figure 3.4: Age Reported at Death in Ireland, Both Sexes Combined, 1950 to 2000*[5]**



There is reason to believe that the problem at older ages is not just limited to age rounding, but that there is a bias to overstate ages. Old age pensions were payable in the United Kingdom of Great Britain and Ireland from 1909 to persons over the age of 70 years, subject to means and other qualifying tests. Unlike England, which had introduced formal registration of births, deaths, and marriages more than 70 years earlier, official registration was only introduced in Ireland from 1864. Accordingly, there was no formal means to verify ages of anyone over 45 years in Ireland in 1909 (Wood (1908)) and, as could reasonably be anticipated, claims for pensions in Ireland far exceeded that expected based on the census of 1901 (Ó Gráda (2002)). It could be expected — and was widely expected at that time (see, for instance, Marr (1909) and the associated discussion) — that a person would report an age at subsequent censuses consistent with their declared age for pension. This would lead to life expectancies at higher ages calculated from census data being exaggerated.

Brown (1930), in reviewing the construction of Irish Life Table 1, disbelieved the exceptionally low mortality at advanced ages reported preferring to believe that "as the pension age approaches the temptation

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[5] Based on data kindly provided by the Central Statistics Office (CSO).

to misstatement of age has still proved irresistible to a considerable section of the community" (p.102). We observe, consistent with this hypothesis, that life expectancies for both Irish males and females at age 75 years show a suspicious jump of more than 20% between 1900-02 and 1910-12 (see Figure 3.1 earlier). In fact, life expectancies for males aged 75 take until 1995-7 to regain the level reported in 1910-12. With each passing calendar year, new age exaggerations to secure a pension could be expected to decline, so the upward bias to estimated life expectancies could be expected also to decline and this trend could be masking some real underlying improvement. This effect could have persisted to some degree even into the 1950s, but is unlikely to be material thereafter.

Even without a monetary incentive, there is a widely observed tendency in censuses and otherwise for age overstatement at advanced ages (Bowerman (1939), Easton (1799), Laslett (1999)). It could be partly because reported dates of birth are misread or mis-keyed from census returns and, while such errors might cancel out and have a negligible effect overall at younger ages where there are comparatively large numbers, the small numbers at higher ages entail that such errors have a large impact.

Whatever the reason, age exaggeration appears to be factor in Ireland. Consider, for example, the 18 men reported to be centenarians in the 1951 census of Ireland. According to the registration of deaths in that and subsequent years, no man died aged 100 or more in 1951, or aged 101 year or more in 1952, or aged 102 or more in 1953, and so on. In fact, we can go all the way to calendar year 2006 and find no man dying aged 155 years or more. There are only two possible explanations for this: either all 18 declared centenarians in 1951 emigrated over the following years or their death certificate reported an age inconsistent with that recorded in the census of 1951.

It is generally believed that the age recorded at time of death is more reliable than age declared at census. Thatcher (1981) reports that investigations of samples of persons reported as centenarians at the time of their death in the England and Wales confirmed the reliability of age recorded on the death certificate. This is in contrast with verification checks on census counts at advanced ages in England and Wales, which, when last done, shows that of the 3,727 centenarians enumerated in the 1981 Census, somewhat less than half (1,644) could independently be verified (Gallop and Macdonald (2005)). Accordingly, it would be better for this reason alone to base mortality estimates on deaths records only. Later we re-estimate mortality rates from death certification data only.

It should be noted that age recorded at death, while more accurate than census data, is still not wholly reliable. Previously, we noted a suspicious age heaping still discernible in the recorded ages at death in Ireland, though it appears to be diminishing with time. A study of the accuracy of reported age at death was recently done in Northern Ireland based on a sample of 1,698 death records (Health Statistics Quarterly (2000)). While dates of birth on the death certificate matched an independent source in 86% of cases, errors were proportionately more likely at advanced ages, with 5% of those reported 90 years and over at the time of death inaccurate by at least two years. This increasing inaccuracy must be borne in mind in any modelling exercise of the crude rates.

Errors in Estimating Crude Mortality Rates

There must be a correspondence between the numerator, being the number of deaths, and the denominator, being the exposed-to-risk, in calculating crude mortality rates. However, age is recorded at censuses in Ireland as date of birth but reported as age last birthday at the census date. Deaths are recorded with either age last birthday at time of death or date of birth, together with the date of death. Deaths are often reported as the number of deaths of a particular age and sex in the respective calendar year. It is not possible to make the exposed-to-risk derived from the census correspond exactly with the reported death data. The inevitable errors that any approximation entails, and their growing significance with increasing age, is perhaps best illustrated by attempting to reconcile the numbers reported at each census.

Take, for example, the 2002 and 2006 censuses. We know the number of, say, males aged 90 in April 2002 and the number aged 94 in April 2006. We also know the number of males that died at each age in each intervening years — so the number dying aged 90 in calendar year 2002, aged 91 in calendar year 2003, up to those dying aged 94 in 2006. Migration flows, which have been particularly high over recent years in Ireland, will also affect the reconciliation, especially at the younger adult ages. To attempt to reconcile these data, we need to make some assumptions about the distribution of deaths over each calendar year and over each year of age. We make three alternative assumptions to illustrate the impact such assumptions have of the attempted reconciliation:

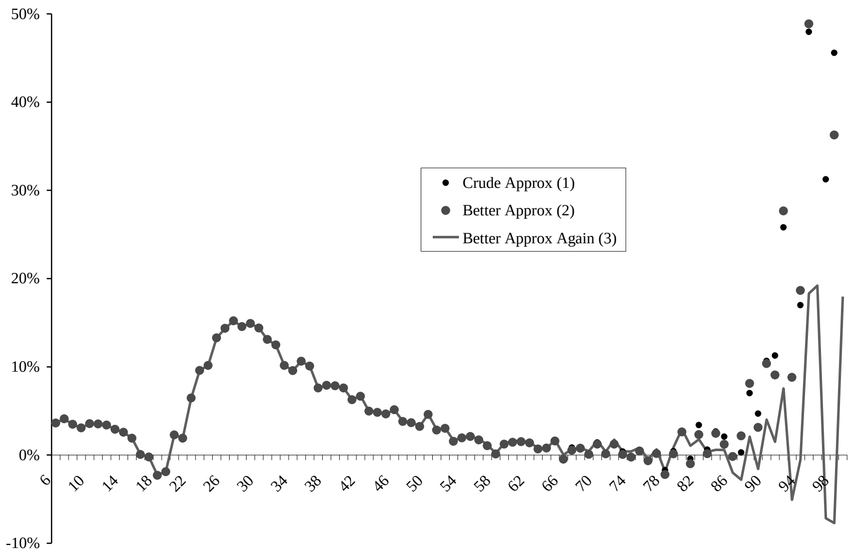
(1) First, the crudest assumption, is that the population aged $x+4$ in the 2006 census can be approximated by the number aged x counted in the 2002 census, reduced by the recorded deaths aged $x+y$ in calendar year $2002+y$, for $y=0, 1, 2, 3$.

(2) Second, a better approximation, is to apply (1) but with the number of deaths from the population aged $x+y$ in year to April $2002+y+1$ estimated as two-thirds the deaths aged $x+y$ in calendar year $2002+y$ and one-third of the deaths aged $x+y$ in calendar year $2002+y+1$, for $y=0, 1, 2, 3$.

(3) Third, and a better approximation again, is to use (1) but now apportioning the deaths in each year to April as determined in (2), by the ratio of the population count of the two adjacent ages that could contribute to the deaths.

We can see the impact of assumptions (1) to (3) on the final result from Figure 3.5.

Figure 3.5: Relative Error in Reconciliation Irish Censuses, 2002 and 2006, Males, Differing Approximations



The assumptions produce a negligible difference in the estimated population in 2006 at ages up to 80 years (the difference between the three approximations being no greater than 1% in this age range). Migration flows is the reason the population estimates do not coincide with the number enumerated. However, for later ages, when the exposed-to-risk population changes rapidly with age, the approximations show increasingly large deviations from one another. Migration is not a significant factor at these later ages, so these deviations are deviations also from the true value. It immediately follows that the approximation used to ensure that the deaths correspond to the exposed-to-risk becomes less reliable as age increases. Accordingly, the resultant estimates of the crude mortality rates also become less reliable with increasing age.

Errors in Statistically Modelling the Crude Rates

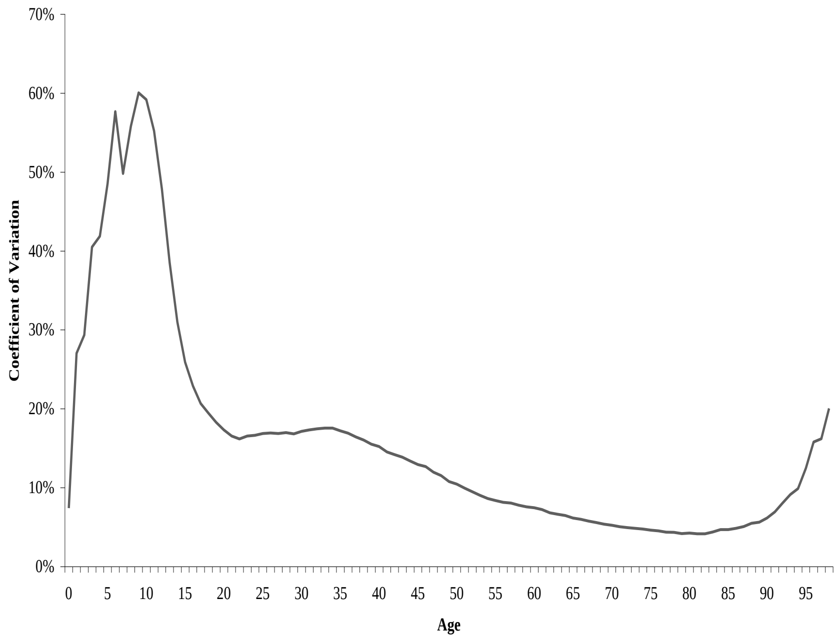
Statistical variation is present in crude mortality rates. This can give rise to mis-estimation of parameters when fitting models to the crude rates. Variation in crude rates arises from the following three sources.

Statistical Variation

There are relatively few survivors to very advanced ages, so the mortality rates at these later ages must be estimated from a smaller sample. This introduces progressively larger random errors in estimating the underlying mortality rate, until at ages of say, 105 years and over, the Irish data is too sparse to provide an acceptable estimate of the underlying mortality rate. Random error, though, is not a significant problem up ages of about 95 or so, even for a country with a population as small as that of Ireland. Figure 3.6 plots the coefficient of variation (standard deviation of the mortality estimator divided by it mean) against age, based on the numbers of males at each age enumerated in the 2002 census and the mortality rate from Irish Life Table 14 Males (2001-2003).

At ages over 95 years or so, we must become progressively more reliant on the graduation method, and less on the estimated crude mortality rates, as the relative error in the mortality rates increase dramatically as the exposed-to-risk declines.

Figure 3.6: Coefficient of Variation of Estimator of Mortality, Based on 2002 Census Numbers of Males, and Mortality of Irish Life Table 14 Males



Heterogeneity and secular variation in underlying mortality rates

Random error is just one form of error that must be guarded against in modelling crude mortality rates. Other types of statistical variation that affect the standard error arise from heterogeneity in mortality rates and stochastic variation in the underlying mortality rate. Heterogeneity in mortality rates, perhaps counter-intuitively, reduces the standard error of the mortality rate (estimated by the usual deaths divided by the initial exposed-to-risk) while stochastic variation in the underlying mortality rate increases the standard error (Benjamin and Pollard (1980), especially Chapter 17). Stochastic variation in the underlying mortality rates can be anticipated to increase with increasing age as mortality rates at later ages show a greater sensitivity to environmental conditions — such as extremes in weather conditions and outbreaks of influenza. Studies with Australian data (Pollard (1970)) suggest that increases in the standard error due to stochastic variation tend to dominate reductions due to heterogeneity, but the overall affect is marginal. If the relationship posited for Australian data (Pollard (1970), p.260, formula 9) were to hold for males in Ireland in 2002, the coefficient of variation show in Figure 3.6 must be increased by no more than 1.6% even at advanced ages.

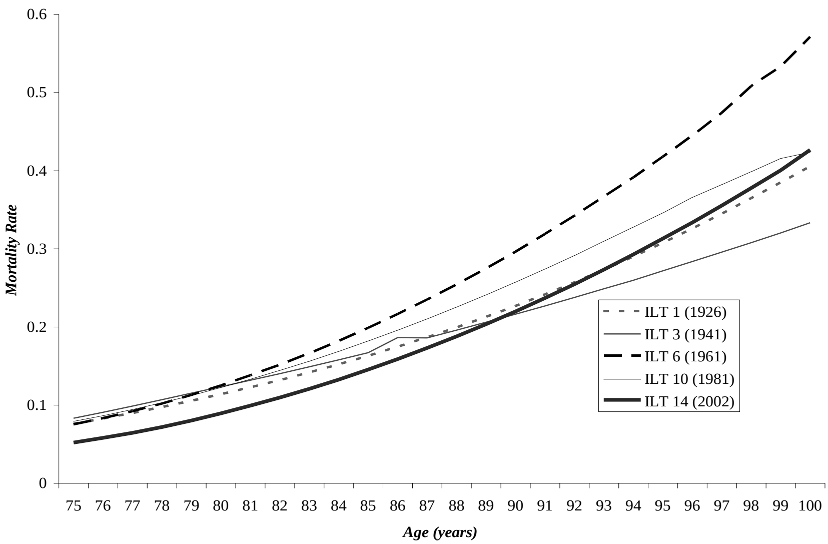
The standard error can be controlled by averaging deaths at a particular age over several calendar years, which reduces both random error and the adjustment required for take account of stochastic variation in the underlying mortality rate. This method of reducing standard error will be exploited later in the paper.

Model Error

The method used to construct all fourteen of the official Irish life tables is based on King's method (King (1909)), a method close in both theory and outcome to cubic spline graduation. For ages 7 to 87 years, the method involves grouping deaths and population count into five-year or ten-year age groups, estimating the mortality rate for the mid-age of the group and using osculatory interpolation (or, with ten-year groupings, Langrangean interpolation) to estimate mortality rates at intervening ages (Geary (1929), CSO (1965, 1986)). For the extremes of age — under 7 years and over 87 years — ad-hoc methods are employed. This method of graduation — by grouping deaths and population numbers — has been designed to remove much of the effects of age heaping, at least for ages up to 87 years.

Mortality rates at very advanced ages are typically estimated by assuming a mathematical relationship between mortality rate and age. Clearly, posting such a relationship introduces the possibility of errors from model misspecification. In estimating mortality rates above age 87 years for the Irish life tables, different forms of the mortality curve have been assumed over the years. For Irish Life Table 1 (ILT 1), corresponding to the experience 1925-7, rates were obtained by fitting a Makeham curve to the (King's method) estimated values of and p_{90} (Geary (1929)). In recent times, a quadratic curve has been fit passing through the King's method estimate of q_{72}, the parameters of the curve found by minimising the weighted squares of differences between King's estimates and the curve at the points $\hat{q}_{77}, \hat{q}_{82}, \hat{q}_{87}, \hat{q}_{92}$, the weights being the square of the number of deaths in the associated quinquennial group (see CSO (1965, 1986)). Figure 3.7 graphs the resultant mortality curves for Irish males from age 75 years, with successive curves separated by approximately two decades.

Figure 3.7: Mortality Curves at Older Ages from Irish Life Tables, Male, 1926-2002



The mortality curves show mortality increasing at quite different gradients with age between the different life tables. In particular, mortality in more recent times is reported to increase with increasing age at a greater rate than in 1926 and in 1941, with the result that, though mortality at age 75 years is significantly lower now than sixty or seventy-five years ago, by age 100 years it is recorded as higher now than at those earlier times. Accordingly, identifying secular trends at advanced ages is compounded in that the curve-fitting approach used to estimate mortality rates at older ages changed with time. The difficulties of comparability over time introduced by changing methods of extrapolation at later ages is also a feature of English Life Tables of the twentieth century where finite difference methods where superseded by fitting a Gompertz curve, superseded in turn by a logistic-type curve, then cubic splines with different assumed limiting ages (Gallop (2002)).

Alternative Method of Estimating Mortality at Advanced Ages

Method of Extinct Generations

Estimating mortality rates at advanced ages is challenging and, as the foregoing remarks make clear, these difficulties are compounded with Irish data. Vincent (1951) suggested an approach to estimating mortality at older ages that overcomes many of the problems identified in the previous section. Vincent's approach, known as the 'method of extinct generations', has been modified and applied by Humphrey (1970) to UK data and updated and further developed by Thatcher (1981, 1987, 1992, 1999a), Thatcher et al. (1998), and Andreev et al. (2003). The underlying idea is simple: data from death registrations can be employed to provide a better assessment of the exposed-to-risk at advanced ages than census data. The merit of the method of extinct generations is sometimes attributed to using the more accurate age recorded on the death certificate than that declared at a census. While this is undoubtedly true for centenarians, the more significant merit of using Vincent's approach comes from achieving a closer correspondence between deaths and exposed-to-risk at ages up to the late 90s, as we shall show later.

Deaths in Ireland are reported as age last birthday at time of death. Let us denote the number of deaths aged x in calendar year y by d_x^y. Ignoring migration, the persons who die aged $x+n$ in calendar year $y+n$ comprise two groups: (i) those born in calendar year $y+n-(x+n)= y-x$ who died after reaching their birthday in calendar year $y+n$ and (ii) those born in calendar year $y-x-1$ who died before their birthday in calendar year $y+n$. If we assume that deaths are uniformly spread over the year of age then the initial exposed-to-risk at time y is closely approximated by

$$E_x^y \cong \sum_{i=0}^{\infty} d_{x+i}^{y+i} \qquad (1)$$

(see Humphrey (1970), p.107). Vincent's idea is to employ E_x^y and estimate the crude mortality rate at age x in calendar year y as:

$$\hat{q}_x = d_x^y \big/ E_x^y \qquad (2)$$

Of course, it is assumed that there is no migration at these advanced ages. No figures are published showing migration flows for Ireland at ages above 75 years or so but those published for persons aged 65 and over show that net migration in this broad age group totalled just 2,200 in the six years to end 2008, despite very high overall net migration at 295,400 persons in these calendar years (calculated from Table 4 in CSO (2008)). We can conclude that net migration at ages over 75 years must be negligible.

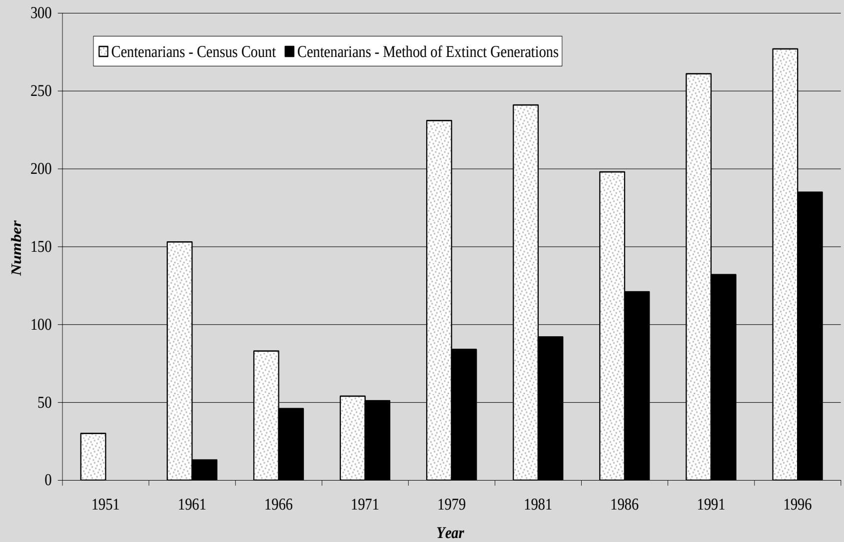
Tables 3.2 and 3.3 in Appendix I set out the number of recorded deaths at each age at or over 99 years for Irish males and females respectively over the calendar years 1950 to 2006, based on data kindly provided by the Central Statistics Office (CSO). Humphrey (1970) reports, from a study of ages at death and estimated mortality rates in England and Wales, that there appeared to be an appreciably more accurate age of death if the birth was registered. In the case of Ireland, with official registration of births introduced in 1864, deaths up to age 75 years in calendar 1939 had an associated formal registration of birth, as do deaths up to age 76 years in 1940, and so on, up to 111 years in 1975. This cut-off is marked by a line in Tables I.1 and I.2. Box II reconsiders the number of centenarians and records of the longest lived in Ireland.

Let ω be the highest attained age over the period studied, so that no person was alive at age $\omega+1$. We might take $\omega = 111$ for females in Ireland (see Appendix). Our data set is a record of deaths in Ireland from 1950 up to and including calendar year 2006, broken down by age and sex. The method of extinct generations can be used to estimate mortality rates at age up to $\omega-a$ in calendar year $2006-a$, for $a=0, 1 ,2,\ldots$.. Accordingly, we can estimate female mortality rates at age 111 years at end of calendar 2006, at ages over 110 years in calendar year 2005,…, and at ages over 85 years in 1980. We require some other method to give estimates of mortality rates at younger ages in more recent calendar years.

Box II: Number of Centenarians and Longest Lived in Ireland

It is of interest to compare the number of centenarians enumerated in Irish censuses with the number of centenarians estimated by the method of extinct generations. The censuses report the number in April of the census year while the method of extinct generations estimates the number as at 1st January who will subsequently die as centenarians. One would expect this latter number to be slightly greater than the former.

Figure 3.8: Number of Centenarians Reported in Irish Censuses, Compared with Number Estimated by Method of Extinct Generations [6]



The two methods provide quite different estimates with the method of extinct generations giving considerably lower numbers. This pattern of higher numbers recorded in censuses than by method of extinction generations has been a noted feature in England and Wales. Thatcher (1981) investigates the 1971 census count of centenarians and shows it to be about double the more reliable estimate of the method of extinct generations.

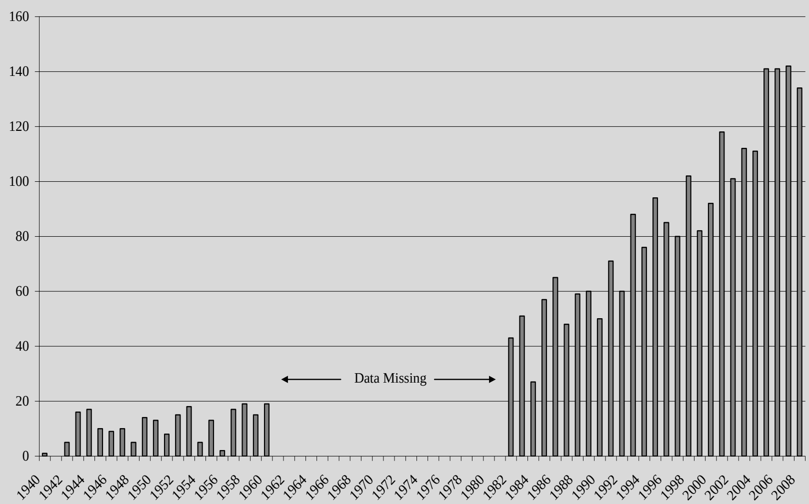
Aside from the census and death data, there is another source of information on the number of centenarians in Ireland. The Centenarian Bounty is a payment made by the President of Ireland to anyone in Ireland reaching their 100th birthday. The scheme started in 1940 with a 'bounty'

[6] Based on data kindly provided by the Central Statistics Office (CSO). In estimating the number of centenarians in 1996, we took ω=110 years (or, equivalently, assumed no-one was alive aged 111 or more at the end of 2006).

payment of £5 but had grown so that it amounts to €2,540 in 2008, which is arranged to be presented on the recipients 100th birthday with a congratulatory letter signed by the President. The scheme applied only to those living in Ireland (whatever their nationality) at first but was extended in March 2006 to include all Irish citizens born on the island of Ireland wherever they are now resident. A further development to the scheme was made in 2000, so that from that time a commemorative coin, especially designed each year, is given on each birthday celebrated after their 100th.

For those in receipt of a state pension the bounty is automatically awarded on their birthday but others must apply. Given the widespread knowledge of the bounty in Ireland and the materiality of the payment in recent years, it can be expected to achieve close to 100% coverage of those centenarians resident in Ireland.[7]

Figure 3.9: Number of Persons in Ireland Reaching 100th Birthday in Calendar Years 1940-1960 and 1982-2008[8]

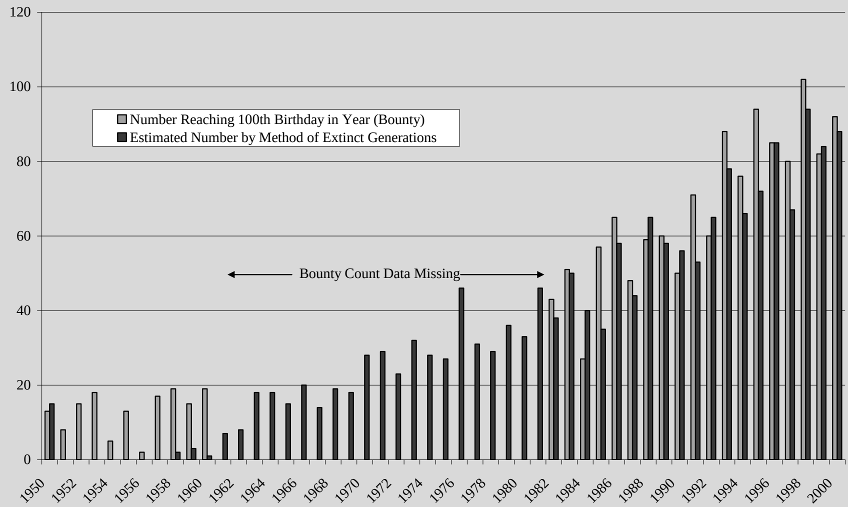


[7] This is in contrast with the number of Queen's messages of congratulations sent to those reaching their 100th birthday and on every birthday from their 105th in the UK and Commonwealth, which is believed "not to provide accurate numbers of the very elderly. However, they can provide a lower bound on numbers at the oldest ages." Gallop (2002) p.4. Where such schemes exist in other countries to celebrate citizens' longevity, such as the President's congratulatory letter in the US and even the silver cup and certificate presented in Japan to centenarians, none can expect to have the same completeness of coverage as that in Ireland.

[8] From data kindly provided by the Office of the President of Ireland (*Áras an Úachtaráin*).

The number of centenarian bounties awarded in any year can provide an independent check on the numbers at advanced ages. Figure 3.10 compares the numbers awarded the bounty in every calendar year from 1950 for which data exists and compares it with the number of 100th birthdays in that year estimated by the method of extinct generations.

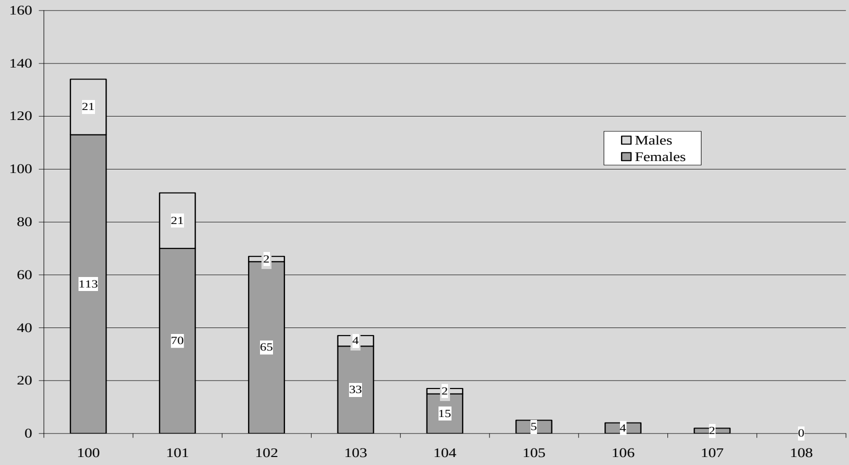
Figure 3.10: Number of Persons in Ireland Reaching 100th Birthday in Each Calendar Year, by Bounty Awards and Estimated by Method of Extinct Generations



We note that the number of bounty payments appears high in the fifties and sixties when age could not be verified by birth registration, as could reasonably be expected. From 1982 to 2000, the numbers match reasonably well: over that period 1,290 bounty payments were made and 1,196 persons are estimated to have reached their 100th birthday by the method of extinct generations.

We note the marked trend of increasing numbers of centenarians in Ireland. The trend is primarily due to the increasing numbers of females reaching extreme ages. Figure 3.11 highlights the numbers centenarians in Ireland in 2008 by age and gender. Of the total 223 centenarians who celebrated a birthday in 2008, 194 (87%) were female.

Figure 3.11: Number of Bounty Awards for 100th Birthday and Medals for Attaining Higher Ages Awarded to Irish Residents in Year 2008[9]



Finally, it is of interest to record that the highest age recorded at death in Ireland over the period 1950 to 2006 was of a female aged 111 years who died in 1984 (and therefore an official record of her birth should exist). This equals the highest verified age of death of a person in Ireland ever (Katherine Plunket who died on 14th October 1932 (Thatcher (1999b)). The highest recorded age of a male death in the period 1950 to 2006 inclusive was 110 years (in 1969) but he was born before formal registrations of birth. The greatest longevity in Irish males with a birth registration is 107 years, with four such cases reported — in years 1978, 1982, 2000 and 2003.[10]

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[9] From data kindly provided by the Office of the President of Ireland (Áras an Úachtaráin).

[10] Up to 1992, the highest verified age at death in England and Wales was 114 years for a female (in 1987) and 112 years for a male (in 1990) (Thatcher (1992)). The highest verified age at death of a human is of the French woman Jeanne Calment who died on 4th August 1997 at the age of 122 years (Robine and Allard (1999) and Guinness Book of Records (2008)). It should be noted that the Irish have always figured prominently in league tables of centenarians and supercentenarians. Easton (1799) gives a list of supposed centenarians that ever lived numbering 1,712, of which no less than 145 were mainly resident in Ireland. The list includes St. Patrick (122 years), St Kevin of Glendalough (120 years), and the oldest reported Irish person, the Countess of Desmond (145 years) who died in 1612 and "was married in the reign of King Edward IV, was in England the same reign, and danced with the Duke of York, the King's brother. Upon the ruin of the house of Desmond, she was

Exposed-to-Risk by Census Method and Method of Extinct Generations Compared

It is of interest to investigate to what extent the initial exposed-to-risk, E_x^y, as determined by the method of extinct generations, differs from a population count of those aged x made during calendar year y, i.e., compare how close a census count is to E_x^y. The census count, occurring at the end of April, is closer to a central exposed-to-risk than the initial exposed-to-risk estimated by the method of extinct generations. The adjustments to the census count required to make it directly comparable with the initial exposed-to-risk of the method of extinct generations (which corresponds to deaths grouped by age last birthday in a calendar year) are:

(1) To the count of those aged x last birthday at the end of April in the calendar year: this population count should be given a weight of 0.722 equivalent years' exposure assuming births of the group are uniformly distributed over the calendar year.

(2) To the count of those aged $x+1$ in April of the calendar year: this population count should be given a weight of 0.056 equivalent years' exposure, again assuming births of the group are uniformly distributed over the calendar year.

(3) To the count of those aged $x-1$ in April of the calendar year: this population count should be given a weight of 0.222 equivalent years' exposure, assuming births of the group are uniformly distributed over the calendar year.

(4) Also one must add one-third of deaths aged x in the calendar year, under the assumption that deaths are spread uniformly over the calendar year.

We compare the ratio of the two initial exposed-to-risk counts for males in Ireland at ages up to 98 years, when the census count has been adjusted as described.

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obliged, at the great age of one hundred and forty, to travel from Bristol to London, to solicit relief from the court, being reduced to poverty. Lord Bacon says, she renewed her teeth twice or thrice. This remarkable lady was a subject for the pens of a variety of authors. She retained her vigour to the last." (pp 5-6). Of course, all of these remarkable feats of longevity must be put down to straightforward exaggeration, (see, for instance, (Bowerman (1939), Laslett (1999)). In fact, early tales in many cultures tell of improbable longevity – *Tír na nÓg* and Oisín's several hundred year visit there, and, excepting Cain and Abel, the lifespan of the first ten men mentioned in the Bible averaged more than 850 years, with Methuseh the longest lived at 969 years (Boldsen and Paine (1995)).

Figure 3.12: Ratio of Adjusted Population Count in Censuses of 1951, 1961 and 1971 to Count by Method of Extinct Generations, Males, Ages 75 Years and Over

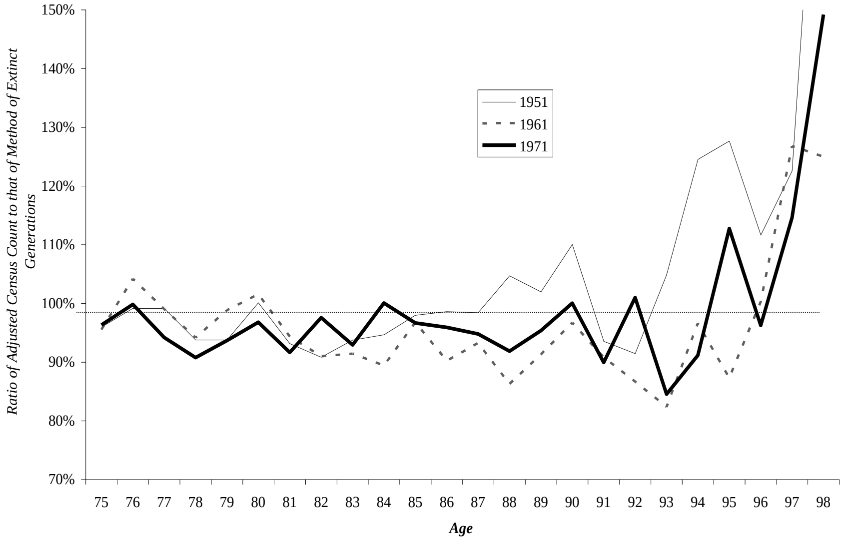


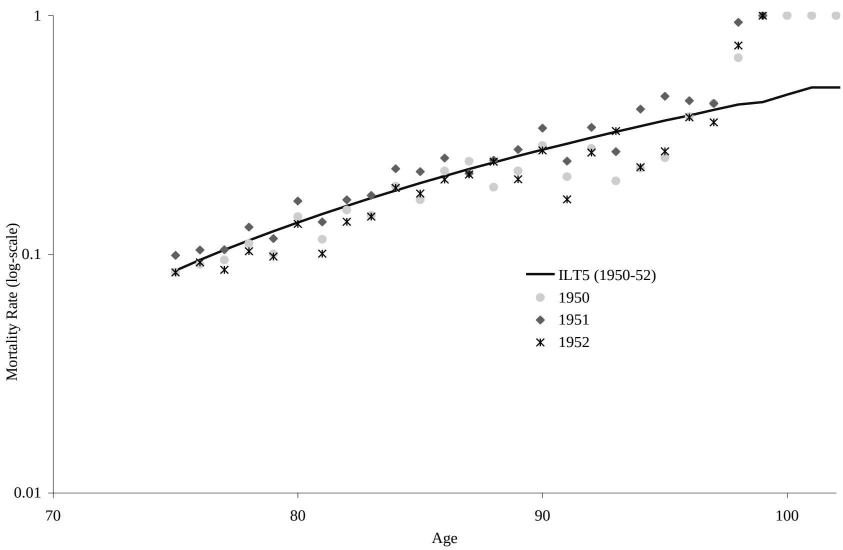
Figure 3.12 shows that the adjusted census count differs from the count estimated by the method of extinct generations by being materially below the latter in the age range 75 to late 90s, and thereafter significantly higher. Using such data to estimate mortality rates would tend to overstate mortality in ages up to the late 90s and, based on Figure 3.12, the overstatement could be of the order of 5–10% for mortality rates in the age range 85–95 years. Accordingly, we can conclude that the method of extinct generations produces estimates materially closer to the true exposed-to-risk corresponding to the death count. Note that this finding is contrary to expectation: Thatcher (1987), *inter alia*, suggests that the merit of the method of extinct generations over the census method lies solely in its correction of the age exaggeration in the latter. Our analysis suggests that correcting for age overstatement is a minor merit, at least up to ages in the late 90s: the major merit in the method of extinction generations is its materially better approximation to the true exposed-to-risk at these advanced ages when the exposed-to-risk changes so rapidly with age (see Figure 3.5 earlier). Corroboration of this insight is found in Beatty and Rodgers (2000) who, using both the method of extinct generations and

reconciliation with administrative data, also report an under-enumeration of the same order of magnitude at these older ages in the Northern Ireland censuses of 1971, 1981 and 1991, reporting that the under-enumeration was zero for males aged 70 years (zero for females ages 75 years) rising smoothly to 15% for males in the group aged 95 years and over (16% for females).[11]

Estimating Mortality in Ireland by the Method of Extinct Generations

In this section, we apply the method of extinct generations to estimate the crude mortality rates at advanced ages. We take as a case study, males aged 75 years and over in the calendar years 1950–1952, corresponding to the experience that Irish Life Table 5 is based on.

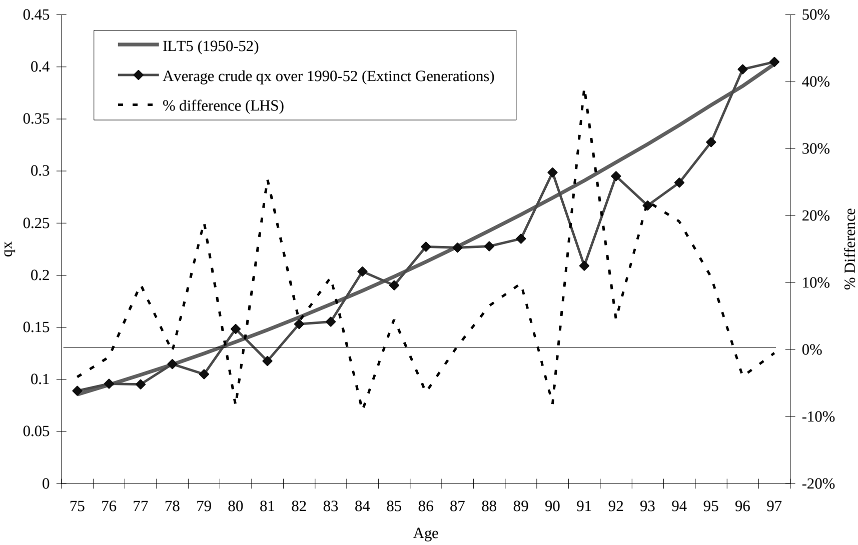
Figure 3.13: Crude Mortality Rates in Years 1950, 1951 and 1952 Estimated using Method of Extinct Generations, ILT 5 (1950-52), Males, Ages Above 75 Years



[11] The numbers found using the method of extinct generations were found to be broadly consistent with the numbers estimated from analysing the number flows on Northern Ireland's Central Health Index and also the count in the person-based database maintained by the DHSS of all claimants of benefits related to pensionable age (believed to have high coverage). However, in contrast, it should be noted that Gallop and Macdonald (2005) report the opposite: that a variant of the method of extinct generations used to estimate the population in each country of the UK over the years 1981, 1991 and 2001 produce numbers lower than the census count at advanced ages, with the divergence the highest for Northern Ireland in 1981 and 1991.

The graph shows that the rates in ILT 5 give a reasonable fit to the crude rates up to about age 90 years but somewhat loosely fitting thereafter, with perhaps the official rates being too high from about 87 years to the late 90s. The exposed-to-risk was under 10 for each age over 98 years, so random errors become significant. The large percentage differences are captured in Figure 3.14.

Figure 3.14: Average of Crude Mortality Rates in Years 1950-1952 Estimated using Method of Extinct Generations, Compared with ILT 5 (1950-52), Males, Ages Above 75 Years



The life expectancy of a 75-year-old male was reported as 6.8 years in ILT5 but, based on the average crude mortality rate over 1950-52, we estimate it to have been slightly higher at 7.0 years. A similar analysis for both males and females was performed from 1951 to 1971 and, in each case, the re-estimated life expectancy using the method of extinct generations was marginally higher. Figure 3.15 graphs the results.

Mortality rates at older ages reported in Irish Life Tables appear to be too heavy. Figure 3.15 illustrates the discrepancy between rates reported in Irish Life Tables and those estimated using the method of extinct generations for a male aged 90 and 95 years using mortality rates averaged over three years centred on the year shown.

Figure 3.15: Life Expectancy at Age 75 years, by Method of Extinct Generations Compared with that Reported in Irish Life Tables

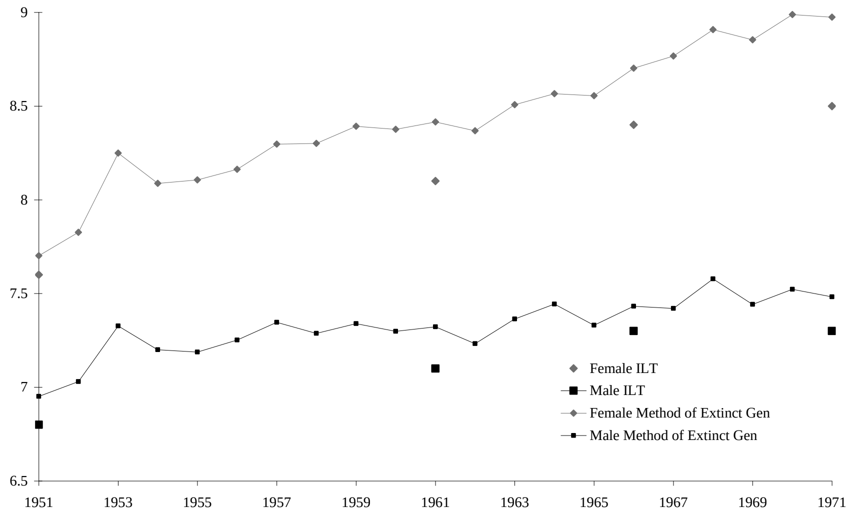
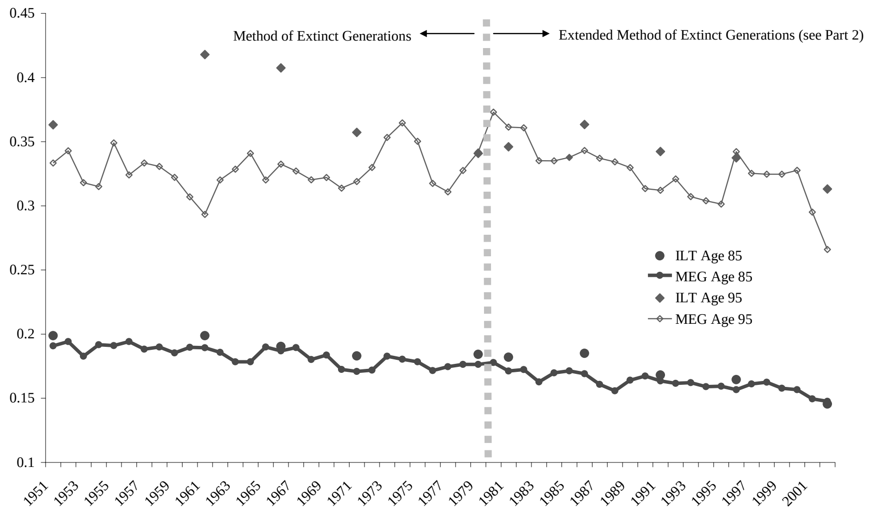


Figure 3.16: Mortality Rates for Male Aged 85 and 95 Years, Estimated by Method of Extinct Generations Compared to that Reported in Irish Life Tables, 1955-1991



The lack of correspondence between deaths and exposed-to-risk using the census method, with a resultant bias to understate the true exposed-to-risk, has led to the rates reported in the Irish Life tables been overstated.

Comparison with International Trends in Mortality at the Highest Ages

Thatcher (1992) provides estimates of the mortality in England and Wales at advanced ages based on a variant of the method of extinct generations over six decades, 1930-1990. He shows that mortality rates seem to have fallen broadly uniformly over the period, with the rate of improvement tending to decline slightly with increasing age. Subsequent analysis up to the 2003 shows the trend decline accelerating in more recent decades (Gallop and Macdonald (2005), see especially Figures 1-4 and Tables 7-8) but with little evidence of improvements after age 100 years. Figure 3.17 (Plate 7) shows a parallel shift downward in the mortality curve at advanced ages in England and Wales over the period and contrasts it with crude mortality rates in Ireland estimated by the method of extinct generations over the same periods. The changes in mortality in Ireland are somewhat erratic. Mortality rates in Ireland were lower than those in England and Wales in the 1950s but higher in the 1980s. We note that there was a slight improvement recorded in Irish rates up to age 90 years but rates disimproved marginally from 91 to 95 years, from which age the trend is obscured by random fluctuations.

Levels and trends of mortality at the highest ages have also been studied for many other countries. The Kannisto-Thatcher Oldest-Old database (within the *Odense Archive of Population Data in Aging*, Odense University Medical School, University of Odense, Denmark and available on-line from the Max Planck Institute for Demographic Research)[12] is a highly structured database with data on all deaths at and over age 80 years in more than thirty low mortality countries, divided by sex, age at death, calendar year of death, and calendar year of birth. Thatcher, Kannisto and Vaupel (1998) provides a detailed study of the level and trend of mortality since 1960, basing the exercise on a subset of the database, using data from just the thirteen countries (out of the thirty) countries that maintained good quality records over the period.[13] The thirteen countries were Austria, Denmark, England and Wales, Finland, France, Iceland,

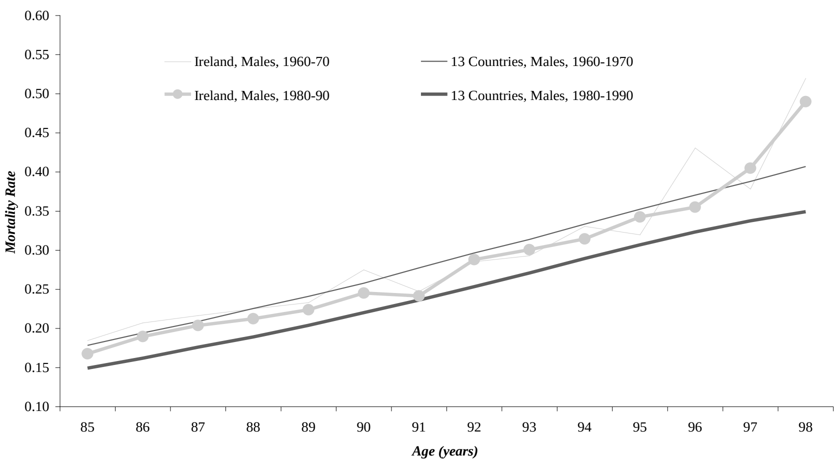
[12] http://www.demogr.mpg.de/; http://www.demogr.mpg.de/databases/ktdb/

[13] England and Wales are included as a country as vital registration in these isles divides the UK into England and Wales, Northern Ireland, and Scotland. Age of death not grouped. Models were fitted by sex, country, each of the three calendar decades, by cohort born 1871-1880 and in various aggregates.

Italy, Japan, the Netherlands, Norway, Sweden, Switzerland and West Germany. In total the study included over 32 million deaths over the period 1960-1990 and included over 120,000 persons that attained a century. The exposed-to-risk was calculated using an extended version of the method of extinct generations and it was assumed that no migration occurred at these advanced ages.

Ireland is included in the Kannisto-Thatcher Oldest-Old database but was not included in the subgroup of thirteen countries studied because the data was deemed to fall below 'good quality' and even 'acceptable quality', being classed as 'conditionally acceptable quality' (Kannisto (1994))[14]. Irish data is believed to biased by an overstatement of age at death at these later ages so "these data give probably a roughly correct description of the mortality trend though at a level artificially lowered by age overstatement" (Kannisto (1994)). Figure 3.18 compares Irish mortality for older males over the decades 1960-70 and 1980-90 as estimated using the method of extinct generations with that of the thirteen country average over the same periods.

Figure 3.18: Comparison of Mortality Rates for Males Ages 85 to 98 years, Ireland and 13 Developed Country Average Over Decades 1960-70 and 1980-90[15]



[14] It is classed above the 'weak quality' of the US, Canada and others.

[15] Mortality Rates for 13 Developed Countries (see text), Table 6.1 in Thatcher et al. (1998). For Ireland as calculated by author using the method of extinct generations.

Figure 3.18 shows again the parallel shift in the mortality curve with time in the 13 countries with Irish rates tending to lie between the two international curves and being significantly less smooth.

Figures 3.17 and 3.18 highlight the need to graduate the Irish crude mortality rates found by the method of extinct generations. Age heaping in the reported age of death at, say, 90 years, has created a crude mortality rate at age 90 years implausibly higher than the crude mortality rate of a 91-year-old. It is also necessary to extend the method of extinct generations in some manner so mortality rates for non-extinct generations can also be estimated. The next chapter, *Mortality in Ireland at Advanced Ages, 1950–2006: Part 2: Graduated Rates,* reports the results of these further investigations into Irish mortality at the highest ages.

Conclusion

We may summarise our initial findings on Irish population mortality at advanced ages over the period from the 1950s to 1980s as follows:

(1) There are significant non-random biases in the underlying data at advanced ages, with age exaggeration in Irish census data and age heaping in Irish death data.

(2) The inevitable lack of correspondence between the exposed-to-risk estimated from census data and the death count from registration data grows in significance with increasing age and is a significant source of error at advanced ages.

(3) Applying the method of extinct generations to estimate crude mortality rates at older ages, eliminates errors from (2) and reduces errors from (1). Distortion in mortality rates from age heaping in death counts remain, particularly at ages 80 and 90 years, and pose a challenge to graduation.

(4) Re-estimating Irish mortality rates using the method of extinct generations show that mortality in Ireland over the period from the 1950s to the 1980s was marginally lower than recorded in the Irish Life Tables, so life expectancies at advanced ages were higher than previously believed. In particular, mortality rates did not increase as steeply with age as reported in the official tables.

(5) The re-estimated crude rates show a very slight decrease in mortality rates up to age 90 years, with no improvement discernible at older ages.
(6) Improvements at advanced ages in Ireland have not been as great as those in England and Wales or other developed countries over the same period. Mortality for males in England in the 1950s was marginally higher than that for males in Ireland at advanced ages but by the 1980s it was lower. Similarly, the rate of improvement in Irish female mortality at higher ages lags those seen in England and Wales over the same period.

The next chapter graduates the crude rates and extrapolates patterns to very advanced ages, using several popular formulae, different calibration techniques, and different evaluation criteria. Extensions to the method of extinct generations so that mortality rates of more recent, still surviving generations, can be estimated are explored and the results reported. Part 2 shows a modest trend of improvement in male and female mortality at advanced ages accelerating in the most recent decades but still lagging those evident in England and Wales.

Appendix I: Deaths Recorded in Ireland at and over 99 years, 1950-2006[1]

Table 3.2: Number of Recorded Deaths at Ages 99 Years and Over, Males, 1950-2006

Year	99	100	101	102	103	104	105	106	107	108	109	110	Total deaths at age 100 and over
1950	6	5	1	1	2	–	1	–	–	–	–	–	10
1951	3	–	–	–	–	–	–	–	–	–	–	–	0
1952	1	–	–	–	–	–	–	–	–	–	–	–	0
1953	4	–	–	–	–	–	–	–	–	–	–	–	0
1954	8	–	–	–	–	–	–	–	–	–	–	–	0
1955	5	–	–	–	–	–	–	–	–	–	–	–	0
1956	8	–	–	–	–	–	–	–	–	–	–	–	0
1957	10	–	–	–	–	–	–	–	–	–	–	–	0
1958	2	–	–	–	–	–	–	–	–	–	–	–	0
1959	5	–	–	–	–	–	–	–	–	–	–	–	0
1960	8	–	–	–	–	–	–	–	–	–	–	–	0
1961	4	–	–	–	–	–	–	–	–	–	–	–	0
1962	4	–	–	–	–	–	–	–	–	–	–	–	0
1963	4	–	–	–	–	–	–	–	–	–	–	–	0
1964	7	3	5	–	–	–	1	–	–	–	–	–	9
1965	3	–	1	1	–	1	–	–	–	–	–	–	3
1966	6	4	–	–	–	–	–	1	–	–	–	–	5
1967	5	–	1	1	1	1	–	–	–	–	1	–	5
1968	4	5	1	1	–	1	–	–	–	–	–	–	8
1969	8	1	1	–	–	1	1	–	–	1	–	1	6
1970	15	3	2	1	–	–	–	–	–	–	–	–	6
1971	7	3	1	1	–	–	–	–	–	–	–	–	5
1972	6	2	4	1	–	1	–	–	–	–	–	–	8
1973	9	5	1	1	–	–	–	–	–	–	–	–	7
1974	8	5	1	1	–	–	–	–	–	–	–	–	7
1975	10	2	1	–	–	–	1	–	–	–	–	–	4
1976	9	–	2	2	–	–	–	–	–	–	–	–	4
1977	13	2	1	2	–	–	–	–	–	–	–	–	5
1978	4	2	3	–	1	–	–	–	1	–	–	–	7
1979	9	5	1	2	1	1	–	–	–	–	–	–	10
1980	10	3	3	–	2	–	1	–	–	–	–	–	9
1981	10	1	–	1	–	–	–	–	–	–	–	–	2
1982	10	1	2	3	–	–	–	–	1	–	–	–	7
1983	5	4	–	1	–	–	–	–	–	–	–	–	5
1984	7	2	4	2	2	–	–	–	–	–	–	–	10
1985	6	2	2	3	1	1	–	1	–	–	–	–	10
1986	7	2	2	2	–	1	1	–	–	–	–	–	8
1987	6	7	2	–	–	–	1	–	–	–	–	–	10

[1] Based on data kindly provided by CSO. The figures for 2006 are provisional. Line through table indicates cut-off from which time birth registration of those recorded deaths was required.

Year	99	100	101	102	103	104	105	106	107	108	109	110	Total deaths at age 100 and over
1988	7	5	3	2	–	–	–	–	–	–	–	–	10
1989	8	3	–	–	–	–	–	–	–	–	–	–	3
1990	7	8	7	1	–	–	–	–	–	–	–	–	16
1991	4	3	4	2	1	–	–	–	–	–	–	–	10
1992	9	2	2	–	–	1	–	–	–	–	–	–	5
1993	7	4	4	3	–	–	–	1	–	–	–	–	12
1994	7	3	6	2	–	–	–	1	–	–	–	–	12
1995	10	2	1	4	1	1	2	–	–	–	–	–	11
1996	9	9	4	–	1	–	1	–	–	–	–	–	15
1997	9	6	7	2	1	–	1	–	–	–	–	–	17
1998	8	7	3	4	1	1	–	–	–	–	–	–	16
1999	7	13	6	1	1	–	2	–	–	–	–	–	23
2000	10	8	4	1	2	–	1	–	1	–	–	–	17
2001	5	5	6	1	2	–	–	–	–	–	–	–	14
2002	9	8	2	2	2	–	2	–	–	–	–	–	16
2003	13	10	8	–	–	–	–	–	1	–	–	–	19
2004	13	8	2	1	3	2	–	–	–	–	–	–	16
2005	9	6	6	3	2	1	–	–	–	–	–	–	18
2006	17	11	2	3	4	1	1	1	–	–	–	–	23

Table 3.3: Number of Recorded Deaths at Ages 99 Years and Over, Females, 1950-2006

Year	99	100	101	102	103	104	105	106	107	108	109	110	111	Total deaths at age 100 and over
1950	11	10	–	3	–	–	–	–	–	–	–	–	–	13
1951	11	–	–	–	–	–	–	–	–	–	–	–	–	0
1952	10	–	–	–	–	–	–	–	–	–	–	–	–	0
1953	13	–	–	–	–	–	–	–	–	–	–	–	–	0
1954	9	–	–	–	–	–	–	–	–	–	–	–	–	0
1955	16	–	–	–	–	–	–	–	–	–	–	–	–	0
1956	13	–	–	–	–	–	–	–	–	–	–	–	–	0
1957	10	–	–	–	–	–	–	–	–	–	–	–	–	0
1958	13	–	–	–	–	–	–	–	–	–	–	–	–	0
1959	16	–	–	–	–	–	–	–	–	–	–	–	–	0
1960	6	–	–	–	–	–	–	–	–	–	–	–	–	0
1961	14	–	–	–	–	–	–	–	–	–	–	–	–	0
1962	14	–	–	–	–	–	–	–	–	–	–	–	–	0
1963	10	–	–	–	–	–	–	–	–	–	–	–	–	0
1964	10	8	6	5	3	–	1	–	–	–	–	–	–	23
1965	17	7	–	2	1	1	–	–	–	–	–	–	–	11
1966	19	8	1	2	3	1	–	–	–	1	–	–	–	16
1967	12	4	3	4	–	–	–	1	–	–	–	–	–	12
1968	14	3	6	2	–	–	–	1	–	–	–	–	–	12
1969	22	5	5	2	1	–	–	–	–	–	–	–	–	13
1970	16	14	5	1	–	–	–	–	–	–	–	–	–	20
1971	13	5	2	3	–	1	–	–	1	–	–	–	–	12
1972	26	5	5	3	1	1	–	–	1	–	–	–	–	16
1973	21	14	4	4	2	–	1	–	–	–	–	–	–	25
1974	30	7	3	8	3	1	–	–	–	–	–	–	–	22
1975	28	4	5	6	1	2	–	–	–	–	–	–	–	18
1976	17	15	6	6	1	1	–	–	–	–	–	–	–	29
1977	26	10	14	4	–	2	–	–	–	–	–	–	–	30
1978	30	10	3	5	2	–	–	–	–	–	–	–	–	20
1979	27	8	7	4	1	–	–	–	–	1	–	–	–	21
1980	26	11	9	3	2	4	1	1	–	–	–	–	–	31
1981	15	15	9	4	2	1	4	–	–	–	–	–	–	35
1982	22	16	8	1	2	–	–	–	–	–	–	–	–	27
1983	17	18	6	8	3	1	2	2	1	–	–	–	–	41
1984	16	13	5	6	4	2	–	1	–	–	–	–	1	32
1985	22	5	8	5	1	2	1	2	–	–	–	–	–	24
1986	28	25	11	7	4	1	1	–	–	1	–	–	–	50
1987	24	11	11	5	1	3	1	–	–	–	–	–	–	32
1988	40	20	12	5	–	3	2	1	–	–	–	–	–	43
1989	32	17	16	3	4	4	–	1	–	–	–	–	–	45
1990	31	16	13	9	2	4	2	1	–	–	–	–	–	47
1991	31	16	9	4	6	3	–	1	–	1	–	–	–	40
1992	30	23	8	5	5	2	1	1	2	1	–	–	–	48
1993	35	24	10	6	7	4	3	–	2	1	–	–	–	57
1994	34	24	15	10	9	4	2	–	–	–	–	–	–	64
1995	46	18	14	10	8	2	1	1	–	–	–	–	–	54
1996	47	17	13	9	6	2	1	–	–	–	–	1	–	49

Year	99	100	101	102	103	104	105	106	107	108	109	110	111	Total deaths at age 100 and over
1997	41	15	20	12	4	3	–	1	–	–	–	–	–	55
1998	46	28	13	12	9	4	–	2	–	–	–	–	–	68
1999	38	29	20	11	2	5	2	3	–	–	–	–	–	72
2000	42	29	16	12	8	6	1	–	1	–	–	–	–	73
2001	46	31	13	5	8	3	3	1	–	–	–	–	–	64
2002	39	31	24	9	6	5	2	1	–	–	–	–	–	78
2003	50	30	19	21	7	3	2	–	–	2	1	–	–	85
2004	50	43	18	14	8	8	3	1	–	1	1	–	–	97
2005	55	34	15	11	7	5	1	2	1	–	1	–	–	77
2006	52	43	27	15	9	4	4	3	–	1	1	–	–	107

Chapter 4

Mortality in Ireland at Advanced Ages, 1950-2006: Graduated Rates

Abstract

We graduate the Irish mortality experience from 1950 to 2003 by mathematical formulae from ages 75 years and upwards. The shape of the mortality curve at advanced ages is shown to be different to that recorded in the official tables, with the curve best fitted with Kannisto's version of Perks's Law. Mortality rates show only a modest trend of improvement in the early decades, below improvements in other developed countries. We evaluate the various approaches suggested to date to extend the method of extinct generations so mortality rates for non-extinct generations can be estimated. It is shown that the key advantage of this method is not in correcting for age misstatements but in achieving a close correspondence between death counts and the exposed to risk. This insight allows a rather straightforward approach to estimating the mortality of non-extinct generations. Applying the approach, we show that there has been an acceleration in the rate of improvement in more recent decades, but secular improvements in Irish mortality at advanced ages still lag those of England and Wales.

Introduction

The last chapter reconsidered the official Irish mortality record since 1950 as documented in the Central Statistics Office (CSO) Irish Life

Tables. Several problems were identified that frustrate the accurate assessment of mortality rates at older ages, including age heaping in census counts and in reported age at death, and age exaggeration in census returns, especially at age 100 years. More materially, the errors in estimating the crude mortality rates due to a lack of correspondence between registered deaths and the exposed-to-risk from census returns were shown to grow in significance with advancing age. Finally, secular trends in mortality could not be reliably identified from the official Irish Life Tables because of the inconsistency created by employing different models to graduate and extrapolate mortality rates at advanced ages from one life table to the next.

Mortality rates were re-estimated in the last chapter using the method of extinct generations to overcome the significant errors introduced at older ages by the census method. The crude mortality rates using the method of extinct generations show that mortality in Ireland over the period from the 1950s to the 1980s was marginally lower than recorded in the Irish Life Tables, so life expectancies at advanced ages were higher than previously believed. In particular, the crude mortality rates did not increase as steeply with age as reported in the official tables. The re-estimated crude rates for both males and females show a very slight decrease in mortality rates up to age 90 years from the 1950s to the 1980s, with no improvement discernible at older ages. The improvements at advanced ages in Ireland have not been as great as those in England and Wales or other developed countries over the same period.

This chapter graduates the crude rates using several popular formulae, different calibration techniques, and different evaluation criteria and extrapolates mortality curves to very advanced ages. The graduation approach recognises the remaining biases in the crude mortality rates arising from age heaping in death counts, particularly at ages 80 and 90 years. Extensions to the method of extinct generations so that mortality rates of more recent, still surviving generations, can be estimated are explored and the results reported.

The layout of the chapter is as follows. The crude rates are graduated by mathematical formula follows this introduction. After exploration with different formulae, parameter estimation approaches and evaluation criteria, the Kannisto model (a two-parameter version of

Perks's Law) is found to be robust and outperform the other models when extrapolated to ages both below and above the fitted age range. The section after evaluates the various approaches suggested to date to extend the method of extinct generations so mortality rates for non-extinct generations can be estimated and proposes a new approach. It is shown that the key advantage of the method of extinct generations is not, as hitherto supposed, in correcting for age misstatements, but in achieving a closer correspondence between death counts and the exposed to risk. Accordingly, special care must be taken in adjusting the census count to constrain extensions to the method of extinct generations. Our study with Irish data suggests a straightforward and more transparent approach to extending the method of extinct generations to estimate the likely range of mortality rates at advance ages in more recent times. We apply this new approach to provide estimates of Irish mortality at advanced ages up to 2001–2003. The final section concludes by summarising the results of our investigations, which shows a modest trend of improvement in male and female mortality at advanced ages accelerating in the most recent decades but still lagging those evident in England and Wales. Overall, we estimate that life expectancy for a 75-year-old male was 7.0 years in 1951 rising to 9.1 years in 2002, higher than the official Irish Life Table estimates of 6.8 and 8.9 years respectively. Similar underestimates are found for female life expectancies. The shape of the curve at advanced ages was also different to that recorded in the official tables, with the rate of increase in mortality rates decelerating more markedly.

Graduation of the Irish Experience

Figure 4.1 (Plate 8) highlights the need to graduate the Irish crude mortality rates found by the method of extinct generations in Chapter 3. Age heaping in the reported age of death at, say, 90 years, has created a crude mortality rate at age 90 years implausibly higher than the crude mortality rate of a 91-year-old. The crude rates in England and Wales form a more regular curve, strictly increasing with age, than the crude Irish rates. Given the problems with the Irish data and the need to extrapolate the curve to very high ages, graduation by mathematical formula is considered preferable over more data-based techniques.

Laws of Mortality

Several mathematical formulae have been suggested that might parsimoniously capture how mortality rates change with advancing years of age. Olshansky and Carnes (1997) and Forfar (2004) give an overview of such so-called 'laws of mortality'. Often the laws express a relationship between the force of mortality at age x, generally denoted μ_x and age, which is the instantaneous rate of mortality at exact age x. We have the identity

$$q_x = 1 - e^{-\int_x^{x+1} \mu_t dt} \qquad (1)$$

The approximation $q_x \cong 1 - e^{-\mu_{x+0.5}}$ or, rearranging,

$$\mu_{x+0.5} \cong -\ln(1 - q_x) \qquad (2)$$

is very good, typically introducing a relative error of less than 0.05% in the approximation. We use (2) when estimating q_x given μ_x or vice versa.
A list of classic laws of mortality would include:

Gompertz's Law (Gompertz (1825)):

$$\mu_x = \exp(\alpha + \beta x) \qquad (3)$$

Makeham's Law (Makeham (1860)):

$$\mu_x = c + \exp(\alpha + \beta x) \qquad (4)$$

Perks's Law (or Logistic Model) (Perks (1932)):

$$\mu_x = c + \frac{\exp(\alpha + \beta x)}{1 + \exp(\alpha + \rho + \beta x)} \qquad (5)$$

Perks's Law — Beard's Version (Beard (1964) (1971)):

$$\mu_x = \frac{\exp(\alpha + \beta x)}{1 + \exp(\alpha + \rho + \beta x)} \qquad (6)$$

Weibull's Law (Weibull (1951)):

$$\mu_x = a x^b \qquad (7)$$

Other models more recently suggested, based on goodness-of-fit for older ages over many mortality experiences, include:

Heligman-Pollard 1 (Heligman and Pollard (1980)):

$$q_x = \frac{GH^x}{1 + GH^x} \qquad (8)$$

Heligman-Pollard 2:

$$q_x = \frac{GH^x}{1 + KGH^x} \qquad (9)$$

Heligman-Pollard 3:

$$q_x = \frac{GH^{x^K}}{1 + GH^{x^K}} \qquad (10)$$

Perks's Law-Kannisto Version (Thatcher et al. (1998)):

$$\mu_x = \frac{\exp(\alpha + \beta x)}{1 + \exp(\alpha + \beta x)} \qquad (11)$$

For brevity, we shall shorten Heligman-Pollard to HP and Perks's Law-Kannisto Version to simply Kannisto's model.

Mortality laws can be grouped by their limiting behaviour into three distinct classes, namely, into (i) those that assume that there is a fixed age limit human life, i.e., there exists an age ω such that $q_\omega = 1$, (ii) those that assume that there is no finite fixed age limit but mortality increases with advancing age to asymptotically reach unity, i.e., $q_x < 1$ for all x but

$\lim_{x \to \infty} q_x = 1$, and (iii) those that assume the mortality peaks or reaches a plateau or asymptote below unity, i.e., there is a constant *c*, such for all *x*, $q_x \le c < 1$. It can easily be shown that none of the above laws are in the first group and that all bar HP 2 (with $O < K < 1$ when it then limits to 1/K) and Perks's Law and the Beard and Kannisto's version of it are in the second group, with these four laws being in the third group.

Fitting the Models

It was decided to fit six models to the crude Irish mortality data at advanced ages determined by the method of extinct generations. Three model types were taken from the class of laws that assumed mortality increases with age asymptotically to unity — the Makeham, HP 1 and HP 3. The other three models chosen — the Logistic, Kannisto, and HP 2 — assume that the mortality rate tends to a plateau somewhat below unity. Figure 4.2 (Plate 9) illustrates the different shape of the six curves, especially evident at advanced ages, when each curve was fitted to the reported Irish female mortality rates in 2001-2003 (CSO (2004)), by minimising the square of the relative error in the age 75 years to 100 years.

Purely statistical techniques cannot be relied on provide a satisfactory calibration of the models in the light of the data anomalies highlighted in Chapter 3. Age heaping and age exaggeration at advanced ages, the extent of which varies with time, ensures that deviations from any fitted model, even the true underlying model, will have a significant non-random component. Indeed, the considerations in Chapter 3 suggest that non-random deviations could be more significant than random deviations due to biases in the data, especially in the earlier calendar decades studied. Accordingly, the conditions are not satisfied to apply uncritically such statistical parameter estimation procedures as maximum likelihood or model selection criteria such as the Akaike Information Criterion. Extensive experiments show that models are rejected at conventional p-levels, with p-values less than 0.05% even when fit to the relatively parse data sets of mortality rates from a single calendar year or year of birth (see section later) and considerably lower than that when the data is aggregated over many calendar years. A more nuanced approach is required to model fitting and evaluation.

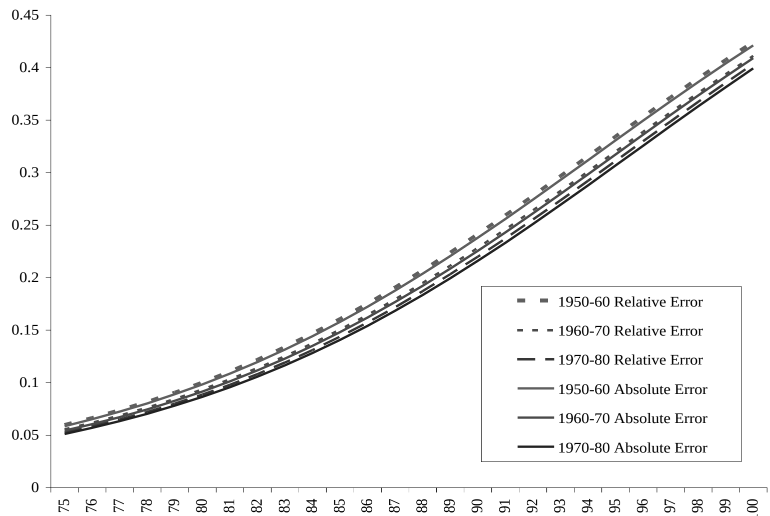
Experimentation by fitting the models over different age ranges and by different techniques revealed that no single model proved more satisfactory than any of the others: the model selected depended on the evaluation criteria. Two main approaches were settled on which, though different, produce essentially the same results. In both approaches, the models' parameters were estimated over a key part of the age range so the resultant curve adhered closely to the crude rates over that range, and the fitted curves were then extrapolated to higher and lower ages and the fit re-evaluated over these longer ranges using several different measures. Specifically, the models were fit in the age range 83 to 100 years. The parameterised model was then used to extrapolate mortality rates back to 75 years of age and over 100 years of age. The models were then assessed on:

(1) Best fit in age range 83–100 years, as determined by weighted least squares.
(2) Best fit when extrapolated back to age 75 years, as determined by weighted least squares. This criterion was introduced to ensure that the fitted formula from age 83 would blend smoothly with mortality rates up to this age. There are sufficient reliable data to enable Irish mortality rates up to the early or mid-80s to be estimated by other means.
(3) Best fit by unweighted least squares in age range 88 to 98 years. Weighted least squares gives considerably more weight to the younger ages in the age range. This criterion was introduced to test whether the fit was reasonable over the whole range 88 to 98 years.

The first method employed for parameter estimation in the fitted age range of 83 to 100 years was weighted least squares. Using a distance metric appears natural in this context, as we wish to monitor the closeness of the fitted model to the crude rates. Of course, estimation by maximum likelihood or minimum chi-square would have produced almost identical parameter estimates (Benjamin and Pollard (1980), p.320). The second method was to estimate the estimate the parameters by minimising the weighted relative square error (as described in detail in the Appendix). The motivation for this alternative estimation procedure was that, as the level of mortality changes by a factor of four times over the fitted age range 83 to 100 years, it was desirable to ensure that the model would fit with equal proportionate closeness to all ages. In the event, the models fit by the two differing procedures were reassuringly very close to one another, see Figure 4.3.

Model calibration was done on ten distinct data sets. Males and females were modelled separately on both a calendar year and cohort basis. Deaths and exposed to risk were aggregated over the 11 calendar years 1950-60, 1960-70, and 1970-80 and also related to cohorts by year of birth in 11-year ranges 1875-1885 and 1885-1895. We set out the results of the modelling exercise when the parameters were estimated by weighted least squares below and, in Appendix I, show the results when the parameters were estimated by minimising the weighted relative square error.

Figure 4.3: Kannisto Models Fit to Irish Female Crude Mortality Rates (by Method of Extinct Generations), 1950-60, 1960-70, and 1970-80 using Weighted Least Squares, Relative and Absolute Error



Of course, if one of the formulae captures the true underlying mortality curve that applied in age range from 75 years onwards, and the crude mortality rates were only subject to random fluctuations, then that model should perform well on all the tests. However, age heaping and other data anomalies ensure that no model is acceptable using purely statistical criteria of goodness-of-fit.

Figure 4.4 (Plate 10) graphs the fitted models based on the aggregated deaths and exposed to risk for males born in years 1885 to 1995 inclusive, determined by the method of extinct generations.

Evaluation of Calibrated Models

Table 4.1a summarises the results when models were fitted to male mortality rates over the age range 83 years to 100 years by weighted least squares and evaluated on criteria (1)-(3) earlier. Each numeric entry in the table gives an index of the goodness-of-fit for that model type (column heading), when fitted to the crude mortality rates over each of the five distinct periods (subgroup headings in first column), using the fit evaluation in row heading. Values in the table have been rebased to aid comparability across the two distinct measures employed (weighted least squares and unweighted least squares).

***Table 4.1a: Evaluation of Models, Irish Males, Aggregated Over Periods, by Year of Death and Year of Birth*[1]**

	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
Year of Death: 1970-80						
Weighted Least Squares, ages 83 -100	83	62	63	62	62	62
Weighted Least Squares, ages 75 -100	513	1267	1388	269	238	276
(Unweighted) Least Squares, ages 88-98	76	29	31	64	58	65
Year of Death: 1960-70						
Weighted Least Squares, ages 83 -100	110	116	118	103	113	103
Weighted Least Squares, ages 75 -100	451	1577	1755	441	368	445
(Unweighted) Least Squares, ages 88-98	136	84	82	116	122	116
Year of Death: 1950-1960						
Weighted Least Squares, ages 83 -100	162	155	156	151	152	151
Weighted Least Squares, ages 75 -100	499	1522	1683	600	589	603
(Unweighted) Least Squares, ages 88-98	406	253	245	358	355	359
Cohort, born 1875-1885						
Weighted Least Squares, ages 83 -100	97	100	101	92	92	92
Weighted Least Squares, ages 75 -100	467	1393	1558	435	442	447
(Unweighted) Least Squares, ages 88-98	85	35	33	64	65	64
Cohort, born 1885-1895						
Weighted Least Squares, ages 83 -100	64	65	67	56	56	56
Weighted Least Squares, ages 75 -100	362	1295	1462	329	335	336
(Unweighted) Least Squares, ages 88-98	47	18	17	34	35	35

[1] Values in the Table have been rebased to aid comparability across the two distinct measures employed (weighted least squares and unweighted least squares). That is, the weighted least measure (whatever the range) has been multiplied by the same scaling constant, and the unweighted least square measure has similarly been rescaled. Of course, the lower the value the better the fit.

The results in Table 4.1a produce a muddled picture. The in-sample fit gives little discrimination between the models, especially considering the data anomalies. The Makeham and Logistic laws can, perhaps, be ruled out as these models do not dove-tail nicely with mortality rates at younger ages outside the fitted range. Each of the other four models have strengths. The same remarks hold true for females as shown in Table 4.1b.

Table 4.1b: Evaluation of Models, Irish Females, Aggregated Over Periods, by Year of Death and Year of Birth

	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
1970-80						
Weighted Least Squares, ages 83 -100	124	82	84	87	89	88
Weighted Least Squares, ages 75 -100	408	2111	2572	721	795	771
(Unweighted) Least Squares, ages 88-98	71	38	36	61	69	65
1960-70						
Weighted Least Squares, ages 83 -100	171	128	128	129	129	130
Weighted Least Squares, ages 75 -100	558	1594	1915	723	754	686
(Unweighted) Least Squares, ages 88-98	130	81	78	123	127	119
1950-1960						
Weighted Least Squares, ages 83 -100	274	220	219	239	238	238
Weighted Least Squares, ages 75 -100	888	1597	1401	667	691	660
(Unweighted) Least Squares, ages 88-98	308	214	182	301	308	291
Cohort, born 1875-1885						
Weighted Least Squares, ages 83 -100	114	100	112	102	102	101
Weighted Least Squares, ages 75 -100	570	1403	1166	547	542	578
(Unweighted) Least Squares, ages 88-98	81	43	49	65	64	70
Cohort, born 1885-1895						
Weighted Least Squares, ages 83 -100	80	76	88	70	70	70
Weighted Least Squares, ages 75 -100	675	1956	1645	655	648	647
(Unweighted) Least Squares, ages 88-98	46	10	18	19	19	19

Note: Values above have been rebased to aid comparability across the two distinct measures employed (weighted least squares and unweighted least squares).

The probabilistic model underlying parameter estimation — whether weighted least squares, maximum likelihood or minimum chi-squared — tend to give considerably higher weightings to ages at the beginning of the age range fit, as the data count is higher at those ages. In the fitting procedure adopted, three-quarter of the weights applied to ages 83 to 87 years inclusive when fit over age range 83 to 100 years. This entails that the calibrated models might not be satisfactory if used at higher ages. Accordingly, we check the overall reasonableness of the models, especially at higher ages, by less formal procedures. Tables 2a and 2b show the life expectancy at ages 75, 85 and 95 years based on the crude mortality rates calculated from the data for males and females respectively, and the percentage deviation when life expectancies were calculated from the fitted models. Tables 2a and 2b also show the estimated mortality rate at age 100 years for each sex.

Tables 4.2a and 4.2b highlight the unacceptable fit of the Makeham and Logistic models when extended back to 75 years of age, as the resultant implied life expectancies understate the life expectancy calculated directly from the data by about 10%. The HP 1 also shows an unacceptably poor fit to life expectancies at ages 75 and 85 years, again giving a significant underestimate. We note that the Kannisto model most closely reproduces the life expectancies at age 75 years estimated directly from the data.

Selection of Kannisto Model

The conclusion from the modelling exercise is that the Kannisto model tended to perform reasonably well when calibrated by different methods and when extrapolated to lower ages than the range fit. Each of the other models was found not to have the same all-round robustness of the Kannisto model. The principle of parsimony also favours the two-parameter Kannisto model, as all the other fitted models except HP 1 more parameters. It would appear that the general shape of the Kannisto curve better approximates the curve of mortality rates at advanced ages.

This general conclusion is supported by Thatcher, Kannisto and Vaupel (1998) who conclude from modelling their considerable dataset of over 32 million deaths at advanced ages over the period 1960-1990 that "the logistic model and its Kannisto approximation are the best of the original six models". The six models they used were Gompertz's Law, Makeham's Law, Perks's Law, the Kannisto model, Weibull's model and HP 1 and the criteria used were primarily goodness-of-fit tests given the quality of the underlying data. The study confirmed the growing consensus that the Gompertz, Makeham, Weibull and Heligman-Pollard I give a relatively poor fit and all tended to predicted mortality rates far too high above age 100 years when fit to crude mortality rates in age range

80-98 and extrapolated.[2] This leaves Perks's Law, and its special case, the Kannisto version. On the principle of parsimony, the conclusion is to fit the Kannisto model and, should the fit not be adequate, only then attempt the more general Perks's Law.

Table 4.2a: Estimates of Life Expectancies, Various Ages, Based on Fitted Models, Irish Males, by Year of Death and Year of Birth

	Based on Data*	Percentage deviation from column (2) when life expectancies calculated from the fitted models					
	(2)	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
Based on Period 1970-80							
Life Expectancy at age 75	7.38	4.4%	-10.6%	-11.2%	-7.9%	-1.7%	-2.7%
at age 85	4.00	-1.4%	0.6%	0.7%	-6.1%	-0.3%	-0.1%
at age 95	2.07	2.1%	-1.3%	-0.5%	-1.7%	2.9%	4.5%
q_{100}	0.499	0.443	0.505	0.498	0.471	0.456	0.449
Based on Period 1960-70							
Life Expectancy at age 75	7.40	2.3%	-11.7%	-12.5%	-9.1%	-2.0%	-3.8%
at age 85	3.89	-0.5%	1.3%	1.3%	-5.5%	2.4%	0.5%
at age 95	2.11	-0.9%	-4.5%	-5.3%	-6.1%	1.3%	-0.3%
q_{100}	0.446	0.447	0.507	0.514	0.484	0.456	0.462
Based on Period 1950-60							
Life Expectancy at age 75	7.20	0.7%	-10.4%	-11.2%	-9.4%	-4.1%	-4.2%
at age 85	3.85	-1.2%	0.3%	0.4%	-6.1%	-0.3%	-0.2%
at age 95	1.92	10.3%	2.4%	1.5%	3.6%	9.5%	9.8%
q_{100}	0.572	0.440	0.517	0.524	0.479	0.459	0.458
Cohort, born 1875-1885							
Life Expectancy at age 75	7.31	1.7%	-10.2%	-11.0%	-8.1%	-2.9%	-3.0%
at age 85	3.92	-0.3%	1.0%	1.1%	-5.5%	0.5%	0.4%
at age 95	2.06	4.7%	-1.8%	-2.6%	-2.4%	3.8%	3.7%
q_{100}	0.424	0.434	0.504	0.511	0.476	0.453	0.454
Cohort, born 1885-1895							
Life Expectancy at age 75	7.44	2.1%	-10.7%	-11.5%	-8.3%	-3.2%	-3.2%
at age 85	4.00	-0.8%	0.8%	0.9%	-5.9%	0.1%	0.1%
at age 95	2.13	2.1%	-2.6%	-3.3%	-3.8%	2.2%	2.2%
q_{100}	0.428	0.433	0.494	0.500	0.470	0.448	0.448

* q_{100} from the crude data is estimated as average of rates q_{99}, q_{100} and q_{101} due to the very uneven development of the crude rates at these ages. Typically, the crude rate at age 99 years was materially higher than age 100.

[2] Weibull's model could be made to give a good fit within the range of ages 85 to 105 but "gives some highly dubious extrapolations" whether below age 85 years or at very high ages (110 to 120 years).

Table 4.2b: Estimates of Life Expectancies, Various Ages, Based on Fitted Models, Irish Females, by Year of Death and Year of Birth

	Based on Data*	Percentage deviation from column (2) when life expectancies calculated from the fitted models					
	(2)	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
Based on Period 1970-80							
Life Expectancy at age 75	9.02	0.5%	-11.0%	-12.5%	-10.1%	-5.4%	-5.3%
at age 85	4.77	-1.3%	0.3%	0.4%	-6.3%	0.2%	-0.1%
at age 95	2.40	2.3%	1.0%	0.9%	-1.0%	7.0%	6.2%
q_{100}	0.419	0.399	0.434	0.437	0.419	0.391	0.394
Based on Period 1960-70							
Life Expectancy at age 75	8.63	1.5%	-9.6%	-10.9%	-9.8%	-5.0%	-4.4%
at age 85	4.60	-1.4%	0.3%	0.4%	-6.1%	-0.1%	-0.2%
at age 95	2.28	4.1%	1.6%	1.1%	1.8%	8.8%	7.5%
q_{100}	0.439	0.409	0.452	0.458	0.426	0.402	0.408
Based on Period 1950-60							
Life Expectancy at age 75	8.11	3.3%	-9.1%	-8.4%	-7.6%	-2.7%	-2.1%
at age 85	4.40	-1.9%	1.0%	-0.2%	-6.4%	-0.5%	-0.7%
at age 95	1.98	14.8%	13.6%	9.0%	10.7%	18.2%	16.8%
q_{100}	0.594	0.421	0.462	0.485	0.446	0.422	0.428
Cohort, born 1875-1885							
Life Expectancy at age 75	8.35	0.8%	-8.9%	-7.6%	-7.1%	-1.8%	-2.6%
at age 85	4.64	-0.8%	0.1%	-0.2%	-6.2%	-0.4%	-0.3%
at age 95	2.42	5.7%	-1.4%	-4.3%	-2.0%	3.9%	5.4%
q_{100}	0.403	0.380	0.439	0.456	0.416	0.396	0.390
Cohort, born 1885-1895							
Life Expectancy at age 75	8.88	-4.5%	-11.2%	-9.8%	-9.5%	-4.5%	-4.5%
at age 85	4.86	0.5%	0.6%	0.5%	-5.9%	-0.2%	-0.2%
at age 95	2.59	9.4%	-2.0%	-4.5%	-2.9%	2.9%	3.0%
q_{100}	0.377	0.343	0.413	0.427	0.394	0.374	0.374

* q_{100} from the crude data is estimated as average of rates q_{99}, q_{100} and q_{101} due to the very uneven development of the crude rates at these ages. Typically the crude rate at age 99 years was materially higher than age 100.

One of the strengths of the Kannisto model (and the more general Perks's Law) reported by Thatcher, Kannisto and Vaupel (1998) is that they provide the best estimates of mortality rates when extrapolated above 98 years. Of course, it was not possible to form any reasonable estimate of Irish mortality at such high ages given the paucity of data. They estimate that q_{120} is between about 0.5 and 0.65 for both males and females (although perhaps the reported standard error is too low (Macdonald (2001)). Extrapolations of the models fit to the Irish data produce estimates of q_{120} within this range for the Kannisto models, and sometimes for the logistic, but are always too high and outside the range for the other models. Figure 4.5 (Plate 11) illustrates the typical pattern in extrapolation the models fit to Irish mortality rates.

The Kannisto model, as detailed earlier, has the form:

$$\mu_x = \frac{\exp(\alpha + \beta x)}{1 + \exp(\alpha + \beta x)} .$$

Accordingly, under this model, the force of mortality is always increasing with age, with its rate of increase declining, that is:

$$\frac{d}{dx}\mu_x = \beta.\left(\mu_x - \mu_x^2\right) \qquad (12)$$

Materially, the model converges asymptotically, with:

$$\lim_{x \to \infty} \mu_x = 1,$$

so that

$$\lim_{x \to \infty} q_x = 1 - e^{-1} \cong 0.632 .$$

Accordingly, one can view the Kannisto model as anchoring extremely advanced mortality rates at a level not inconsistent with large scale studies.

We conclude that Irish mortality at advanced ages is best modelled using the Kannisto model. Of course, over some time periods studied, the Kannisto model might be bettered by other models by certain criteria (for example the Logistic when fitted by minimising weighted relative error often outperforms its restricted version of the Kannisto on the goodness-of-fit criteria) but the requirement to apply a single model over many time periods so trends can be more easily identified favours the more robust and structured Kannisto model. Though it matters little (with estimated mortality rates over the age range 80-100 years always within 2% of each other), calibrating the model by minimising weighted relative error rather than weighted absolute error was considered marginally preferable. We shall minimise weighted relative error in subsequent model fitting.

Level and Trends in Irish Mortality Rates at the Highest Ages, 1950-1980

We now summarise the finding from graduating the mortality experience using the Kannisto model, fitted to the crude mortality rates determined by the method of extinct generations by minimising the sum of the weighted relative errors in the range 83 to 100 years. We shall refer to this graduation simply as the graduated Irish experience.

Figures 4.6 and 4.7 (Plates 12 and 13) well illustrate the small but clear trend of improvement in the Irish male and female mortality experience over the three decades 1950-80 and compares it with the experience in England and Wales over the same period. Table 4.3 compares how our graduated experience compares with the official Irish Life Tables at selected ages over the different decades.

Table 4.3: 1000q_x from Selected Irish Life Tables, Males, Ages 80 and over[3]

Source	*Period*	*Mortality Rate at Age x years, where x=*				
		80	*85*	*90*	*95*	*100*
Males						
ILT 10	1981-82	122	182	257	346	424
Graduated Experience	**1970-80**	**120**	**180**	**255**	**339**	**419**
ILT 8	1970-72	122	183	262	357	408
Graduated Experience	**1960-70**	**116**	**181**	**264**	**356**	**442**
ILT 6	1960-62	125	199	296	418	571
Graduated Experience	**1950-60**	**120**	**189**	**276**	**370**	**455**
ILT 5	1950-52	136	199	274	363	467
Females						
ILT 10	1981-82	87	139	209	294	394
Graduated Experience	**1970-80**	**89**	**144**	**220**	**311**	**403**
ILT 8	1970-72	97	153	226	315	415
Graduated Experience	**1960-70**	**96**	**151**	**228**	**320**	**411**
ILT 6	1960-62	104	164	243	339	454
Graduated Experience	**1950-60**	**100**	**161**	**241**	**334**	**424**
ILT 5	1950-52	117	171	235	309	395

[3] From various Irish Life Tables by Central Statistics Office and author's calculations.

We may summarise our findings over the period 1950 to 1980 as follows:

1. There has been a small but discernible trend of improvement in male and female mortality at advanced ages (of about 0.5% per year), with the rate of improvements declining with increasing age. Female mortality has been improving marginally faster than male mortality.
2. Mortality at age 100 years has been declining and it is estimated to be 0.42 for Irish males and 0.40 for Irish females in the 1970s.
3. Improvements at advanced ages in Ireland have not been as great as those in England and Wales over the same period. Mortality for males in England in the 1950s was marginally higher than that for males in Ireland at advanced ages but in the 1970s was about 5% lower. Irish female mortality was some 7% higher than that of females in England and Wales in the 1950s but the gap has widened so that in the 1970s Irish female mortality at advanced ages is about 10% higher.
4. Irish data is consistent with the hypothesis that there is no maximum lifespan and that mortality rates plateau at very advanced ages at a level below unity. The best fit model, when extrapolated to very advanced ages, tentatively suggests that the mortality rate is only 0.61 at age 125 years, (and almost identical for males and females) and the extrapolated mortality rate at this age is declining very slowly with the passage of time.

Finally, Irish mortality is at a higher level and showing more modest rates of decline that those in many developed countries as illustrated in Figure 4.8 (Plate 14).

Extending the Method of Extinct Generations

Methods to Estimate Exposed-to-risk when Generations are Not Yet Extinct

Define E_x^y as the initial exposed to risk at age x in calendar year y corresponding to the deaths aged x last birthday in calendar year y given by d_x^y. The method of extinct generations gives:

$$E_x^y = \sum_{i=0}^{\omega} d_{x+i}^{y+i} \qquad (13)$$

To apply (13) requires that by calendar year $y+\omega$, the cohort aged x in calendar year y have all died. Let us assume that everyone will die before, say, their 112th birthday. Our database comprises all deaths in Ireland, subdivided by age and sex in each calendar year from 1950 to 2006. Accordingly, the method of extinct generations allows us to estimate the exposed to risk and therefore the mortality rate at age 111 in 2006, at age 110 in 2005, …, at age 90 in 1985. We desire a way to extend the method of extinct generations to cohorts not yet extinct so we can estimate the mortality rate of, say, a 90-year-old in more recent calendar years than 1985.

Let p (for 'present') be the most recent calendar year for which we have the number of deaths. The cohort may not be all dead by the end of calendar year p and so we need an estimate of the number alive at that time, i.e., the number of survivors in the cohort at the start of calendar year $p+1$. So (13) must be modified to:

$$E_x^y = \sum_{i=0}^{i=p-y} d_{x+i}^{y+i} + E_{x+(p+1-y)}^{p+1} \qquad (14)$$

The crude mortality rate at age x in calendar year y is then given:

$$q_x^y = d_x^y \big/ E_x^y \qquad (15)$$

Three methods have been proposed to date to extend the method of extinct generations. Two involve methods to estimate E_z^{p+1}, namely, the survivor ratio method (see, for instance, Thatcher (1992)) and the Das Gupta method (Das Gupta (1990)). Another method, proposed by Andreev (1999), known as the Mortality Decline method, estimates a log-linear age-specific decline in mortality from previous cohorts to estimate the survivor count of an unexpired cohort. Our earlier analysis suggests another approach, also based on extrapolating mortality rates, which we now describe.

We can estimate the number in each cohort still alive at the end of the period based on the assumption that the Kannisto model adequately models late-life mortality. This novel approach may be summarised by the following recursive procedure:

(1) Fit a Kannisto mortality curve to the last extinct cohort and use this as an initial estimate for the mortality curve for those born one calendar year later. [Alternatively, in times of rapid mortality change, fit a Kannisto curve to each of the last *n* extinct cohorts and extrapolate the trend in the two fitted parameters to identify the Kannisto curve most likely to provide a reasonable fit to those born one year later.]

(2) Apply the Kannisto curve in (1) to estimate the exposed to risk $E^{p+1}_{x+(p+1-y)}$, that is,

$$\hat{E}^{y}_{x} = \frac{\sum_{i=0}^{i=p-y} D^{y+i}_{x+i}}{1 - {}_{p+1-y}\, p_x} \quad (16)$$

Where the mortality function is estimated from the Kannisto curve from (1). Hence, from (14), we have:

$$\hat{E}^{p+1}_{x+(p+1-y)} = \hat{E}^{y}_{x} - \sum_{i=0}^{i=p-y} D^{y+i}_{x+i} \quad (17)$$

(3) Now, with an initial estimate of $E^{p+1}_{x+(p+1-y)}$, we can calculate the crude mortality rate for the birth cohort in subsequent calendar year.

(4) Fit another Kannisto curve to the crude mortality rates so obtained, to update the best estimate of mortality curve. If there is a significant difference between the initial estimate and this update then repeat procedure from (2) using the updated estimate. Stop the iterative procedure when there is an immaterial difference between two successive iterations.

Evaluation of Different Methods to Extend the Method of Extinct Generations

A study of the relative performance of the different extensions was made in Thatcher et al. (2002), which compares their relative performance in 9 countries over a period of 35 years. The study evaluates the performance of the survivor ratio method, the Das Gupta method and the Mortality Decline method and concludes that errors from each method tend to underestimate the observed population at higher ages by the order of 5 to 15%. This is confirmed by Andreev (2004), which shows that the survivor method understates the population aged 90 years and over in

England and Wales by 8.4% over the period 1980-1995 when compared with the count by the method of extinct generations, and errors of this magnitude are not unusual (see especially Table 1 therein).

However, we note from Chapter 3 (see Figure 3.12) that the census count, suitably adjusted, can approximate the required exposed-to-risk to within 0-10% (averaging at about 5%) in an Irish context. We summarise the findings of our reconciliation attempts in Chapter 3 and in Table 4.4 and Figure 4.9. The table and graph show how close the census count, suitably adjusted, can approximate the required exposed-to-risk.

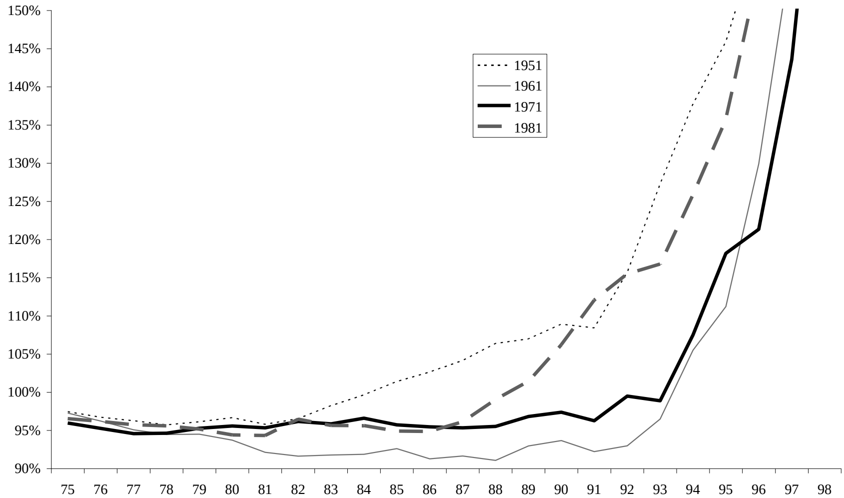
Figure 4.9 and Table 4.4 show that using the census data, suitably adjusted, is probably a more reliable way of estimating the exposed-to-risk in more recent times, than the three proposed extensions to the method of extinct generations. The adjustments to the census data are key because, as discussed in the previous chapter, the rationale for using the method of extinct generations is that it achieves a closer correspondence between the death data and the exposed to risk – not, as formerly believed, because it corrects age misstatements. Note that using census data tends to underestimate the exposed-to-risk by about 5%, no doubt largely due to an undercount of those at advanced ages. (The exception is the older age groups in 1951 which are perhaps attributable to residual age exaggeration for pension purposes.)

Table 4.4: Ratio of Adjusted Population Count in Censuses of 1951, 1961, 1971 and 1981 to Count by Method of Extinct Generations, Males and Females, Various Age Groups[4]

Age Group	1951	1961	1971	1981
Males				
75 Years & over	0.975	0.973	0.960	0.965
80 Years & over	0.967	0.937	0.956	0.944
85 Years & over	1.014	0.926	0.957	0.949
90 Years & over	1.089	0.937	0.974	1.062
Females				
75 Years & over	0.983	0.962	0.926	0.951
80 Years & over	0.972	0.931	0.920	0.936
85 Years & over	1.015	0.914	0.904	0.923
90 Years & over	1.091	0.908	0.871	1.010

[4] See Figure 3.10 and discussion in Chapter 3.

Figure 4.9: Ratio of Adjusted Population Count in Censuses of 1951, 1961, 1971, 1981 to Count by Method of Extinct Generations, Males, Various Age Groups at and over Age Point in Graph



Thatcher et al. (2002) report that the three proposed methods produce considerably better estimates of the exposed-to-risk if the initial estimates are scaled so that they are made match an independently estimated population count (say, at or above age 90 years). Such constrained methods tended to reduce the error to a 1-5% range but tend to overstate the exposed-to-risk. They concluded that in all cases the survivor ratio method when constrained to match the official estimates of population at and over age 90 years was best. A more recent study, Andreev (2004), reports that a development of Das Gupta's method performs even better for larger populations and, in particular, does not exhibit a bias like the constrained survivor ratio. However, for smaller populations — even larger than that of Ireland — the variant of the Das Gupta method shows little or no improvement over the constrained survivor method.

We attempted to estimate the number surviving in each cohort based on the assumption that the Kannisto model adequately models late-life mortality, as described in the previous subsection. Figures 4.10a and 4.10b plot the two parameters that produce the best fitting Kannisto curve for those born in each of the years 1875 to 1895, for females and males respectively. The fitting procedure is the same as that described and applied earlier. The parametric form of the Kannisto model used for fitting purposes was $\mu_x = {ae^{bx}}\Big/{1+a(e^{bx}-1)}$ to aid comparisons with models fit in Thatcher et al. (1998).

Figure 4.10a: Parameters of Best Fit Kannisto (Weighted Relative Error), Ages 83-100, Irish Female Cohorts Born 1875-1895

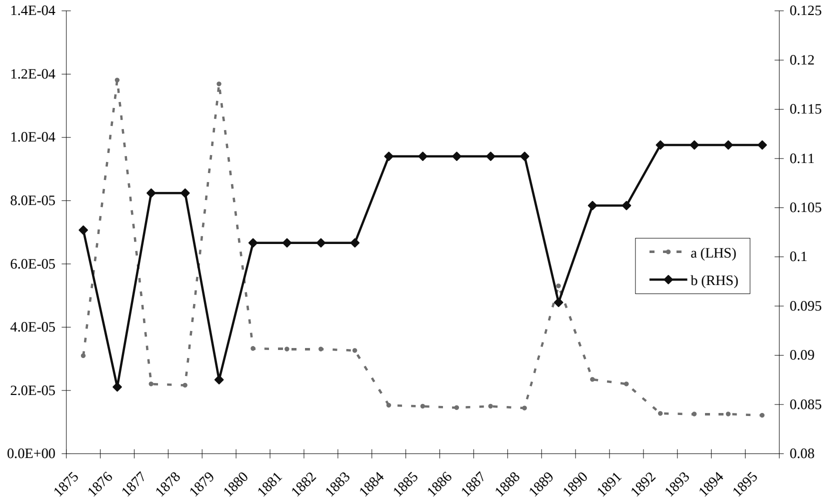
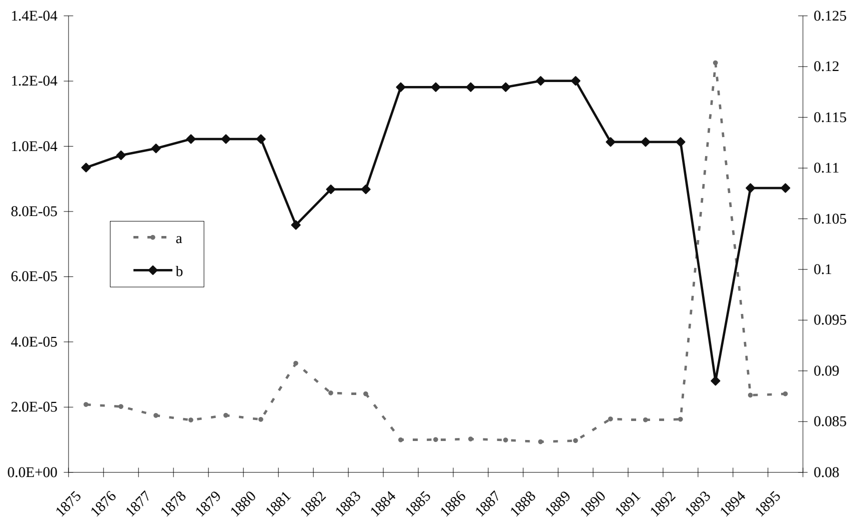


Figure 4.10b: Parameters of Best Fit Kannisto, Ages 83-100, Irish Male Cohorts Born 1875-1895



Figures 4.10a and 4.10b show that parameters *a* and *b* are dependent parameters, negatively correlated with each other, so that a fit can be found with an unusually high parameter *a* coupled with an unusually low *b* or vice versa. So the best fit parameters are very sensitive to the underlying data, with the possibility that small changes in the crude mortality rates can have a large impact on parameter estimates. For instance, take the fit to females born in 1879. The minimum chi-squared value of the best fit was 43.3 (which was close to the average across all fits), which, with 16 degrees of freedom has a p-value of 0.025%. However, taking values of *a* and *b* closer to the average observed over the twenty cohorts, with *a*= 2.15E-05 and *b*= 0.1068, the chi-squared value is 58.4 (which has an associated p-value of 0.000097%). Given the paucity of data points and the problems of age heaping identified with the data, the proposed method based on fitted Kannisto curves to estimate the numbers surviving in each cohort is simply not robust enough with the Irish data. [The analysis also shows the need to group Irish data over many calendar years or years of birth to reduce random error in the crude rates producing a rogue fit. These considerations, and the very gradual improvements with time identified, have informed the decision to group death data into 11 years groups.]

The conclusion from this subsection is that the most promising method of extending the method of extinct generations must, in some way, constrain the numbers estimated surviving at the end of the period by independent estimates. However, it appears a feature of census counts on the island of Ireland that they undercount the population at advanced ages.[5] From experiments with Irish data, we estimate that the desired survivor count can be estimated to within about 5% of the true number by suitably adjusting the census count, with a bias towards an under-estimate. This contrasts with the constrained survivor ratio, the best of the existing methods, which, when studied in other national datasets, tended to overstate the exposed-to-risk by, on average, between 1% and 5%.

[5] In the Republic of Ireland, the actual census count, unadjusted, is reported. This can be expected to always under-count the true population.

Estimating Irish Mortality at Advanced Ages in Recent Decades

The survivor ratio method attempts to estimate E_{x+1}^{p+1} based on estimating the ratio $R_x^p(k)$, defined as

$$R_x^p(k) = E_{x+1}^{p+1} \Big/ \sum_{i=0}^{k} d_{x-i}^{p-i} \qquad (18)$$

A reasonable approximation to $R_x^p(k)$ might be to use the observed ratio the calendar year earlier, that is $R_x^{p-1}(k)$, and, applying it gives:

$$E_{x+1}^{p+1} \cong R_x^{p-1}.(\sum_{i=0}^{k} d_{x-i}^{p-i}) \qquad (19)$$

So (19) approximates E_{x+1}^{p+1} using information known before time $p+1$ and hence derive mortality rates at each age up to time p.

In fact, two variants of the survivor ratio method are employed in practice:

(i) Rather than take $R_x^p(k) \cong R_x^{p-1}(k)$, we can average the ratio over the m immediately preceding cohorts so as to create a more stable ratio, i.e.,

$$R_x^p(k,m) \cong 1\!\!\big/_{m} \sum_{i=1}^{m} R_x^{p-m}(k) \qquad (20)$$

(ii) In the case that mortality is believed to be changing over the period then we could put

$$R_x^p(k,m) \cong c.R_x^{p-1}(k,m) \qquad (21)$$

for some constant c, with $c > 1$ in the case of mortality decline.

Thatcher et al. (2002) suggests employing both (i) and (ii), by taking $m = k = 5$ and determining c by the constraint that the population estimated at 90 years of age and over at time $p+1$ be made to match independent estimates of the surviving population. This variant of the constrained survivor ratio method is now used in estimating mortality at the highest ages in both the Human Mortality Database and the Kannisto-Thatcher Database.

For the Irish data, we must take appropriate values of *m*, and *k* to estimate $R_x^p(k,m)$, together with the age at which reconciliation to independent population estimates are made so *c* can be determined. For smaller populations, like that of Ireland, the estimate of $R_x^p(k,m)$ is subject to larger random fluctuations for any even *k* and *m*, suggesting that these parameter be increased for smaller populations. Simulations show that the coefficient of variation of $R_x^p(5,5)$ is about 10% in the early to mid-90s year of age in populations the size of Ireland, rising rapidly with age. The crude mortality rates deduced from the constrained survivor method also depend on how the survivor estimates are scaled to the population estimate.

In contrast to the constrained survivor method, it would appear more straightforward to simply estimate the surviving population at the end of the period. Our previous investigations suggest that, by suitably adjusting the census count, we can estimate the surviving population to within about 5% of the true surviving count at ages over 83 years. Indeed, the 5% is more a bias towards an under-count (no doubt largely due to an undercount in the census itself), rather than a varying amount. It follows that we can get achieve a reasonable extension to the method of extinct generations by simply (i) adjusting the census count as detailed in Chapter 3 so that it better corresponds to the surviving numbers, (ii) estimating the resultant crude mortality rates, and (iii) fitting a suitable curve to the crude rates as outlined earlier.

Ireland's most recent census was in April 2006. Accordingly, the most recent independent population estimate that can be used to constrain survivor estimates is at the start of 2006. We directly estimated the number of survivors of each cohort at the start of 2006 by adjusting the census count at each age as detailed in Chapter 3 and assuming that the resultant adjusted census number represented alternatively (a) 100% of the survivors or, alternatively, (b) 95% of the survivors (so multiplied by 105%). We then calculated the crude mortality rates and fitted the six mortality curves to the crude rates as outlined earlier. Full details of the goodness-of-fit statistics of the models, in the same manner as earlier fits, are given in the Appendix, Tables A.5 and Table A.6. Once again, the Kannisto model produces acceptable fits across the different datasets.

Conclusion

Table 4.5 summarises the mortality rate of males and females in Ireland at advanced ages over the five decades from the 1950s to the 1990s. We note that there is an insignificant difference over the 1990s in the two alternative approaches in estimating the survivors at the end of the period. The best fitting Kannisto curve is shown, from which the mortality rates are derived. Full details of the different models fit and the statistics of the goodness-of-fit are given in Table 4A.5 and Table 4A.6 in Appendix I.

Table 4.5: Irish Mortality at Advanced Ages, Estimated by Kannisto Model

Period	*Fitted Parameter Values of Kannisto*		*$1000q_x$ where x=*				
	$a.10^5$	*b*	*80*	*85*	*90*	*95*	*100*
Males							
1990-2000 (105%E)	2.18	0.1074	104	160	234	318	402
1990-2000 (100%E)	2.19	0.1074	105	161	235	319	404
1980-90	2.99	0.1049	115	173	247	331	412
1970-80	3.15	0.105	120	180	255	339	419
1960-70	1.36	0.115	116	181	264	356	442
1950-60	1.12	0.118	120	189	276	370	455
Females							
1990-2000 (105%E)	0.385	0.1225	66	113	182	272	370
1990-2000 (100%E)	0.403	0.1221	67	114	183	273	370
1980-90	1.29	0.110	80	127	193	275	363
1970-80	0.806	0.117	89	144	220	311	403
1960-70	0.888	0.117	96	151	228	320	411
1950-60	0.932	0.117	100	161	241	334	424

Note: The parametric form of the Kannisto model used for fitting purposes was $\mu_x = ae^{bx} / 1 + a(e^{bx} - 1)$.

We may summarise and update our earlier findings to cover the entire period 1950 to 2000, as follows:

1. The trend of improvement in male and female mortality at advanced ages has very modest in early decades but has accelerated in the most recent decades.
2. The rate of improvement declines with increasing age.
3. Female mortality has been improving marginally faster than male mortality until the early nineties. At very advanced ages no or little improvement is discernible.
4. The Kannisto model gives a reasonable fit to Irish mortality at advanced ages.
5. Irish data is consistent with the hypothesis that there is no maximum lifespan and that mortality rates plateau at very advanced ages at a level below unity. The best fit model, when extrapolated to very advanced ages, tentatively suggests that the mortality rate is only 0.61 at age 125 years, (and almost identical for males and females) and the extrapolated mortality rate at this age is declining very slowly with the passage of time.
6. Mortality rates at most advanced ages are marginally lower than recorded in the Irish Life Tables, so life expectancies at advanced ages are slightly higher than previously believed. The more recent Irish Life Tables are more accurate than earlier tables. Table 4.6 contrasts our estimate of mortality at advanced ages in Ireland in 2001-2003 with Irish Life Table 14. Details of the fit, and alternative models, are given in Tables 4A.7 and 4A.8 in Appendix I.

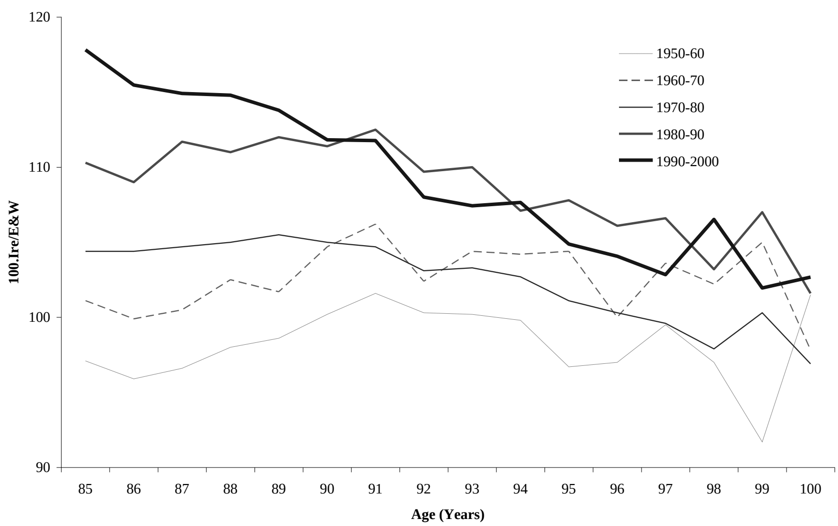
Finally, the improvement in Irish rates has lagged that observed in England and Wales, as outlined in Figure 4.11. Mortality rates in the 1950s were lower in Ireland than England and Wales in the 1950s but are now higher.

Table 4.6: Irish Mortality at Advanced Ages, 2001-03, Estimated by Kannisto Model

Period	*Fitted Parameter Values of Kannisto*		*$1000q_x$ where x=*				
	$a.10^5$	*b*	*80*	*85*	*90*	*95*	*100*
Males							
2001-03 (105%E)	1.24	0.1118	88	139	210	296	385
2001-03 (100%E)	1.46	0.1104	91	143	214	299	387
ILT 14 (2001-03)	-	-	89	145	220	313	424
Females							
2001-03 (105%E)	0.283	0.1243	57	100	164	251	350
2001-03 (100%E)	0.288	0.1245	59	103	168	257	356
ILT 14 (2001-03)	-	-	58	104	167	249	349

Note: The parametric form of the Kannisto model used for fitting purposes was $\mu_x = ae^{bx} \big/ 1 + a(e^{bx} - 1)$.

Figure 4.11: Ratio of Irish Male Mortality to that of England and Wales, Each Decade 1950-2000[6]



[6] Sources: For England and Wales, ungraduated rates in Thatcher (1992) and, for period 1994-1998 (which approximates 1990-2000), from Gallop and Macdonald (2005). Irish rates were graduated by author.

Appendix I: Evaluation of Mortality Models Fit by Weighted Relative Square Error

We experimented with other methods of fitting and evaluating the models. One approach used was to fit the models by minimising the weighted *relative* square error rather than the absolute weighted square error used previously, that is to replace $(\hat{q}_x - q_x)^2$ with $\left(\frac{\hat{q}_x - q_x}{q_x}\right)^2$ in the sum to be minimised, so the quantity to be minimised was:

$$\sum_x \frac{E_x}{q_x(1-q_x)}(\hat{q}_x - q_x)^2 \qquad \text{(II.1)}$$

The motivation for this change was that the level of mortality changes by a factor of four times over the fitted age range 83 to 100 and we wished to ensure that the model would fit with equal proportionate closeness to the lower ages as the higher ages. For computational convenience, we actually minimised the following:

$$\sum_x \frac{E_x}{\hat{q}_x(1-\hat{q}_x)}\left(\frac{\hat{q}_x - q_x}{q_x}\right)^2 \qquad \text{(II.2)}$$

As before, we fitted the models in the age range 83 years to 100 years (inclusive), extrapolating the model back to 75 years, and evaluating the models on (i) weighted relative error in fitted range 83 to 100 years, (ii) weighted relative error in range 75 to 100 years, (iii) unweighted error in range 88 to 98 years, (iv) comparing life expectancies at ages 75, 85 and 95 years calculated from the fitted models with that calculated directly from the underlying crude mortality rates and (v) comparing mortality rate estimates at age 100 years. The models fitted by this latter approach tended to better approximate the estimated life expectancy at age 75 years, but Makeham's law together with HP 2 and HP 3 produce the worse comparisons. The fitted Kannisto model tends to more closely approximate crude life expectancies. The results of this alternative model fitting approach are set out in the Tables 4A.1-8. This procedure produces almost identical parameter estimates to the weighted least squares approach used in the main body of the paper and, accordingly, leads to the identification of the Kannisto model as preferable over the other models.

Table 4A.1: Evaluation of Models, Irish Males, Aggregated Over Periods, by Year of Death and Year of Birth. Relative Error

	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
1970-80						
Weighted Least Squares, ages 83 -100	45	45	50	73	115	53
Weighted Least Squares, ages 75 -100	94	100	239	202	740	636
(Unweighted) Least Squares, ages 88-98	101	108	36	38	239	83
1960-70						
Weighted Least Squares, ages 83 -100	80	80	96	104	119	76
Weighted Least Squares, ages 75 -100	187	155	251	210	608	533
(Unweighted) Least Squares, ages 88-98	138	134	102	94	264	138
1950-60						
Weighted Least Squares, ages 83 -100	101	99	104	123	135	99
Weighted Least Squares, ages 75 -100	243	191	257	249	647	570
(Unweighted) Least Squares, ages 88-98	385	369	275	276	457	383
Cohort, born 1875-1885						
Weighted Least Squares, ages 83 -100	74	71	84	99	129	71
Weighted Least Squares, ages 75 -100	261	183	212	265	828	686
(Unweighted) Least Squares, ages 88-98	85	76	71	41	256	84
Cohort, born 1885-1895						
Weighted Least Squares, ages 83 -100	54	46	60	76	118	53
Weighted Least Squares, ages 75 -100	223	118	191	211	764	630
(Unweighted) Least Squares, ages 88-98	54	67	59	23	256	54

Table 4A.2: Estimates of Life Expectancies, Various Ages, Based on Models, Irish Males, Aggregated Over Periods, by Year of Death and Year of Birth. Relative Error

	Based on Data*	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
1970-80							
Life Expectancy at age 75	7.38	-0.1%	0.4%	-7.4%	2.6%	8.3%	7.5%
at age 85	4.00	-0.5%	-0.6%	-0.2%	-2.8%	-5.7%	-2.6%
at age 95	2.07	8.1%	8.8%	-3.6%	-7.6%	-16.3%	1.6%
q_{100}	0.499	0.419	0.415	0.510	0.510	0.551	0.447
1960-70							
Life Expectancy at age 75	7.40	1.8%	0.2%	-6.1%	1.1%	6.8%	5.9%
at age 85	3.89	0.1%	-0.2%	0.5%	-1.6%	-4.4%	-1.2%
at age 95	2.11	0.6%	0.1%	-10.8%	-10.3%	-18.5%	-1.1%
q_{100}	0.446	0.442	0.446	0.543	0.515	0.555	0.451
1950-60							
Life Expectancy at age 75	7.20	2.3%	0.6%	-6.0%	1.3%	6.9%	6.0%
at age 85	3.85	-2.7%	-2.1%	-1.9%	-3.4%	-6.4%	-3.0%
at age 95	1.92	6.1%	6.4%	-5.9%	-4.1%	-12.9%	6.0%
q_{100}	0.572	0.455	0.459	0.562	0.526	0.564	0.459
Cohort, born 1875-1885							
Life Expectancy at age 75	7.31	3.2%	1.6%	-5.2%	2.5%	8.5%	7.3%
at age 85	3.92	-1.9%	-0.8%	-0.6%	-2.2%	-5.2%	-1.9%
at age 95	2.06	-0.9%	1.0%	-10.6%	-9.5%	-18.4%	-0.4%
q_{100}	0.424	0.450	0.446	0.546	0.515	0.558	0.451
Cohort, born 1885-1895							
Life Expectancy at age 75	7.44	3.2%	0.3%	-5.7%	2.1%	8.0%	6.9%
at age 85	4.00	-2.1%	-0.3%	-0.9%	-2.7%	-5.7%	-2.4%
at age 95	2.14	-1.8%	6.1%	-10.7%	-10.2%	-19.2%	-1.3%
q_{100}	0.424	0.446	0.412	0.536	0.509	0.553	0.446

* q_{100} from the crude data is estimated as average of rates q_{99}, q_{100} and q_{101} due to the very uneven development of the crude rates at these ages. Typically the rate at age 99 years was materially higher than age 100.

Table 4A.3: Evaluation of Models, Irish Females, Aggregated Over Periods, by Year of Death and Year of Birth. Relative Error

	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
1970-80						
Weighted Least Squares, ages 83 -100	59	47	51	80	142	46
Weighted Least Squares, ages 75 -100	135	144	613	169	429	146
(Unweighted) Least Squares, ages 88-98	83	101	37	44	288	121
1960-70						
Weighted Least Squares, ages 83 -100	83	70	79	115	163	85
Weighted Least Squares, ages 75 -100	190	171	508	229	528	487
(Unweighted) Least Squares, ages 88-98	137	188	82	91	314	145
1950-60						
Weighted Least Squares, ages 83 -100	159	154	133	179	197	160
Weighted Least Squares, ages 75 -100	336	318	337	389	843	625
(Unweighted) Least Squares, ages 88-98	312	274	190	200	389	349
Cohort, born 1875-1885						
Weighted Least Squares, ages 83 -100	98	69	77	150	233	67
Weighted Least Squares, ages 75 -100	399	213	506	495	1268	309
(Unweighted) Least Squares, ages 88-98	104	134	41	62	397	127
Cohort, born 1885-1895						
Weighted Least Squares, ages 83 -100	95	56	69	140	260	54
Weighted Least Squares, ages 75 -100	263	161	707	308	957	190
(Unweighted) Least Squares, ages 88-98	88	55	10	39	481	54

Note: Values above have been rebased to aid comparability across the two distinct measures employed (weighted least squares and unweighted least squares).

Table 4A.4: Estimates of Life Expectancies, Various Ages, Based on Models, Irish Females, Aggregated Over Periods, by Year of Death and Year of Birth. Relative Error

	Based on Data*	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
1970-80							
Life Expectancy at age 75	9.02	-0.9%	-2.6%	-10.7%	-1.3%	3.1%	-0.2%
at age 85	4.78	-3.0%	-0.7%	0.0%	-3.2%	-6.5%	-0.5%
at age 95	2.40	-1.0%	8.1%	-2.6%	-7.9%	-17.9%	10.3%
q_{100}	0.419	0.403	0.365	0.444	0.452	0.501	0.358
1960-70							
Life Expectancy at age 75	8.63	0.2%	-2.0%	-9.5%	-0.3%	4.0%	4.0%
at age 85	4.60	-2.7%	-0.5%	-0.3%	-3.4%	-6.4%	-3.0%
at age 95	2.28	1.2%	11.7%	-2.5%	-7.1%	-16.1%	2.3%
q_{100}	0.439	0.411	0.368	0.464	0.467	0.510	0.407
1950-60							
Life Expectancy at age 75	8.11	2.1%	1.9%	-5.8%	2.1%	6.4%	4.9%
at age 85	4.40	-3.2%	-3.6%	-2.3%	-4.3%	-7.2%	-3.0%
at age 95	1.98	13.6%	8.8%	2.1%	2.1%	-7.0%	17.2%
q_{100}	0.594	0.424	0.451	0.518	0.493	0.531	0.411
Cohort, born 1875-1885							
Life Expectancy at age 75	8.35	3.9%	0.9%	-10.3%	3.7%	8.4%	2.6%
at age 85	4.64	-3.4%	-0.6%	0.3%	-4.4%	-7.6%	-0.5%
at age 95	2.42	-2.7%	12.5%	-1.7%	-11.6%	-20.7%	11.9%
q_{100}	0.403	0.412	0.348	0.444	0.473	0.519	0.354
Cohort, born 1885-1895							
Life Expectancy at age 75	8.88	1.6%	-1.3%	-12.7%	1.0%	5.5%	0.0%
at age 85	4.86	-3.7%	-0.6%	0.5%	-4.3%	-8.5%	-0.3%
at age 95	2.59	-5.9%	10.0%	-2.7%	-13.0%	-24.1%	10.0%
q_{100}	0.377	0.401	0.333	0.420	0.451	0.511	0.337

*q_{100} from the crude data is estimated as average of rates q_{99}, q_{100} and q_{101} due to the very uneven development of the crude rates at these ages. Typically, the rate at age 99 years was materially higher than age 100.

Table 4A.5: Evaluation of Models, Irish Males and Females, Aggregated Over Periods, by Year of Death and Year of Birth. Relative Error

	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
Males						
1990-2000 (105%E)						
Weighted Least Squares, ages 83 -100	6	5	8	31	86	6
Weighted Least Squares, ages 75 -100	8	12	365	50	445	34
(Unweighted) Least Squares, ages 88-98	19	37	7	12	266	54
1990-2000 (100%E)						
Weighted Least Squares, ages 83 -100	7	5	7	34	90	6
Weighted Least Squares, ages 75 -100	8	10	346	61	463	41
(Unweighted) Least Squares, ages 88-98	19	38	7	13	281	51
1980-1990						
Weighted Least Squares, ages 83 -100	17	18	18	39	92	29
Weighted Least Squares, ages 75 -100	28	34	230	106	529	549
(Unweighted) Least Squares, ages 88-98	106	112	30	38	256	78
Females						
1990-2000 (105%E)						
Weighted Least Squares, ages 83 -100	6	7	15	59	42	84
Weighted Least Squares, ages 75 -100	9	9	18	269	205	2128
(Unweighted) Least Squares, ages 88-98	3	11	74	53	141	181
1990-2000 (100%E)						
Weighted Least Squares, ages 83 -100	6	7	17	72	47	89
Weighted Least Squares, ages 75 -100	10	10	19	353	235	2369
(Unweighted) Least Squares, ages 88-98	3	15	81	55	151	176
1980-1990						
Weighted Least Squares, ages 83 -100	23	22	31	42	95	37
Weighted Least Squares, ages 75 -100	79	69	421	68	218	56
(Unweighted) Least Squares, ages 88-98	33	34	28	22	265	67

Note: Values above have been rebased to aid comparability across the two distinct measures employed (weighted least squares and unweighted least squares).

Table 4A.6: Estimates of Life Expectancies, Various Ages, Based on Models, Irish Males and Females, Aggregated Over Periods, by Year of Death. Relative Error

	Based on Data*	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
Males							
1990-2000 (105%E)							
Life Expectancy at age 75	8.14	-0.6%	-1.1%	-9.3%	1.2%	5.9%	1.4%
at age 85	4.37	-0.8%	-0.1%	0.7%	-3.1%	-6.3%	0.1%
at age 95	2.32	2.6%	6.7%	-3.9%	-11.3%	-20.0%	8.9%
q_{100}	0.429	-0.6%	-1.1%	-9.3%	1.2%	5.9%	1.4%
1990-2000 (100%E)							
Life Expectancy at age 75	8.09	-0.4%	-0.9%	-9.2%	1.5%	6.0%	1.6%
at age 85	4.36	-0.9%	-0.1%	0.5%	-3.3%	-6.8%	-0.1%
at age 95	2.36	0.5%	5.1%	-5.7%	-13.2%	-21.9%	6.7%
q_{100}	0.429	0.404	0.382	0.465	0.486	0.530	0.378
1980-1990							
Life Expectancy at age 75	7.59	0.0%	0.6%	-7.6%	2.5%	7.4%	7.3%
at age 85	4.13	-0.8%	-1.0%	-0.3%	-3.1%	-5.9%	-3.0%
at age 95	2.13	7.7%	8.2%	-2.9%	-7.5%	-20.0%	1.2%
q_{100}	0.444	0.412	0.408	0.495	0.499	0.582	0.439
Females							
1990-2000 (105%E)							
Life Expectancy at age 75	10.47	0.0%	-0.4%	-0.6%	2.7%	2.3%	6.5%
at age 85	5.45	-0.1%	-0.7%	-1.8%	-3.2%	-4.0%	-6.3%
at age 95	2.75	-0.8%	-5.7%	-13.9%	-17.5%	-16.5%	-16.3%
q_{100}	0.372	0.370	0.399	0.460	0.470	0.456	0.440
1990-2000 (100%E)							
Life Expectancy at age 75	10.39	0.2%	-0.2%	-0.3%	3.5%	2.5%	7.3%
at age 85	5.43	-0.2%	-1.0%	-2.0%	-2.8%	-4.2%	-5.9%
at age 95	2.75	-1.0%	-6.5%	-14.4%	-17.4%	-17.0%	-16.3%
q_{100}	0.372	0.370	0.402	0.463	0.470	0.458	0.440
1980-1990							
Life Expectancy at age 75	9.77	-2.4%	-2.1%	-7.6%	-1.2%	1.8%	-1.0%
at age 85	5.14	0.0%	-0.1%	-0.4%	-2.3%	-5.6%	-2.5%
at age 95	2.68	1.8%	1.7%	-9.3%	-10.4%	-21.6%	-11.1%
q_{100}	0.397	0.363	0.363	0.442	0.430	0.500	0.434

*q_{100} from the crude data is estimated as average of rates q_{99}, q_{100} and q_{101} due to the very uneven development of the crude rates at these ages. Typically the rate at age 99 years was materially higher than age 100.

Table 4A.7: Evaluation of Models, Irish Males and Females, Aggregated Over Periods, by Year of Death. Relative Error

	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
Males						
2001-03 (105%E)						
Weighted Least Squares, ages 83 -100	18	18	28	46	59	13
Weighted Least Squares, ages 75 -100	36	35	225	115	297	136
(Unweighted) Least Squares, ages 88-98	34	59	128	131	344	31
2001-03 (100%E)						
Weighted Least Squares, ages 83 -100	18	20	28	25	59	13
Weighted Least Squares, ages 75 -100	40	30	200	40	303	145
(Unweighted) Least Squares, ages 88-98	32	60	137	74	323	32
Females						
2001-03 (105%E)						
Weighted Least Squares, ages 83 -100	6	7	15	59	42	84
Weighted Least Squares, ages 75 -100	9	9	18	269	205	2128
(Unweighted) Least Squares, ages 88-98	3	11	74	53	141	181
2001-03 (100%E)						
Weighted Least Squares, ages 83 -100	4	5	12	86	64	88
Weighted Least Squares, ages 75 -100	11	29	38	286	250	1972
(Unweighted) Least Squares, ages 88-98	11	14	63	64	117	177

Note: Values above have been rebased to aid comparability across the two distinct measures employed (weighted least squares and unweighted least squares).

Table 4A.8: Estimates of Life Expectancies, Various ages, Based on Models, Irish Males and Females, Aggregated Over Periods, by Year of Death. Relative Error

	Based on Data*	Kannisto	Logistic	Makeham	HP 1	HP 2	HP 3
Males							
2001-03 (105%E)							
Life Expectancy at age 75	9.10	-0.9%	-1.0%	-5.1%	1.8%	3.9%	3.0%
at age 85	4.73	1.2%	0.2%	0.2%	-3.3%	-4.7%	0.3%
at age 95	2.74	-7.2%	-12.1%	-18.8%	-24.5%	-27.1%	-6.9%
q_{100}	0.378	0.385	0.413	0.476	0.495	0.504	0.381
2001-03 (100%E)							
Life Expectancy at age 75	8.93	-1.2%	-0.1%	-4.9%	-0.7%	4.4%	3.2%
at age 85	4.65	1.2%	0.8%	0.1%	-0.6%	-4.1%	0.2%
at age 95	2.71	-7.0%	-12.3%	-19.4%	-16.7%	-26.6%	-7.4%
q_{100}	0.381	0.387	0.419	0.484	0.449	0.504	0.387
Females							
2001-03 (105%E)							
Life Expectancy at age 75	11.35	0.0%	-0.4%	-0.6%	2.7%	2.3%	6.5%
at age 85	5.91	-0.1%	-0.7%	-1.8%	-3.2%	-4.0%	-6.3%
at age 95	2.96	-0.8%	-5.7%	-13.9%	-17.5%	-16.5%	-16.3%
q_{100}	0.331	0.370	0.399	0.460	0.470	0.456	0.440
2001-03 (100%E)							
Life Expectancy at age 75	11.15	-0.6%	-1.3%	-1.6%	2.7%	2.9%	5.5%
at age 85	5.81	-0.2%	-0.2%	-1.3%	-2.1%	-2.1%	-5.6%
at age 95	2.92	-1.9%	-4.2%	-12.1%	-16.6%	-15.1%	-15.5%
q_{100}	0.333	0.356	0.372	0.427	0.445	0.430	0.416

Chapter 5

Future Life Expectancies in Ireland

(Co-authored with Rabia Naqvi)

Abstract

Mortality forecasts for the Irish population are published following each census by the Central Statistics Office (CSO) as part of their Labour Force and Population Projections. The projections rely on identifying and extrapolating past trends in mortality improvements. However, since the calendar year 2011, there has been a significant slow-down in mortality improvements and, in fact, mortality rates observed at ages above 90 years increased in Ireland — a reversal of the long-term trend decline that must cause much unease to public health policymakers. The recent change in trend poses challenges when forecasting mortality rates. This paper sets out the approach eventually adopted by the CSO in the recent mortality projections, contrasts it with other extrapolative methods, including the increasingly popular stochastic and coherent methods. Comparing the outputs with these models gives a measure of the uncertainty of the future mortality forecasts for Ireland. The mortality projection for Ireland is also compared with the cohort-adjusted approach employed by the Office of National Statistics (UK) for mortality projections for Northern Ireland, Scotland, and England and Wales. We report that there are only minor differences in projected life expectancies, despite the differences in approaches and assumptions used, so we can conclude that the official mortality rates for Ireland (CSO

(2018)) and Northern Ireland (ONS (2017b)) are not inconsistent. Previous CSO mortality projections have been adopted by the actuarial profession in Ireland and others over the last decade for reserving for pension liabilities, for estimating the value of pensions, and to help judge the sustainability of the Social Insurance Fund. This detailed analysis of the CSO's most recent projections, and comparison with other mortality projections for Ireland, will help those considering its adoption for their purposes and gives a measure of the uncertainty surrounding the forecast. We conclude by setting out the implied cohort life expectancy in Ireland, based on the CSO mortality projections, to help individuals' planning for their future lifetime.

Introduction

Shortly following each quinquennial census in Ireland, the Central Statistics Office (CSO) publish population and labour force projections to aid planning of resources for the future needs of the population (e.g., CSO (2018), CSO (2013), CSO (2008)). Projecting the future mortality rates of the population form part of this exercise and, though the ultimate population and labour force forecasts are considerably less sensitive to this assumption than others (such as migration levels and fertility rates), the expert group advising the CSO devote care to this element as, over the last decade, the projections made by the CSO have been widely adopted in applications where future mortality rates are required. So, for instance, professional guidance for actuaries in Ireland when estimating the amount or value of pensions requires allowance to be made for future mortality improvements in line with the CSO rates of mortality improvements (see Society of Actuaries in Ireland (2018), SAI (2015), SAI (2014), SAI (2008)). Mortality projections have a significant impact on the results in these applications as noted in *The actuarial review of the Social Insurance Fund 2015*:

> … mortality improvement rates into the future are projected in line with the CSO Population and Labour Force Projections 2016-2046. These population projections allow for a more Irish specific view of the rate of future mortality improvements into the long term – an area of significant judgement – and materially impacting the projections.

Department of Employment Affairs and Social Protection (2017), p.43

Mortality projections following the 2016 census have recently been published together with a brief outline of the method and parameters adopted (CSO (2018)). Both authors of this paper were members of the expert group advising the CSO, and outline here more fully the factors considered before the basis on mortality projections was eventually adopted. We discuss the key issues as we view them, contrast the official projections with alternative approaches, and provide a measure of the uncertainty in the projections. We conclude by giving estimates of the remaining life expectancy (the 'cohort' life expectancy) of those alive in Ireland today based on how mortality rates are expected to evolve in future years based on the CSO 2018 projections.

The method the CSO apply to projecting mortality rates is unchanged over the last decade, and is described, including a comparison with alternative methods, in Whelan (2008). The projections in 2013, succinctly outlined in CSO (2013), followed the same general methodology but with updated parameters, (see Hall (2013a) for a full discussion). The forecasting method used by the CSO is from the popular group of 'targeting methods', where short-term trends in mortality improvements are projected to converge over the following 25 years to the underlying long-term trend of improvement observed in the past. A key issue with the CSO 2018 projections (CSO (2018)) is that short-term trends in population mortality improvements are less clear-cut than previously — it appears that there has been a significant slowing in the rate of improvements since the previous forecasts. However, the pattern of change is very uneven at the older ages in recent years, where, surprisingly, increases in mortality rates were recorded at some ages. Also, the current short-term trends in male and female mortality rates, if used unadjusted in the forecasting methodology, produced forecasts where the gender differential in future life expectancies falls below long-established historic norms. Accordingly, the recent CSO 2018 projections required more judgement in deciding what short-term trend in mortality improvements across the age spectrum and between the genders to input into the forecasting model than the more straightforward data-driven estimates that sufficed in the 2008 and 2013 projections.

The objective of this paper is to set out these and other considerations that helped inform the latest official mortality projections. There are many applications where allowance should be made for future changes in mortality rates and longevity (e.g., in planning future healthcare needs, in pension planning), some requiring a best estimate approach but others perhaps demanding a more cautious approach (such as establishing the solvency of an annuity or pension provider). So, alongside the CSO 2018 mortality forecasts, we highlight the potential range of future life expectancies using various stochastic models so the probability of life expectancies being above or below a given number can be estimated.

Indeed, the confidence with which life expectancies can be forecast could become a significant policy issue the next time the CSO is due to project the rates in five year's time. The Government commits to an actuarial assessment of life expectancies in 2022, to study of the ratio between years of life of working and expected years of life in retirement, and "at that point, informed by the review and assessment, a notice period of 13 years will be given in respect of any planned changes to the State pension age before implementation occurs" (Government of Ireland (2018) p.9 and also p.12). We contrast the methods employed and the current range of estimates of projected life expectancies on the island of Ireland made by the Central Statistics Office, by the United Nations new probabilistic model, and by the latest projections from the Office of National Statistics for Northern Ireland. We also survey the demographic and actuarial literature and apply a benchmark stochastic model for forecasting life expectancies and the associated uncertainty to Irish data. We note the extent to which the forecasts changed from the previous time made. Accordingly, we provide three distinct measures of the uncertainty surrounding forecasts of future life expectancies in Ireland: (1) the range of results obtained from different credible modelling approaches applied to Irish data; (2) the confidence bounds to estimates generated by stochastic models applied to Irish and related mortality data; and, (3) the extent to which estimates of future life expectancies in Ireland have changed in recent iterations of the models.

This paper is structured as follows. The section following this introduction overviews the trends in mortality improvement in Ireland in both the long and short-term, putting them in the context of broader international developments. It highlights a significant slowdown in the rate of improvement since 2011, especially at older ages, so that the previous CSO projections following the 2011 census (CSO (2013)) proved too optimistic in the short-term. The section after surveys the wide range of available projection methodologies. Subsections consider and critique each main approach in more depth — the CSO approach adopted for the 2018 projections, the ONS approach to forecasting for Northern Ireland adopted in 2017, the Lee-Carter stochastic model applied to Irish data, and the coherent Bayesian stochastic approach applied by the United Nations to Ireland. The next section after outlines the difference between the period life expectancies forecast by the models and the more relevant cohort life expectancies that estimate the expected remaining lifetime of individuals. Estimates of cohort life expectancies for those living in Ireland are given. The conclusion summarizes the results and the implications.

Historical Trends in Mortality Rates and Life Expectancies in Ireland

Long-term Trends

A trend of falling mortality rates with the passage of time has been observed in Ireland since the second half of the nineteenth century. The trend declines in mortality rates led to life expectancies at birth increasing by an average 0.26 years for males and 0.30 years for females with the passage of each calendar year over the twentieth century. Mortality improvements over the last century and longer were not, of course, uniform over either calendar year or year of age. At the start of the last century mortality improvements were more pronounced at the younger ages with little or no improvements discernible at older ages. As the century progressed, improvements were evidenced at all ages and most especially at the older ages in the last few decades (see Whelan (2008) for an overview, Hall (2013b) for an analysis by cause of death and Whelan (2009b), Whelan (2009c) for an analysis of trends at older ages).

Gains in Irish life expectancy came primarily from reductions in infant and child mortality during the first half of the 20th century but gains in the latter half have been due to decline in mortality rates in the final decades of life (most notably from a decline in mortality due to diseases of the circulatory system). This pattern has been called 'the aging of mortality improvements' and, as Table 5.1 illustrates, this pattern, where gains in life expectancy are more pronounced at the older ages, has continued into the early part of the 21st century.

***Table 5.1: Gains in Life Expectancy in Ireland, from Birth and from Age 65 years, by Gender, 1926-2015*[1]**

	Males			Females		
	Gains in Life Expectancy from			Gains in Life Expectancy from		
Period	*Birth*	*Age 65 years*	*Ratio of gains due to improvements after age 65*	*Birth*	*Age 65 years*	*Ratio of gains due to improvements after age 65*
1911-1926	3.8	-0.2	-5.3%	3.8	0.0	0.0%
1926-1936	0.8	-0.3	-37.5%	1.7	-0.3	-17.6%
1936-1946	2.3	-0.5	-21.7%	2.8	0.0	0.0%
1946-1961	7.6	0.6	7.9%	9.5	1.3	13.7%
1961-1971	0.7	-0.2	-28.6%	1.6	0.6	37.5%
1971-1981	1.3	0.2	15.4%	2.1	0.7	33.3%
1981-1991	2.2	0.8	36.4%	2.3	1.4	60.9%
1991-2002	2.8	2.0	71.4%	2.4	1.6	66.7%
2002-2011	3.3	2.3	69.7%	2.5	1.9	76.0%
2011-2015	1.2	0.5	41.7%	0.7	0.3	42.9%

The broad pattern of mortality improvement over the long term is not unique to Ireland: it is similar in most developed countries. Much of our current understanding of mortality improvements over the twentieth century and, indeed, since early civilisations, is summarised in surveys such as Oliver Lancaster's *Expectations of life: A study in the demography, statistics and history of world population* (Lancaster (1990)) or James Riley's more accessible *Rising life expectancy: A global history* (Riley (2001)). Riley (2001) presents a persuasive case that, in the sweep of human history, mortality reductions can be attributed to six broad (and overlapping) factors: nutrition, wealth and income, behaviour, education, public health, and medicine. The key point is that the mix can be quite different in different countries — especially countries playing catch-up such as many in sub-Saharan Africa — even though the resultant pace of mortality decline has been similar. Recent comparative studies of mortality trends across European countries over the last few decades highlight the increasing homogeneity in mortality improvement patterns leading to a convergence in life expectancies across Western Europe (see, for instance, Avdeev et al. (2011), Meslé (2004), Meslé et al. (2002)).

[1] Authors' calculations from Figures in Table 3 of CSO (2015).

Indeed, Meslé et al. (2002) argue the reason that some, mainly eastern, European countries do not exhibit such convergence is solely due to behaviorial and public health factors, principally due a a failure to curb mortality rates from lifestyle diseases. Further studies (such as Klenk et al. (2016), Leon (2011), Parr et al. (2016), Wilmoth (1998), Wilmoth (2000)) suggest that this observation also holds further afield.

Short-term Trends

Mortality rates vary significantly over the lifespan, with the mortality rate of a man aged 80 years being about 800-times greater than the mortality rate of a 10 year-old boy. Indeed, according to the latest published Irish life tables (CSO (2015)), current mortality rates imply that there is now a probability of less than 15% of an Irish person dying before their 65th birthday. Accordingly, analysis of trends in mortality rates should concentrate more on trends in mortality rates at older ages, as these are now having a greater impact on future life expectancies.

Figure 5.1 (Plate 15) graphs age-standardised mortality rates for ages 65-89 years in Ireland, Northen Ireland, England and Wales, the US, and Japan since 1980. Three different trends are common across all countries: a period of particularly rapid decline in the period 2000-2011, preceded and proceded by periods of less rapid improvements. Japan is of particular interest as it shows, despite having lower mortality rates over almost the entire period, the trend decline has been at least as steep as the other nations, and steeper since 2011 for both sexes. Life expectancy in Japan is the highest in the world and, with no signs of mortality improvements slowing, humankind is unlikely to be approaching any biological limit to human life as yet (see Oeppen and Vaupel (2002)).

Figure 5.1 (Plate 15) graphs a selection of a growing body of data that suggests there has been a significant shift in the trend of mortality improvements internationally since about 2011. The change in trend is not accounted for by one-off events causing unusually heavy mortality, such an influenza outbreak or unusual bad weather conditions (see, for example, Adams et al. (2006), Denney et al. (2013), Ng et al. (2014), Olshansky et al. (2005), Preston et al. (2018), IFoA (2017)). Analysis of subgroups of populations also report similar findings with for instance, the The Continuous Mortality Investigation of mortality underlying insurance contracts and pension schemes in the UK reporting that average mortality improvements over six years since 2011 have been 0.5% p.a. for males and 0.1% p.a. for females, significantly lower than for any other recent six-year period (C.M.I. (2018)).

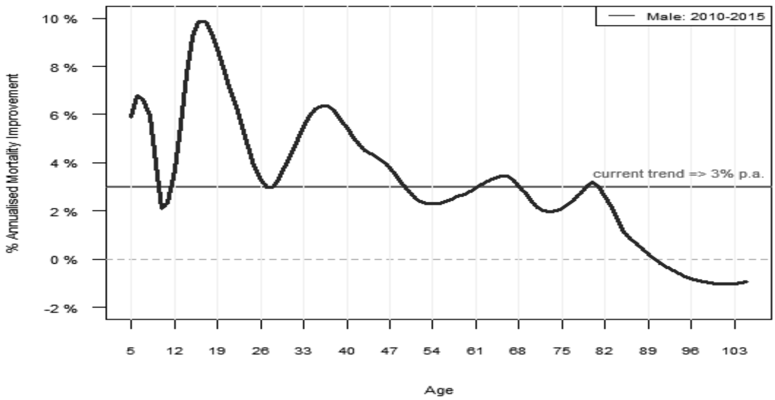
The pattern of mortality improvement by age in Ireland over the period 2010 to 2015 is presented in Figure 5.2 in greater detail. There is a broad, albeit uneven, pattern of mortality improvements reducing as age increases, with those aged above 90 years (both male and female) recording increasing mortality rates over the period. The recent trend of inceasing mortality rates at advanced ages is surprising, as it reverses the trend of slow but contant improvements at these ages over the last half-century (see Whelan (2009b)). There are, of course, issues with estimating mortality rates at these later ages due to age rounding and population mis-estimates (see Whelan (2009a)) but, having experimented with the many ways to overcome these potential problems (e.g., method of near-extinct generations and curve-fitting using the known shape of mortality at these ages), we can report that the averse pattern remains. This recent trend of mortality rates increasing at older ages must cause unease to public health officials. A more detailed analysis of recent trends are advanced ages in Ireland is given in Appendix 1.

Table 5.2 gives the annual rates of improvement over each quinquennial age group over the last decade, last five years, and last three years ending in 2015. As mentioned earlier, it is more important to estimate improvements in mortality rates at older ages accurately rather than younger ages, as its is a older ages where the vast majority of deaths occur. Accordingly, a better average rate of improvement in mortality to apply is an average weighted by deaths, which is shown in the last row of Table 5.2.

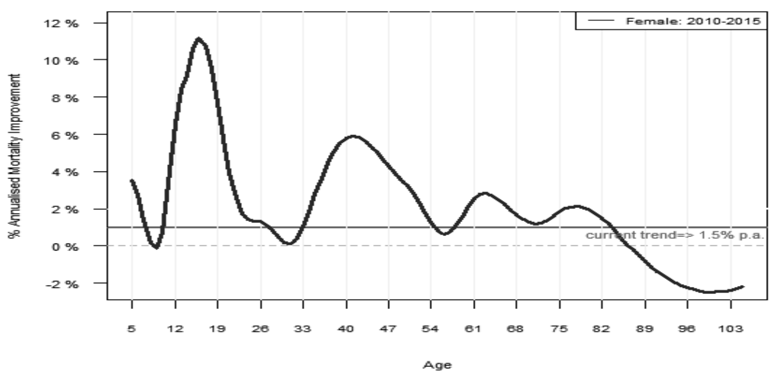
The previous mortality projections by the CSO were published in 2013 (CSO (2013)) which projected a continuation of then short-term rate of improvements of 3% per annum for males and 2.5% per annum for females (see Hall 2013a). Table 5.2 shows that, in fact, the weighted rate of improvement since turned out somewhat lower, averaging about 2.6% p.a. for males and 1.6% p.a. for females.

***Figure 5.2: Percentage Annual Rate of Mortality Improvement by Gender and Age, Ireland, 2010-2015*[2]**

(a) Males



(b) Females



[2] Authors' calculations based on data supplied by the CSO (see CSO (2018)) and CSO (2013).

Table 5.2: Annualised Improvement of Mortality Rates in Ireland over Different Age Groups and Periods Ending 2015[3]

	Males			Females		
Age	**2005-2015**	**2010-2015**	**2012-2015**	**2005-2015**	**2010-2015**	**2012-2015**
0-4	1.6%	1.7%	-0.3%	3.1%	0.9%	2.0%
5-9	8.0%	6.4%	14.0%	2.4%	2.7%	9.8%
10-14	3.4%	5.2%	6.0%	7.6%	7.1%	13.5%
15-19	6.6%	9.7%	6.2%	8.0%	10.6%	15.9%
20-24	3.7%	6.2%	7.6%	3.6%	2.5%	9.4%
25-29	1.2%	3.0%	1.9%	1.4%	1.1%	2.3%
30-34	2.1%	5.0%	5.7%	0.7%	0.5%	0.6%
35-39	1.8%	6.3%	5.4%	2.2%	4.6%	3.9%
40-44	1.9%	4.7%	5.2%	3.5%	5.8%	6.3%
45-49	2.1%	3.7%	4.8%	3.2%	4.2%	4.5%
50-54	2.1%	2.4%	3.1%	1.9%	2.2%	2.1%
55-59	2.1%	2.5%	2.1%	1.4%	0.7%	-0.1%
60-64	2.7%	3.1%	3.5%	2.4%	2.8%	3.5%
65-69	2.8%	3.2%	2.8%	1.9%	1.9%	1.2%
70-74	2.9%	1.9%	1.8%	1.8%	1.2%	0.3%
75-79	2.9%	2.3%	1.8%	2.4%	2.0%	1.6%
80-84	2.0%	2.1%	1.7%	1.9%	1.2%	1.6%
85-89	0.7%	0.4%	-0.2%	1.2%	-0.3%	0.3%
90-94	0.2%	-0.2%	-0.8%	0.2%	-1.1%	-0.3%
95-99	0.0%	-0.3%	-0.8%	0.2%	-1.1%	-0.5%
100-104	0.0%	-0.2%	-0.5%	0.3%	-0.7%	-0.3%
105-109	-0.1%	-0.1%	-0.2%	0.0%	-0.4%	-0.2%
110+	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Averages:						
10- 89 Unweighted	2.6%	3.8%	3.7%	2.8%	3.0%	4.2%
10- 89 Weighted by deaths	2.9%	3.0%	2.6%	2.4%	1.5%	1.6%

[3] Authors' calculations based on data supplied by the CSO (see CSO (2018), CSO (2013), CSO (2008)).

Methods to Project Mortality

Projections of mortality rates are typically extrapolative: projections depend on identifying and forecasting trends in mortality rates observed in the past. The evolution of mortality rates over the past in different countries share common features, notably:

(1) a near-log-linear decline of mortality rate at any particular age with time, and,
(2) the rate of decline of the mortality rate with age diminishes with increasing age.

Extrapolative techniques, generally employed by national statistical agencies and others, find and fit such relationships to past data and project mortality rates assuming the relationships to hold into the future. For a survey of the different approaches to forecasting mortality see, for instance, Booth and Tickle (2008), Pitacco et al. (2009), Stoeldraijer et al. (2013), Wong-Fupuy and Haberman (2004). Past mortality projections have tended to systematically underestimate mortality trends (Murphy (1995), Oeppen and Vaupel (2002), Keilman (2008), Waldron (2005)) and so understate future life expectancies. This has been largely due to forecasters predicting a levelling off or slowdown in the rate of mortality improvements while rates of improvement tended, in actuality, to increase.

The 'targeting' method adopted by the CSO since the 2008 projections (CSO (2008)) is a relatively straightforward version of the extrapolative approach: identified short-term trends are forecast over the short-term future and the short-term trend is blended over the future twenty-five years into a long-term rate of improvement similar to the rate of improvement observed over the long-term in the past. The Office of National Statistics (ONS (2017b)) forecasts mortality rates separately for Northern Ireland, Scotland, England and Wales in a similar manner, and produces similar forecasts to the CSO, but there are some secondary but important differences (see later). Whelan (2008) considers the CSO approach, the historic patterns in Irish mortality rates, and contrasts it with other popular approaches at that time.

The extrapolative approach employed by the CSO and other national statistical agencies, though based on relationships found in mortality rates in the past, still requires the input of experts. The forecast mortality rates depend crucially on the time period in the past that is used to determine the short-term rate of improvement input to the model, and a similar dependency exists between the long-term rate input and the long-term period used in the past. So, for instance, if the short-term trend of improvement is estimated for males using the period 2010–2015 then the (weighted) trend would be 3.0%, while if the period used is 2012–2015 then the trend is 2.6% (from Table 5.2 earlier). More significantly, if the long-term rate of improvement is estimated over, say the period from 1926 (that is, since Irish Life Table 1) or over the period since 1900 then the former period will give a different (higher) long-term rate of improvement as, in general, mortality improvements have been increasing in the more recent calendar years.

Expert judgement is exercised in the actual rates of improvement decided on, even though it may be later 'objectively' justified by a judicious selection of the periods from which to extrapolate. A second, and related, criticism of extrapolative methods is that expert judgement needs to be exercised also when forecasting mortality rates of subgroups within the same population or for two related populations. For instance, mortality forecasts are done separately for males and females in Ireland and there is an obviously, but not explicitly stated, constraint on how future mortality rates might be allowed diverge between the sexes. In particular, it is difficult to envisage an expert group standing over projections that forecast male mortality rates below female rates, as whatever the observed trends, the resultant relationship between the projected rates for the sexes is inconsistent with gender differentials observed in the past.

Unease with such implicit use of expert judgement in determining acceptable projected mortality rates has led to the development of more explicit, and more data-intensive, extrapolation techniques in the last couple of decades. First, since the seminal work of Lee and Carter (Lee and Carter (1992)), there has been particular interest in building stochastic models of mortality projections that combine future mortality forecasts with probability distributions, so that the probability that rates will be higher or lower than any particular forecast is also part of the output of the model. Second, 'coherent' projection methods have been developed over the past decade that explicitly treat the requirement of limiting the divergence between projected mortality rates of related groups exposed to similar factors influencing mortality by jointly modelling the future mortality of the related groups (Danesi et al. (2015), Li and Lee (2005), Shair et al. (2017)). Finally, combining both stochastic modelling and coherent projections with a world mortality database, the recent United Nations ('UN') forecasts of period life expectancy by country and region use a Bayesian hierarchical model (Raftery et al. (2014)), which is one of the more sophisticated and comprehensive implementation of the current art of extrapolative mortality projections. Other projections methodologies, such as the performance-weighted average of many projection models employed recently by Kontis et al. (2017) provide another way to capture the uncertainty about future trends. Reassuringly, the ensemble of 21 projection models for mortality and life expectancy employed in Kontis et al. (2017) produce broadly similar projected life expectancy at birth, country-by-country, to the recent UN forecasts.

In the next several subsections, we outline, discuss, and provide estimates of future life expectancies in Ireland based on several extrapolative techniques, including stochastic and coherent methodologies. We review the CSO approach, used in the previous 2013 projections and the current 2018 projections. We contrast the method and results with those for Northern Ireland published recently by the Office of National Statistics (ONS (2017b)) . Then we describe, fit and critique the Lee-Carter stochastic model to Irish mortality data and use it to forecast future life expectancies, together with 95% confidence bounds. The latest UN projections for life expectancy in Ireland, with their confidence bounds, are also analysed and compared with the CSO projections.

CSO Method

Crude Irish mortality rates over the most recent three calendar years are graduated to avoid the adverse effects of random fluctuations, and the resulted graduated rates are taken the base table for projections (denoted $q_{x,0}$, as the mortality rate at age x in year 0). In the exercise, particular attention is paid to graduating mortality rates at the higher ages, where there are known data issues and where random fluctuations are more material. Graduating at higher ages is done using the Kannisto formula and methods of near-extinct generations (see Whelan (2009b), Whelan (2009c)). The recent 2018 CSO projections were based on the graduated mortality experience over the three calendar years 2014-2016 (so centred on 2015).

Recent trends were then studied from analysing the change in mortality rates for each sex at each age over the previous three years, five years, and longer periods.

The method used for projecting mortality rates is to multiply the mortality rate from the base table by a cumulative reduction factor, $CRF(t,x)$, where x denotes age and t denotes the future time in years from the base year, so:

$$q_{x,t} = q_{x,0} \times CRF(t,x)$$

This projection methodology assumes that short-term rates of improvement will converge to common “target" or long-term rate of improvement at each age and for both genders, by a target year (taken to be the 25th year of projection) and continue to improve at that constant rate thereafter. Accordingly, the cumulative reduction factor is defined recursively as follows:

$$\begin{aligned} &CRF(x,1) = RF(x,1) \\ &CRF(x,t) = CRF(x,t-1).RF(x,t) \\ &t > 1 \end{aligned}$$

where

$$\begin{aligned} &RF(x,t) = 1 - \left(\frac{t}{25} f_{long} + \frac{25-t}{25} f_{short} \right) \\ &t \le 25, x \le 90 \end{aligned}$$

with

$$\begin{aligned} &RF(x,t) = 1 - f_{long} \\ &t \ge 25, x \le 90 \end{aligned}$$

For age 100 years and over, no improvements in mortality rates is assumed, so:

$$\begin{aligned} &RF(x,t) = 1 \\ &t > 0, x \ge 100 \end{aligned}$$

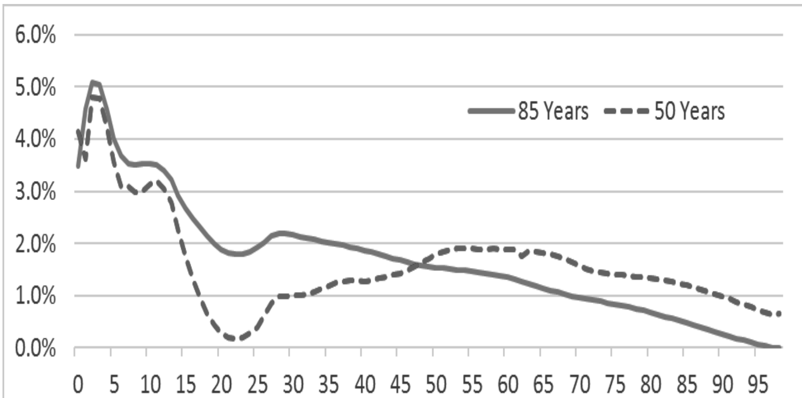
For ages between 90 and 100 year, the rate of improvement are derived by linear intrapolation between the rates at 90 years and 100 years, that is:

$$\begin{aligned} &RF(x,t) = RF(90,t).\frac{100-x}{10} + 1.\frac{x-90}{10} \\ &90 < x < 100 \end{aligned}$$

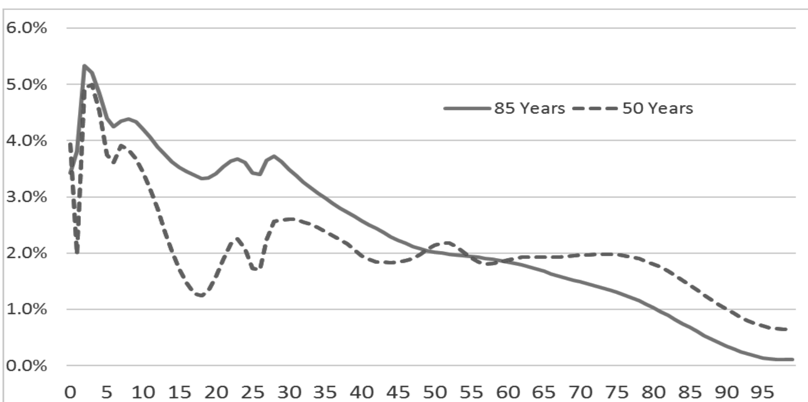
The long-term rate of improvement assumed to comtinue each year from the 25th projection year remains unalterd at 1.5% per annnum, the same as the two previous projections (CSO (2013), CSO (2008)). This rate is close to the long-term rate of both sexes at adult ages over the half century ending 2011 (that is the period before the short-term rate is estimated), as illustrated in Figure 5.3.

***Figure 5.3: Annualised Fall in Irish Mortality per Annum, Over 50 and 85 years Ending 2011, by Age*[4]**

(a) Males



(b) Females



4 Authors' calculations based on age specific mortality rates published by the CSO in Irish Life Tables 1 (1925-1927), Irish Life Tables 6 (1960-1962) and Irish Life Tables 16 (2010-2012).

The short-term rates of improvements for the previous projections were estimated to be 3.0% p.a. for males and 2.5% p.a. for females based on the average rate of improvement over 4 years to 2010 at each age (see Hall (2013a)). All other parameters were the same as for the current 2018 projections, as summarised in Table 5.4.

Table 5.4: CSO 2013 Projection Basis[5]

Base Year: 2010

Short-term Rates of Improvement

Age	Male	Female
$0-90yrs.$	$3.0\%\ p.a$	$2.5\%\ p.a.$
$91-99yrs.$	estimated by linear interpolation between assumed rate and 0% p.a. improvement at 100yrs.	estimated by linear interpolation between assumed rate and 0% p.a. improvement at 100yrs.
$100+yrs.$	$0.0\%p.a$	$0.0\%p.a$

Long-term Rates of Improvement (from 2036 onwards)

Age	Male	Female
$0-90yrs.$	$1.5\%\ p.a$	$1.5\%\ p.a$
$91-99yrs.$	estimated by linear interpolation between assumed rate and 0% p.a. improvement at 100yrs.	estimated by linear interpolation between assumed rate and 0% p.a. improvement at 100yrs.
$100+yrs.$	$0.0\%p.a$	$0.0\%p.a$

The age-specific structure of mortality improvement underwent significant changes by the time of the current 2018 projections, as outlined earlier in Table 5.2 earlier. The weighted average rate of improvement over the 5 years to 2015 was 3.0% p.a. for males but only 1.5% p.a. for females. Over the three years ending 2015, the weighted average fall in mortality rates slowed to 2.6% per annum for males but was largely unchanged for females at 1.6% per annum.

[5] CSO (2013), with further details in Hall (2013a).

Historically, this difference in life expectancy at birth has favoured females over males by around 2 to 7 years in most countries over most periods (see Kalben (2000)). If we use the weighted average rate of improvement over the 5 years to 2015 of 3.0% p.a. for males and 1.5% p.a. for females then the projected gender differential in life expectancy at birth would breach the lower historical threshold of 2 years from calendar 2036 onwards.

It was decided for the 2018 projections to adopt 2.5% per annum as the short-term rate of improvement for males and 2.0% p.a. for females. This entailing a 0.5% p.a. reduction for both genders from the 2013 projection trend rate. The resultant projection basis ensured that the gender differential in life expectancy at birth is preserved within historic limits (being 2.7 years in the calendar year 2051). The basis adopted for the CSO 2018 projections is summaried in Table 5.5.

Table 5.5: CSO 2018 Projection Basis

Base Year: 2015

Short-term Rates of Improvement

Age	Male	Female
$0 - 90 yrs.$	2.5% p.a.	2.0% p.a.
$91 - 99 yrs.$	estimated by linear interpolation between assumed rate and 0% pa. improvement at 100yrs.	estimated by linear interpolation between assumed rate and 0% pa. improvement at 100yrs.
$100 + yrs.$	$0.0\% p.a$	$0.0\% p.a$

Long-term Rates of Improvement (from 2041 onwards)

Age	Male	Female
$0 - 90 yrs.$	$1.5\% p.a$	$1.5\% p.a$
$91 - 99 yrs.$	estimated by linear interpolation between assumed rate and 0% pa. improvement at 100yrs.	estimated by linear interpolation between assumed rate and 0% pa. improvement at 100yrs.
$100 + yrs.$	$0.0\% p.a$	$0.0\% p.a$

It is of interest to compare projected life expectancies in Ireland under the 2013 and 2018 CSO projection bases, if only to see the impact that changed mortality trends in a five-year period can have on projected life expectancies. In Figures 5.4 and 5.5 (Plates 16 and 17), the projected life expectancies from each projection are graphed for future calendar years from birth and at age 65 years, for males and females separately. The impact on observed and projected life expectancies due to the slowdown in mortality improvements over the last few years is obvious in the graphs, especially so for female life expectancies.

The difference in the forecast period life expectancies due to the evolving trends over the five years is summarised in tabular from below. Most of the differences, as could be expected, come in estimating life expectancies from age 65 years.

Table 5.6: Projected Period Life Expectancy at Birth and at Age 65, by Gender and CSO Projection Basis

	Male Life Expectancy (years)		**Female Life Expectancy (years)**		**Gender difference (years)**	
	From Birth	*From Age 65 years*	*From Birth*	*From Age 65 years*	*From Birth*	*From 65 years*
Projected Values 2030						
CSO 2013 Projections	82.8	21.0	86.5	23.6	3.7	2.6
CSO 2018 Projections	82.6	20.6	85.7	22.8	3.1	2.2
Difference	***-0.2***	***-0.4***	***-0.8***	***-0.8***	***-0.6***	***-0.4***
Projected Values 2045						
CSO 2013 Projections	85.0	22.7	88.3	25.1	3.3	2.4
CSO 2018 Projections	84.8	22.3	87.6	24.4	2.8	2.1
Difference	***-0.2***	***-0.4***	***-0.7***	***-0.7***	***-0.5***	***-0.3***

Finally, we conclude this subsection by noting the sensivity of projected life expectancies to the parameters in the projection basis used by the CSO.

Table 5.7: Sensitivity of Life Expectancies to Key Parameters in Mortality Projection Basis

	Cohort Life Expectancy in 2015 (years)		**Period Life Expectancy in 2030 (years)**		**Period Life Expectancy in 2045 (years)**	
	From Birth	*From Age 65 years*	*From Birth*	*From Age 65 years*	*From Birth*	*From Age 65 years*
Male						
Central Projection Basis	89.8	20.4	82.6	20.6	84.8	22.3
Initial Decline – Up 1% p.a.	90.7	21.1	83.5	21.3	85.9	23.2
Initial Decline – Down 1% p.a.	88.9	19.8	81.6	19.9	83.7	21.5
Long-term Decline – Up 0.5% p.a.	91.8	20.6	82.8	20.8	85.6	23.0
Long-term Decline – Down 0.5% p.a.	87.5	20.2	82.3	20.4	84.0	21.7
Female						
Central Projection Basis	92.2	22.9	85.7	22.8	87.6	24.4
Initial Decline – Up 1% p.a.	92.9	23.5	86.6	23.5	88.6	25.1
Initial Decline – Down 1% p.a.	91.5	22.2	84.8	22.1	86.7	23.6
Long-term Decline – Up 0.5% p.a.	94.0	23.0	85.9	23.0	88.3	24.9
Long-term Decline – Down 0.5% p.a.	90.2	22.6	85.5	22.7	86.9	23.8

Comparing Irish Mortality Projections with those of Northern Ireland and the UK

The mortality assumptions underlying the most recent populations forecasts in the UK (the 2016-based National Population Projections) are set out in ONS (2017a) and ONS (2017b). Similar, to the approach by the CSO, the ONS use a targeting approach, blending current short term rates of improvement by age and gender to long-term uniform rates over the next 25 years. Projecions are done overall for the UK and by each constituent country (Northern Ireland, Scotland, England, and Wales), with the parameters for current trends used for Scotland being different to the other nations, reflecting its different pattern of mortality improvements over the period 1961-2015.

The key assumptions in the mortality projections for Northern Ireland and the UK overall can be summarised as:

- Long-term rate of improvement after 25 years: 1.2% per annum, for those aged under 92. For those aged between 92 and 110 the rate declines from 1.2% to 0.1% and remains at 0.1% for those aged over 110 years.
- Currently observed short-term rates of improvement, separately estimated by age and sex, were used for the first year of projection and were assumed to converge to the long-term rates over a 25 year period. Current rates of improvement were all positive and higher for males across most ages (and all ages over 50 years). Convergence from current rates of improvement to the long-term rates are assumed at the same pace for males and females, and for those born between 1940 and 1960 the convergence is by cohort.

So the reduction in mortality assumed under the two approaches are different, and perhaps the rates used are best compared in graphical and tabular form, as given below.

Figure 5.6: Graph of Cumulative Reduction Factor (CRF(t,x)) against Age x and Future Year

(a) Irish Males ***(b) UK (excl Scotland) Males***

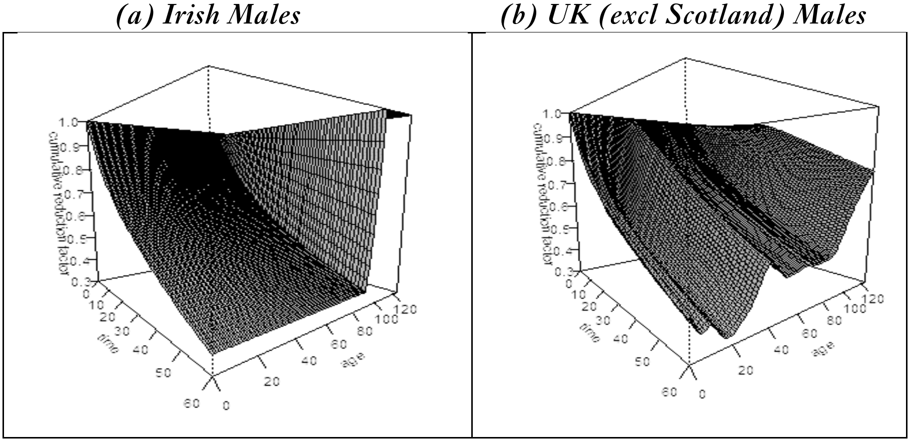


Table 5.8: Assumed Percentage Reduction in Mortality Rates by Selected Ages and Calendar Periods

	Male					
	2015-2016	**2016-2017**	**2030-2031**	**2030-2031**	**Cumulative 2015-2040 (25 year)**	**Cumulative 2016-2041 (25 year)**
Age last birthday	**Ireland[a]**	**UK (excl. Scotland)[b]**	**Ireland[a]**	**UK (excl. Scotland)[b]**	**Ireland[a]**	**UK(excl. Scotland)[b]**
0	2.5%	2.5%	1.9%	1.7%	39.4%	36.4%
5	2.5%	3.6%	1.9%	2.0%	39.4%	43.8%
10	2.5%	3.2%	1.9%	1.9%	39.4%	41.7%
30	2.5%	3.0%	1.9%	1.8%	39.4%	40.3%
50	2.5%	2.0%	1.9%	1.5%	39.4%	32.3%
60	2.5%	1.8%	1.9%	1.5%	39.4%	32.3%
70	2.5%	2.3%	1.9%	1.5%	39.4%	30.4%
80	2.5%	2.0%	1.9%	1.4%	39.4%	31.9%
90	2.5%	0.9%	1.9%	1.4%	39.4%	29.2%

	Female					
	2015-2016	**2016-2017**	**2030-2031**	**2030-2031**	**Cumulative 2015-2040 (25 year)**	**Cumulative 2016-2041 (25 year)**
Age last birthday	**Ireland[a]**	**UK (excl. Scotland)[b]**	**Ireland[a]**	**UK (excl. Scotland)[b]**	**Ireland[a]**	**UK (excl. Scotland)[b]**
0	2.0%	2.6%	1.7%	1.7%	35.5%	37.2%
5	2.0%	3.2%	1.7%	1.9%	35.5%	41.7%
10	2.0%	2.8%	1.7%	1.8%	35.5%	38.7%
30	2.0%	1.3%	1.7%	1.2%	35.5%	27.0%
50	2.0%	1.9%	1.7%	1.5%	35.5%	32.1%
60	2.0%	1.5%	1.7%	1.3%	35.5%	29.2%
70	2.0%	1.8%	1.7%	1.3%	35.5%	28.2%
80	2.0%	1.8%	1.7%	1.3%	35.5%	29.4%
90	2.0%	0.4%	1.7%	1.3%	35.5%	26.2%

The more significant differences in the projection methodologies employed is that Irish mortality rates at higher ages are projected to fall more rapidly that the ONS projections for the UK excluding Scotland. From 25 years onwards the Irish mortality rates up to age 90 are projected to fall by 1.5% per annum while in the UK the corresponding assumed rate 1.2% per annum.

Another difference in the forecasting approaches is that the UK projections allow for a cohort effect. Indeed, forecasting mortality rates by cohort has been a feature of official projections in the UK since a pattern of improvement by birth year was observed during an exploratory analysis of past trends in 1995 (Office of Population Censuses and Surveys (1995)). In particular, a so-called "golden cohort" was identified as those born between calendar years 1923 and 1938, that had higher rates of improvement than previous and subsequent generations. There ensued a debate in the actuarial literature as to whether forecasting is better done incorporating year of birth alongside age and calendar year, with arguments in favour of using such cohort projections outlined in Richards (2008), Richards et al. (2007), Willets (2004), Willets et al. (2004). However, the pattern was less convincing in Irish data (see Whelan (2008)). Whelan (2009a) argued that the pattern in the UK could well to attributed to data-mining, as the hypothesis of a cohort effect was prompted by the data, which was then used to verify the hypothesis and, as such, could be an unreliable pattern to project. Evidence was provided that even the Great Famine in Ireland did not appear to have produced a discernible cohort pattern in mortality in the generations born before, during, or after it. Recent mortality data in the UK has shown that the "golden cohort" no longer appear to experience significantly higher rates of improvement than other generations so mortality is no longer projected by cohort for this group (ONS (2017a)). However, UK forecasters still project by cohort for those born between the calendar years 1940 and 1960.

Despite the differences in short-term and long-term trends assumed, and the method used to converge the rates over the next 25 years, the recent mortality projections for Ireland, Northern Ireland, and the rest of the UK are surprisingly close as illustrated in Figure 5.7 (Plate 18). The different models for Ireland and Northern Ireland forecast life expectancy at age 65 to be within one year of one another out to 2050 (that is an initial difference of 0.2 years and 0.3 years for males and females respectively in 2015, is projected to rise to 0.4 years for males and 0.6 years for females in 2030, and further increase to 0.7 years for males and 0.8 years for females in 2045).

Another element that the experts advising on the UK projections and those advising on the projections for Ireland did not agree on was the long-term rate of improvement in mortality — that is the rate of improvement after 25 years in the future, where 1.2% was used in the

UK central assumption and 1.5% in the Irish assumptions. This is a key parameter in forecasting (see Table 5.8 earlier). The differences are due to analysing different periods in the past and using different weights to average the observed rates of improvement. ONS (2017a) states that the age-standardised rates of improvement from 1961 to 2014 (a period of 53 years) was 1.6% per annum for males and 1.3% for females; but was around 1.4% per annum for both sexes over the last three-quarters of a century and was about 1.2% per annum for both seses over the 20th century in the UK. Whelan (2008) looks at the patterns for Ireland since 1926 (Irish Life Table 1) and shows how it varies by age and, similar to the UK over the same period, suggests 1.5% per annum as reasonble for all ages up to age 90 years.

It is enlightening to see experts in other countries having similar issues with agreeing a long-term rate for mortality improvements. For over two decades now, there has been a heated debate between the Office of the Chief Actuary in the United States, who periodically investigates the financial soundness of the US social security system, and an advisory panel of experts as to what is a reasonable assumption on the long-term rate of mortality improvements (as like Ireland and UK, US projections use a single long-term rate to which all age-and-sex specific rates are assumed to converge 25 years in the future). Future mortality, especially at older ages, is a key driver of the cost of maintaining the US social security (that is the, the Old-Age and Survivors Insurance Trust Fund and the Disability Insurance Trust Fund) and this assumption has is one of the most debated, as the most recent report states:

> No other assumption has been the subject of a more persistent and unresolved disagreement between the Trustees and successive Technical Panels than that of the assumed ultimate rate of improvement in mortality rates.
>
> 2015 Technical Panel on Assumptions and Methods (2015)

The Technical Panel argue that long-term mortality improvements should be 1% per annum (the 2011 Technical Panel suggested 1.25%) while the Office of the Chief Actuary assumes 0.71%. The gap between the two has been narrowing over the last two decades as the Office of the Chief Actuary has increase its estimate.

Stochastic Methods — Lee-Carter Model Applied to Irish Data

Lee and Carter (1992) is a seminal paper in stochastic mortality forecasting, where point projections of mortality rates are accompanied by confidence intervals that give a measure of their reliability based on the underlying probability model. The relative simplicity of the model, coupled with early success, has ensured that even now, a quarter of a century later, the Lee-Carter model or one its subsequent adaptations remains a benchmark against which other stochastic models are compared (Booth and Tickle (2008), Macdonald et al. (2018), Stoeldraijer et al. (2013)). In the original model, the central mortality rates for age x at time t (denoted m_x (t)) are assumed to have the following structure:

$$\ln m_x(t) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}$$

where the α_x, β_x are age-specific parameters, κ_t describes the trend in the mortality rate over time (the so-called mortality index), and $\varepsilon_{x,t}$ are independent, identically distributed normal random variables with zero mean, and the constraints to ensure a unique solution generally being:

$$\sum_t \kappa_t = 0 \quad \sum_x \beta_x = 1$$

Mortality projection under the Lee-Carter method requires only the extrapolation of the mortality index, κ_t, since α_x, β_x are estimated from past data and held constant for the duration of the projection. The β_x measures the sensitivity at each age to changes in the overall mortality index. So, for projection purposes, this can be seen as a single parameter model based on κ_t, an underlying constant exponential rate of decline which is modified at each age by the β_x coefficient. A point to be borne in mind when interpreting the forecast rates and their uncertainty, is that the estimated β_x at high ages is low as, in the past, higher ages have experienced relatively lower mortality improvements. The uncertainty in future mortality rates in the model is proportional to β_x, which can lead to uncertainty being very low for high ages.

Lee and Carter (1992) reports that the mortality index κ_t is approximately linear for the United States over the period 1900–1987, and several sub-periods studied and, excluding the flu epidemic of 1918, the variance of κ_t also appears constant. The stability of κ_t over long periods in the past gave them confidence to base predicted future mortality rates on their model. The evolutuion of κ_t over the future was modelled as a random walk with constant drift and variance (fitted to past values), and extrapolated. Their model predicted period life expectancy of a person born in 2065 in the US would be about 10 years higher at 86 years, with a 95% confidence band of (80.45 years, 90 years) at a time when the US Government Actuary was predicting just 80.45 years.

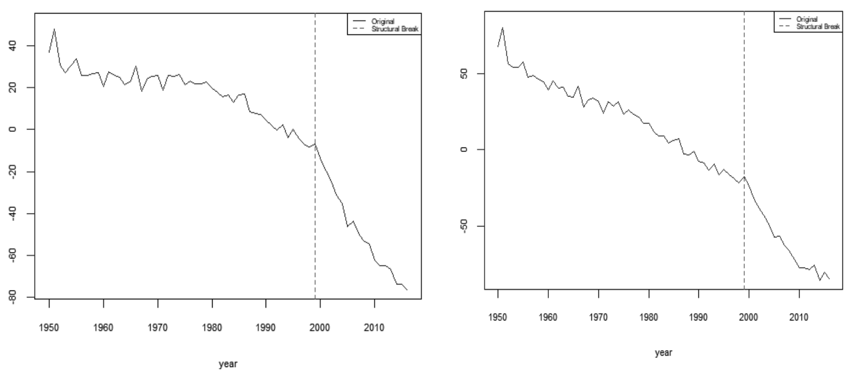
The Lee-Carter model essentially just relies on a near-log-linear decline of mortality rate at any particular age with time and, as such a pattern is evident in most countries, other demographers applied the model to other countries (Tuljapurkar et al. (2000)), and so the Lee-Carter model became widely used in forecasting mortality rates and their associated uncertainty. In fact, the Lee-Carter model can be seen as the stochastic version of the method used by the CSO in mortality projections prior to its adoption of the current method (see Whelan (2008)).

There have been developments of the original Lee-Carter model. (Booth et al. (2006)) compare the performance of four extensions to the orignal model, using data from 1986, and report no significant differences in forecast accuracy for life expectancy, but some are more accurate in estimating mortality rates. More recent extensions such as (Cairns et al. (2009), Renshaw and Haberman (2006)) introduce additional terms to deal with the so-called cohort effect postulated to exist in the UK and elsewhere (see earlier).

One key issue when applying Lee-Carter model, or one of its more recent extensions, to forecasting is the stability or otherwise of the observed trend of κ_t over past periods. Recent empirical studies report that the mortality index estimated depends to high degree on the past period studied, and in many countries over the last half-century, there is evidence of structural breaks in the historic κ_t series. Fitting the Lee-Carter model and testing for structural changes in estimated mortality indices in the period 1950-2006 for 18 developed countries, Coelho and Nunes (2011) detected the presence of significant structural change the mortality development of males, coincident with an accentuated decline in the overall rate of mortality for almost every country, including Ireland (where a break was identified in calendar year 1999). Similar evidence supporting structural change in female mortality development has been reported for only for a few countries, but those countries include Ireland (with a break also identified in calendar year 1999). It should be noted that Coelho and Nunes (2011) considered only the possibility of a single structural break during the period of the data. O'Hare (2012) studies extensions to the Lee-Carter model, including extensions to deal with the postulated cohort effect, and also reports structural breaks in the mortality index in several countries over the period 1950-2000. These empirical findings caution on the use of the Lee-Carter model, and its more recent variants, to forecast mortality rates in Ireland, as the forecast rates will depend on the past period modelled. We fit the Lee-Carter model to male and female mortality rates over the period 1950-2015 and graph the estimated κ_t overleaf.

Figure 5.8: Mortality Index for Ireland from Fitting Lee-Carter Model, 1950-2015

(a) *Males* (b) *Females*



Consider the graph of κ_t for males above (the same comments hold for females). We see a change of slope over the period, with the slope of κ_t over the period 1950-1999 being considerably lower that the slope of κ_t from 1999 to 2016. If we use the data on κ_t since 1999 to estimate the drift and variance of the random walk for future projections, then we estimate a much faster fall in mortality over future time than using the data 1950-1999 or since 1950. Indeed, this result is typical for most developed countries as mortality improvements have tended to accelerate in recent decades (see, for instance, Coelho and Nunes (2011). The conclusion is that rate of change of mortality projected in the future using the Lee-Carter model depends on the past period selected. Indeed, some researchers (such as Booth et al. (2002), Denuit and Goderniaux (2005)) suggest selecting a 'best fitting' period that ensured linearity of the trend component and extrapolating from that.

Nonetheless, it is of interest to compare forecasts made by the CSO using the targeting-based approach described earlier, to those made under the Lee-Carter approach and its associated confidence intervals. Figure 5.9 (Plate 19) graphs the projections of life expectancy at birth and at age 65 by each future calendar year generated by the unmodified Lee-Carter forecast model when fit to Irish mortality rates over the period 1980 to 2016, together with their 95% confidence interval. In the graphs the corresponding life expectancies forecast by the CSO in the 2013 and 2018 projections are shown. The CSO 2013 projections can be interpreted as projections allowing for accentuated mortality decline from 2000, while the 2018 projections can be interpreted as projections incorporating a further trend change, i.e. incorporating the recent attenuation in rates of mortality improvement.

In both cases, median life expectancy projections produced by the CSO targeting-based approach result in higher life expectancy outcomes relative to the (anticipated underestimated) outcomes of the unmodified Lee-Carter forecast model, withthe discrepancy being more pronounced for life expectancy at age 65 for males. The Lee-Carter model also forecasts an unchanging gender differential in life expectancy at birth, and a slightly increasing gender differential in life expectancy at age 65, contrary to recent trends of a reduction of the gender differential.

Coherent Methods

One issue with models, stochastic or otherwise, that treat populations separately is that forecasts of mortality for either sub-groups within the population or of other related populations can produce inconsistencies in the long term (Hyndman et al. (2013)). Coherent methods seek to overcome this issue so that projections for related populations maintain related, e.g. differences in mortality by gender within a single population can be expected to persist within observed limits in the future and projections for similar countries should not differ radically. Full joint modelling has been considered in the Li-Lee method (Li and Lee (2005)), an adaptation of the Lee-Carter method. This method limits the divergence of projections calculated for separate groups by using two components: a factor common to the entire population and another factor specific to each sub-population. The Li-Lee method is based on the following extension to Lee-Carter model:

$$\ln m_x(t,i) = \alpha_{x,i} + \beta_{x,i}\kappa_{t,i} + B_x K_t + \varepsilon_{x,t,i}$$

where the change in mortality over time described by new term $B_x K_t$ is the “common" factor for each sub-population. The term β_(x,i) κ_(t,i) denotes the specific factor of ith subpopulation which allows for differences in the rate of change in subpopulation i's death rates and the rate of change implied by the common factor. Alternatively, Jarner and Kryger (2011) considers joint modelling of a population’s mortality with a larger reference population. Other approaches can also be found, see Shair et al. (2017) for an evaluation of two more recent coherent models.

Apart from being studied in academic literature, the coherent multi-population approach has recently found its way to official population projections in the Netherlands and Canada. Moreover, recent work has sought to constitute coherent forecasting within a Bayesian paradigm. That is, to say for an unknown quantity θ and sample information x, the likelihood function $L(x|\theta)$ provides empirical information on θ (being the probability of observing the sample given θ). The prior distribution $\pi(\theta)$ represents the initial uncertainty on θ. Bayesian inference on θ is made in terms of the posterior distribution $\pi(\theta|x)$, where

$$\pi(\theta|x) \propto \pi(\theta). L(x|\theta)$$

Essentially, a Bayesian framework allows knowledge and opinions to be expressed in terms of a prior distribution, which may be transformed to the posterior distribution, $\pi(\theta|x)$, by incorporating empirical evidence, $L(x|\theta)$.

Several Bayesian treatments of mortality projections have been proposed by many authors (Czado et al. (2005), Girosi and King (2008), Kogure et al. (2009), Raftery et al. (2012), Raftery et al. (2013)). Girosi and King (2008) developed a Bayesian framework that incorporate covariates to improve mortality projections, by pooling information from similar cross-sections, e.g. age-groups, countries. Most recently, a sophisticated Bayesian model has been used by the United Nations to predict the future paths of male and female period life expectancy for each country in a coherent manner (Raftery et al. (2014)). The Bayesian framework allows the experience of another population — or, indeed, all other populations — to be readily incorporated into the modelling process by adjusting the parameters of the prior distributions.

Coherent (Bayesian) Forecasting — the Recent UN Model for Ireland
The UN Population Division issued stochastic population projections for the first time for all countries in the world in 2014 (Bijak et al. (2015)). Mortality forecasts underlying these projections were accomplished using a stochastic Bayesian hierarchical model with gains in life expectancy at birth forecast using a deterministic double logistic function with parameters drawn from a common world population (Raftery et al. (2014)) and then male life expectancies were derived from female life expectancies by projecting the gap between the sexes. The UN forecasts in a stochastic and coherent manner the life expectancies for 159 countries, comprising about 90% of the world's population (so excluding some 38 countries with AIDS epidemics because of their very different mortality patterns and 30 countries with populations under 100,000).

It is of interest to contrast the CSO mortality projections for Ireland with the latest UN forecasts. In Figure 5.10 (Plate 20), we graph the life expectancy at birth under both projection approaches, including the 95% confidence intervals of the UN approach. It should be noted that the UN adjusted their standard model for Ireland as it found that the rate of mortality improvement since 1950 was out of line with similar countries and so adjustments were made to the default projection trajectory (UN (DESA), pp.26–27).

The CSO predict the higher life expectancy at birth in 2030 at 0.3 years higher for males and 0.5 years higher for females, increasing to 0.6 years for females and remaining unchanged for males in 2045 (see Table 5.13 later). It is notable that while UN median projections of life expectancy for females have remained stable over the projections years, greater variability is evident in case of males. UN median projections of life expectancy at birth for males have come to be more aligned, since the 2012 iteration, with those produced by the targeting approach — this might be due to greater coherence between genders being imposed within the UN model in later iterations. Importantly, the gender differential in life expectancy at birth is projected to decrease by both models, and is closely matched in 2030 and 2045. The difference between the models in life expectancy at at age 65 in 2030 is 0.7 years for both males and females, increasing to 0.8 and 0.9 years for males and females respectively in 2045. For females, UN median estimates of life expectancy at birth and age 65 present some challenges — the estimates generated are lower than the estimates produced by the unmodified Lee-Carter model (see earlier).

Cohort Life Expectancy in Ireland

The latest published population life table for Ireland, Irish Life Table 16 (CSO (2015)), provides estimates of the 'period' life expectancy at different ages, for both males and females, which serves as a useful tool to make comparisons of trends over time, and between geographical areas. However, the period life expectancy does estimate reliably how much longer an individual might survive on average, as the Background Notes to Irish Life Tables No 16 make clear:

> Period expectation of life at a given age for 2010-12 is the average number of years a person would live if he or she experienced age-specific mortality rates for that time period throughout his or her life. It is therefore not the number of years someone of that age could actually expect to live because death rates are likely to change in the future.
>
> CSO (2015), first paragraph of Background Notes

The cohort approach to life expectancy directly addresses the problem of how long an individual at a particular age can be expected to live on average in the future. The cohort life expectancy is estimated by adjusting recently experienced mortality rates at each age by projecting future changes to these mortality rates as the individual ages. So, for example, a girl aged 5 years now will be aged 55 years in five decades' time so, in estimating the cohort life expectancy, the current mortality rate of a 55-year-old woman is adjusted to reflect how that mortality rate is expected to change over the next half-century. Projected mortality rates are estimated for each future age at each future period and these projected mortality rates are then used in the calculation of the cohort life expectancy (rather than the historic mortality rates as used to calculate the period life expectancy).

The mortality projection method used by the CSO in population and labour force projections can be applied to to estimate the remaining cohort life expectancy for a person alive in Ireland at the current time. We have estimated the period and cohort life expectancies in Ireland in the calendar year 2020. Such cohort life expectancies have not been published before, despite there importance to an individual planning for the future, such as helping to estimate how much to save for retirement. Irish period and cohort life expectancies on the CSO mortality projection basis used in CSO (2018) are shown at birth and each decennial age in Table 5.9, and are set out in full in Appendix II. It can be seen that there are substantial differences between cohort and period life expectancy due to expected improvements in mortality over future time periods.

Table 5.9: Projected Period and Cohort Life Expectancies in Ireland in 2020 from 2018 CSO Projection Basis, by Gender and at Selected Ages[6]

	Males			**Females**		
Age In 2020	*Period LE in 2020*	*Cohort LE in 2020*	*Gap*	*Period LE in 2020*	*Cohort LE in 2020*	*Gap*
0	80.5	90.4	9.9	84.2	92.7	8.5
10	70.9	79.6	8.7	74.4	82.1	7.7
20	61.0	68.6	7.6	64.5	71.2	6.7
30	51.3	57.7	6.4	54.6	60.3	5.7
40	41.7	46.8	5.1	44.8	49.4	4.6
50	32.2	36.1	3.9	35.2	38.6	3.4
60	23.3	25.9	2.6	26.0	28.4	2.4
70	15.2	16.6	1.4	17.4	18.8	1.4
80	8.4	9.1	0.7	9.9	10.5	0.6
90	4.0	4.1	0.1	4.7	4.8	0.1
100	1.8	1.8	0.0	2.1	2.1	0.0

[6] Authors' calculation.

Table 5.10 shows how estimates of period and cohort life expectancies in the calendar year 2020 have changed from the previous estimates five years ago to the current CSO estimates.

Table 5.10: Selected CSO Projected Period and Cohort Life Expectancies in 2020, by Gender and Projection Basis

	Males			Females		
Projection Basis	*Period LE in 2020*	*Cohort LE in 2020*	*Gap*	*Period LE in 2020*	*Cohort LE in 2020*	*Gap*
From Birth						
2013 Projections	80.5	90.5	10.0	84.8	93.2	8.4
2018 Projections	80.5	90.4	9.9	84.2	92.7	8.5
From Age 65 Years						
2013 Projections	19.5	21.5	2.0	22.3	24.2	1.9
2018 Projections	19.1	21.1	2.0	21.6	23.4	1.8

Finally, it is of interest to compare estimates of cohort life expectancies by the CSO method, with those of the UN for Ireland and those of the ONS for Northern Ireland.

The cohort life expectancy at age 65 for both males and females have been calcilated from latest available UN life table data. UN life table data is presented in an abridged form in roughly 5 year age groups (up to end age interval 85+ years), each by quinquenal period from 1950 to 2100; the survivor function, l_x, is also available seperately in similar form but with end age interval 100+ years . Several methods exist to extricate cohort life expectancies from such available abridged life table data, including polynomial interpolation, osculatory interpolation, cubic spline interpolation. By using osculatory interpolation, namely Karup-King's third difference method (King (1914), Siegel and Swanson (2004)) and further cohort-wise interpolation by ordinary least squares method with yearly steps, we constructed cohort life tables from the available data published by the UN. In Table 5.11 we set out the estimated cohort life expectancy at age 65 for males and females and compare the extent of differences between the projection methods.

Table 5.11: Projected Period and Cohort Life Expectancies at Age 65 in 2020, by Gender and Projection Method

	Males			Females		
Method	*Period LE in 2020*	*Cohort LE in 2020*	*Gap*	*Period LE in 2020*	*Cohort LE in 2020*	*Gap*
From Age 65 years						
Target Method - CSO	19.1	21.1	2.0	21.6	23.4	1.8
Target Method - ONS (Northern Ireland)	19.0	20.7	1.7	21.2	22.8	1.6
Coherent Method (Bayesian) - UN	18.5	20.1	1.6	21.1	22.4	1.3

Finally, we conclude by indicating the sensitivity of the period and cohort life expectancies estimated using the CSO approach to changes in the parameters for short-term and long-term rates of mortality decline.

Table 5.12: Sensitivity of Estimates of Life Expectancies Estimated under the CSO Approach to Changes in the Parameters for Short-Term and Long-Term Rates of Mortality Decline

		Cohort Life Expectancy in 2020 from age 65 (years)	*Period Life Expectancy in 2020 from age 65 (years)*
Male	Central Projection Basis	21.1	19.1
	Initial Decline – Down 1% p.a.	20.3	18.8
	Long-term Decline – Down 1.2% p.a.	20.3	19.1
Female	Central Projection Basis	23.4	21.6
	Initial Decline – Down 1% p.a.	22.7	21.3
	Long-term Decline – Down 1.2% p.a.	22.6	21.5

Conclusion

This paper outlines several mortality projection methodologies favoured by official statisticians and academic demographers, and calculated future life expectancies when the different models are applied to Irish data. Table 5.13 summarises some key outputs from these models. It shows that the CSO 2018-based projections forecast higher life expectancies than either of the Lee-Carter model applied to Irish data, or latest UN forecasts for Ireland, and a higher increase in life expectancies than the ONS for Northern Ireland. However, as detailed earlier in subsections treating each methodology, the differences are small in a probablistic sense — that is, given the large uncertainty inherent in such forecasts, the forecast rates are reasonably close.

Irish mortality data (like data from other regions) on which the models are calibrated show quite a mixed pattern of changing trends — accelerating and slowing and, at some advanced ages sometimes showing no improvement or even negative trends. Accordingly, the confidence intervals around the above central estimates are wide and widen with each year ahead forecast. It is at ages above age 65 years that most of the uncertainty arises in estimating life expectancies, as changes to the already very low mortality rates at younger ages has comparative minor impact on life expectancy. Figure 5.11 (Plate 21) graphs the expected trajectory of period life expectancies at age 65 years under each of models as calendar years roll on.

There are proposals to link the State pension age with life expectancies from the calendar year 2035 (Government of Ireland (2018)), with a review of the State pension age already planned for the calendar year 2022. A central issue in this review will be how reliably future life expectancies can be estimated. It must not be supposed that the extra mortality data gathered over the next four years or refinements in forecasting techniques in the meantime will help narrow the uncertainty inherent in modelling future mortality. There is more than enough data already on the course of human mortality — from across the regions of the world and across the recent millennia. We ignore most of the past data as it is irrelevant to the future — as todays causes of deaths have changed from the age-old biblical causes of "by the sword, by famine, by plague, and by the wild animals of the earth". Nor will developments in statistical forecasting technique help much, as the past is only a limited guide to the future, in human mortality as much as in the rest of human destiny.

Table 5.13: Period Life Expectancy in Ireland at Birth and at Age 65 by Gender, Observed in 2015, and Projected to 2030, and to 2045 using Different Projection Methods

	Male Life Expectancy (years)		**Female Life Expectancy (years)**		**Gender difference (years)**	
	From Birth	*From Age 65 years*	*From Birth*	*From Age 65 years*	*From Birth*	*From Age 65 years*
Observed 2015 Ireland	79.4	18.2	83.3	20.9	3.9	2.7
Projected Values to 2030						
Target Method - CSO	82.6	20.6	85.7	22.8	3.1	2.2
Stochastic Method (Lee-Carter)	81.6	19.4	85.5	22.2	3.9	2.8
Coherent Method (Bayesian) - UN*	82.2	19.9	85.2	22.1	3.0	2.2
Projected Values to 2045						
Target Method - CSO	84.8	22.3	87.6	24.4	2.8	2.0
Stochastic Method (Lee-Carter)	83.4	20.7	87.3	23.6	3.9	2.9
Coherent Method (Bayesian) -UN*	84.5	21.5	87.0	23.5	2.6	2.0
Observed 2016 Northern Ireland	78.8	18.4	82.3	20.6	3.5	2.2
Projected Values to 2030						
Target Method – ONS for Northern Ireland	81.4	20.2	84.5	22.2	3.1	2.0
Projected Values to 2045						
Target Method – ONS (Northern Ireland)	83.3	21.6	86.3	23.6	3.0	2.0

Mortality rates fell markedly in the past century and longer due to significant improvements in nutrition, housing, public health, education, and medicine. This, in turn, was achieved only by a significant allocation of resources by the individual and the state to achieve this end. Future improvements will require further significant resource allocation and these resources must be directed towards those of older ages (often termed "economically" unproductive). The state plays a significant role in providing income, health care and other services to this subgroup in Ireland, so any changes to such provision can be expected to have an impact on mortality trends.

The suggestion currently mooted is that the future State pension age be set relative to future life expectancy so that the proportion of working life to years in retirement be kept roughly constant, perhaps in the ratio 2:1 (Government of Ireland (2018)). If such a scheme is agreed upon, then it can be construed as a social contract — that the state commits to directing resources to achieving the forecast increases in life expectancies at older ages. Such an understanding would require annual monitoring of mortality improvements against the target rates, and corrective actions in the form of resource allocations if there is significant deviation. Viewed in such a way, the projections of life expectancies earlier are reasonable targets, believed achievable with a reasonable allocation of resources. With this perspective, the trend in mortality rates in Ireland at ages 90 and over in the last few years would raise an alarm as previous gains in life expectancies are being lost. This also alters the emphasis from mortality forecasting to the more important exercise of monitoring mortality improvements against reasonable targets to help in the allocation of resources. As the British demographer John Hajnal remarked:

> … as little forecasting as possible should be done … Forecasts should flow from analysis of the past. Anyone who has not bothered with analysis should not forecast.
>
> Hajnal, J. (1955) The prospects of population forecasts. JASA, 50, 309–22 (p. 321)

Appendix I: Trends in Irish Mortality at Late Ages

Declines in mortality at older ages have contributed significantly to gains in life expectancy over recent decades, as noted earlier. However, for ages above about 85 years, data issues become increasingly significant and pose challenges in estimating mortality rates and therefore trends in mortality rates at these ages. Problems with the data include age rounding, population mis-estimates, and deaths not corresponding exactly to exposed to risk estimates (see Whelan (2009a) for a full discussion on the Irish data, and Cairns et al. (2016) for an international overview).

There are several methods to improve the reliability of mortality estimates at late ages, providing better estimates than using the traditional census estimates for the exposed to risk. Vincent (1951) provides an early method using death data to estimate the exposed to risk, but his method of "extinct generations" can only be employed retrospectively when all the members of a given generation have died. However, "nearly extinct" cohort methods can use the age pattern of deaths in past cohorts to be predict future deaths in current cohorts. Such "near extinct" methods include the Das Gupta method and the more general Survivor Ratio method (Terblanche and Wilson (2015)). We employed the Das Gupta method to help improve estimates of mortality rates at ages above 85 years in Ireland over recent calendar years.

Das Gupta Method (unconstrained)

The Das Gupta (Das Gupta (1990)) method relies on estimating the 'death ratio' ${}_m r_{x,t}$. Define $D(t,x)$ as the number of deaths occurring during year t amongst those aged x last birthday at the start of the year, then:

$$D(t,x) = {}_m r_{x,t} \times D(t-1, x-1)$$

where the experience of several previous cohorts is used to estimate the death ratio ${}_m r_{x,t}$. So,

$${}_m r_{x,t} = \frac{\sum_{j=1}^{m} D(t-j, x)}{\sum_{j=1}^{m} D(t-j-1, x-1)}$$

$$\Rightarrow P(t,x) = \sum_{i=1}^{\omega-x+1} \left(\prod_{j=1}^{i} {}_m r_{x+j-1,t+j-1}\right) \times D(t-1,x-1)$$

Moreover, it follows that the probability of dying at age x last birthday can be expressed as:

$$q_{x,t} = \frac{1}{1 + \sum_{i=2}^{\omega-x+1}\left(\prod_{j=2}^{i} {}_m r_{x+j-1,t+j-1}\right)}$$

Survivor Ratio Method (unconstrained)

This method assumes that the ratio of the survivors of a cohort to the sum of deaths experienced by cohorts in the previous several years is relatively constant and that once this ratio has been estimated, it can be applied to the current cohort to estimate the survivors.

Defining $P(T,x)$ as the number of persons aged x last birthday at the start of the current year T, we have:

$$P(T,x) = s_{x,T} \times D(T-i, x-i)$$

where the 'survivor ratio' $s_{x,T}$

$$s_{x,T} = s_{x,T-1} \times \frac{P(T-1,x)}{\sum_{i=1}^{k} D(T-i-1, x-i)}$$

In both cases, the experience of several previous cohorts is used to estimate the ratio to reduce the effect of random fluctuations.

Constraining estimates produced by either method above to available external population estimates can help to improve performance — unconstrained methods typically produced errors between 5%–15% while constrained methods typically produced errors between 1%–5% (Thatcher et al. (2002)). Applying the (unconstrained) Das Gupta method and comparing resulting population estimates to ages over 90 years in year 2000 against population estimates obtained by method of extinct cohorts to evaluate performance, the total relative error was 13.6% for males and 8.3% for females.

Irish Data

Checks on the reliability of Irish data at late ages are initiated by considering the trajectory of crude estimates of probabilities of death over a range of ages, computed from CSO data using the census as follows:

$$q'_{x\_CSO} = \frac{m'_x}{1 + {m'_x}/{2}}$$

where m'_x is the central death rate (for base year y) calculated as follows:

$$m'_x = \frac{d_x^{(y-1)} + d_x^{(y=2015)} + d_x^{(y+1)}}{3 \times \theta_x^{(y=2015)}}$$

with $\theta_x^{(y=2015)}$ being the population estimates in mid-calendar year y.

These rates were compared against corresponding indirect estimates derived by the Das Gupta method and the latest estimates from Irish Life Table No. 16 (relating to period 2010-2012) to identify any irregularities, which inform the next correction steps. Results from the data reliability checks and graduation are presented below for males and females respectively.

Irish Males

Figure A5.1 (Plate 22) graphs male mortality rates in Ireland in 2015 estimated using the census method and Das Gupta method and compares the rates with those of the most recent graduated experience, Irish Life Table 16 centred in year 2011.

The graph highlights an issue with the crude rates estimated by the census method at ages above 94 years, where it shows an implausible decline in mortality rates. Applying the Das Gupta method produces a more reasonable trajectory of increasing mortality rates. Table A5.1 sets out the data underlying Figure A5.1 (Plate 22).

The final column of Table A5.1 (Plate 22) labelled 'imputed' rates is a weighted average of the (CSO) census estimate and Das Gupta estimate (allowing for estimation error) for ages 95 to 98 years. We now fit various models of late-life mortality rates to the census rates and to the imputed rates, see Figure A5.2 (Plate 23). These models include here variants of the Heligman-Pollard model, the Kannisto, the Log-Quadratic, and Makeham's Law (see Whelan (2009c) for further details including details of the fitting procedures).

Table A5.1: Comparison of Mortality for Males in Ireland, Late Ages, using Census Method and Das Gupta method, 2014-16

	CSO Data		Probability of Death			Relative Error (%)		
Age x	Population Estimate, $\theta_x^{(y=2015)}$	Deaths, d'_x	q'_{x_CSO}	$q'_{x\_DasGupta}$	q_{x_ILT16}	$\frac{q'_{x\_DasGupta}}{q'_{x\_CSO}}$	$\frac{q_{x\_ILT16}}{q'_{x\_CSO}}$	Imputed Probability of death, q^*_x
80	8308	1430	0.0558	0.0719	0.0639	28.9%	14.6%	0.0558
81	7653	1556	0.0656	0.0808	0.0712	23.3%	8.6%	0.0656
82	6651	1585	0.0764	0.0904	0.0792	18.3%	3.7%	0.0764
83	6001	1531	0.0816	0.0989	0.0881	21.3%	8.1%	0.0816
84	5500	1508	0.0874	0.1088	0.0980	24.4%	12.1%	0.0874
85	4521	1509	0.1054	0.1239	0.1089	17.6%	3.3%	0.1054
86	3640	1445	0.1241	0.1449	0.1208	16.8%	-2.7%	0.1241
87	2928	1357	0.1434	0.1608	0.1340	12.1%	-6.6%	0.1434
88	2513	1281	0.1566	0.1708	0.1484	9.1%	-5.3%	0.1566
89	2063	1110	0.1646	0.1867	0.1641	13.4%	-0.3%	0.1646
90	1675	1052	0.1895	0.2014	0.1813	6.3%	-4.4%	0.1895
91	1139	894	0.2314	0.2235	0.2000	-3.4%	-13.6%	0.2314
92	786	694	0.2566	0.2481	0.2201	-3.3%	-14.2%	0.2566
93	584	606	0.2949	0.2600	0.2419	-11.8%	-18.0%	0.2949
94	438	459	0.2974	0.2854	0.2652	-4.0%	-10.8%	0.2974
95	351	345	0.2815	0.3160	0.2902	12.2%	3.1%	0.3056
96	256	216	0.2466	0.3376	0.3168	36.9%	28.5%	0.3103
97	165	151	0.2647	0.3360	0.3448	26.9%	30.3%	0.3146
98	140	106	0.2241	0.3406	0.3675	52.0%	64.0%	0.3057

The mortality curves are close to one another within the data range, where there is much greater variably in the progression of the estimated mortality rates from the data than across the fitted mortality curves. The imputed data gives a better fit but the fitted curves in either (a) or (b) remain very close to one another. Summary details of the fit of the curves to the rates are given in Table A5.2.

Table A5.2: Evaluation of Fit of the Mortality Curves to (a) Male Census Rates and (b) Male Imputed Rates, 2014-2016.

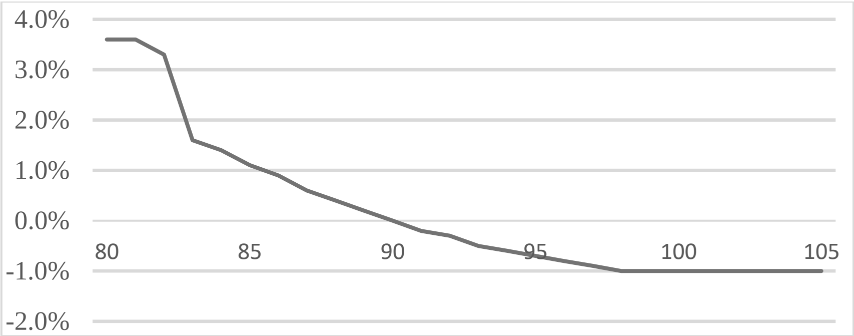
(a) Census rates	**Logistic**	**Kannisto**	**LogQuad**	**Makeham**	**HP 1**	**HP 2**	**HP 3**
Sum of relative square error, ages 75- 98	0.450037	0.460079	0.350540	0.661269	0.571981	0.311196	0.399189
Sum of absolute square error, ages 75- 98	0.024841	0.026200	0.018152	0.035228	0.031445	0.016842	0.021735
(a) Imputed rates	**Logistic**	**Kannisto**	**LogQuad**	**Makeham**	**HP 1**	**HP 2**	**HP 3**
Sum of relative square error, ages 75- 98	0.177786	0.132725	0.157800	0.303377	0.175399	0.130391	0.158049
Sum of absolute square error, ages 75- 98	0.011593	0.008173	0.009074	0.015065	0.012602	0.007171	0.009104

It is difficult to decide on the best fitting curve, given that they are all so close to one another. The widely observed deviation of mortality from the exponential increase at very old ages is a phenomenon called "late-life mortality slow-down (Manton et al. (2008), Wachter (1999)). The Kannisto model, a logistic-type curve with a horizontal asymptote has been shown to give a good fit to mortality rates at higher ages for many populations, including the population of Ireland (Whelan (2009c)). Accordingly, we adopt the Kannisto curve as it was also adopted in estimating the base table for the 2013 projections (see Hall (2013a)).

Figure A5.3 (Plate 24) graphs the Kannisto model fitted to (a) the census rates non-adjusted (Kannisto 1 curve) and (b) the imputed rates (Kannisto 2 curve) for the three calendar years centred 2015. The results are compared with base rates for the CSO 2013 projection exercise, which relate to the mortality rates centred on calendar year 2010.

We can now estimate the annualised rate of improvement of mortality rates for Irish males at these advanced ages, by comparing the rates given by the Kannisto 2 curve for the three calendar years centred 2015 to the base rates of the CSO 2013 projection, relating to calendar year 2010 (see Hall (2013a)).

Figure A5.4: Annualised Rate of Improvement, 2010-2015, Irish Males at Older Ages



Irish Females

Mortality rates for Irish females in the three calendar years centred in 2015 are shown graphically in Figure A5.5 (Plate 25), when the crude rates are calculated using the census method (CSO) and the Das Gupta method. The graph also compares the mortality rates with those of Irish Life Table 16, centred in calendar year 2011.

The crude rates are increasing with age, unlike the pattern seen earlier for Irish males, so there is no need to experiment with imputed rates before graduating them. We can see from the graph that mortality rates in 2015 were higher than the rates for 2011 for ages over age 89 years. Table A5.3 sets out the rates graphed above.

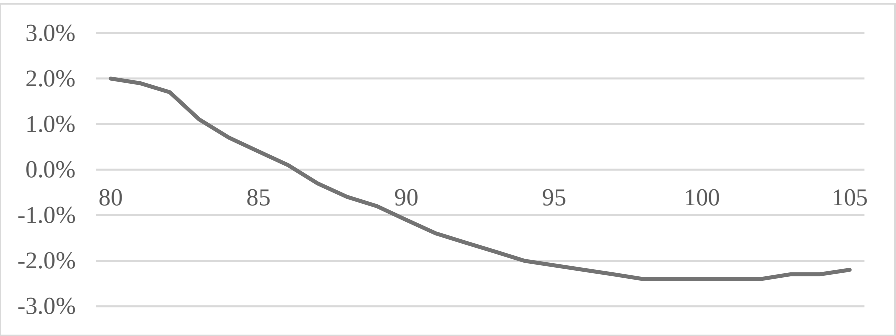
Table A5.3: Comparison of Mortality for Females in Ireland, Late Ages, using Census Method and Das Gupta Method, 2014-2016

	CSO Data		Probability of Death			Relative Error (%)	
Age, x	Population Estimate, $\theta_x^{(y=2015)}$	Deaths, d'_x	q'_{x_CSO}	$q'_{x\_DasGupta}$	q_{x_ILT16}	$\frac{q'_{x\_DasGupta}}{q'_{x\_CSO}}$	$\frac{q_{x\_ILT16}}{q'_{x\_CSO}}$
80	10335	1172	0.0371	0.0481	0.0417	29.7%	12.4%
81	9625	1319	0.0447	0.0561	0.0474	25.7%	6.2%
82	8671	1416	0.0530	0.0637	0.0540	20.2%	2.0%
83	8201	1485	0.0586	0.0712	0.0616	21.6%	5.1%
84	7834	1625	0.0668	0.0816	0.0702	22.1%	5.0%
85	7063	1600	0.0728	0.0899	0.0798	23.5%	9.7%
86	6278	1768	0.0897	0.1022	0.0907	14.0%	1.2%
87	5478	1783	0.1029	0.1185	0.1028	15.2%	-0.1%
88	4880	1740	0.1122	0.1299	0.1162	15.8%	3.6%
89	4307	1829	0.1322	0.1420	0.1308	7.4%	-1.1%
90	3682	1705	0.1433	0.1619	0.1466	13.0%	2.3%
91	2823	1680	0.1805	0.1883	0.1635	4.3%	-9.4%
92	2613	1497	0.1743	0.2043	0.1813	17.2%	4.0%
93	1691	1264	0.2216	0.2196	0.1999	-0.9%	-9.8%
94	1281	1108	0.2520	0.2431	0.2189	-3.5%	-13.2%
95	1017	837	0.2412	0.2542	0.2379	5.4%	-1.4%
96	774	699	0.2617	0.2835	0.2567	8.3%	-1.9%
97	505	506	0.2862	0.3199	0.2747	11.78%	-4.01%
98	402	385	0.2753	0.3369	0.2916	22.39%	5.92%

Similar to the analysis for males, we graduated the female mortality rates at these late ages using several mortality laws and selected the Kannisto model, although all models give very similar graduated rates. The best fitting Kannisto to the 2014-2016 experience is plotted against the Kannisto fit to the 2009-2011 experience used as the base table for the CSO 2013 projections (Hall (2013a)). (See Figure A5.6: Plate 26).

The graph shows that female mortality rates over age 89 years were higher in 2015 than five years earlier. We graph the annualised difference in the mortality rates in Figure A5.7.

Figure A5.7: Annualised Rate of Improvement, 2010-2015, Irish Females at Older Ages



Conclusion of Appendix

We have studied the trends in mortality rates at advanced ages in Ireland over recent years, using various methods to correct well-known issues with the data. Estimates of rates of change of mortality rates, whether calculated from crude mortality rates estimated using the census method (see Table A5.2), or after graduation (of crude rates or of imputed values), all conclude that mortality rates for both males and females have increased over the last years for those aged over 90 years. The increase in mortality is higher for females. This recent trend of inceasing mortality rates at advanced ages is surprising and must cause alarm to public health officials, as it reverses the trend of slow but contant improvements at these ages in Ireland over the last half-century (see Whelan (2009b)).

Appendix II: *Table A5.4 Life Expectancy Tables: Projected Period and Cohort Life Expectancies in Ireland in 2020 on CSO 2018 Projection Basis, by Gender and Single Year of Age*

	Males			Females		
Age in 2020	**Period LE in 2020**	**Cohort LE in 2020**	**Gap**	**Period LE in 2020**	**Cohort LE in 2020**	**Gap**
0	80.5	90.4	9.9	84.2	92.7	8.5
1	79.8	89.6	9.8	83.4	91.8	8.4
2	78.8	88.5	9.7	82.4	90.8	8.4
3	77.8	87.4	9.6	81.4	89.7	8.3
4	76.8	86.3	9.5	80.4	88.6	8.2
5	75.8	85.2	9.4	79.4	87.5	8.1
6	74.9	84.1	9.2	78.4	86.4	8.0
7	73.9	83.0	9.1	77.4	85.4	8.0
8	72.9	81.9	9.0	76.4	84.3	7.9
9	71.9	80.8	8.9	75.4	83.2	7.8
10	70.9	79.6	8.7	74.4	82.1	7.7
11	69.9	78.5	8.6	73.4	81.0	7.6
12	68.9	77.4	8.5	72.4	79.9	7.5
13	67.9	76.3	8.4	71.5	78.8	7.3
14	66.9	75.2	8.3	70.5	77.7	7.2
15	65.9	74.1	8.2	69.5	76.6	7.1
16	64.9	73.0	8.1	68.5	75.5	7.0
17	63.9	71.9	8.0	67.5	74.5	7.0
18	62.9	70.8	7.9	66.5	73.4	6.9
19	62.0	69.7	7.7	65.5	72.3	6.8
20	61.0	68.6	7.6	64.5	71.2	6.7
21	60.0	67.5	7.5	63.5	70.1	6.6
22	59.1	66.4	7.3	62.5	69.0	6.5
23	58.1	65.3	7.2	61.5	67.9	6.4
24	57.1	64.2	7.1	60.5	66.8	6.3
25	56.2	63.1	6.9	59.5	65.7	6.2
26	55.2	62.1	6.9	58.6	64.6	6.0
27	54.2	61.0	6.8	57.6	63.5	5.9
28	53.3	59.9	6.6	56.6	62.5	5.9
29	52.3	58.8	6.5	55.6	61.4	5.8
30	51.3	57.7	6.4	54.6	60.3	5.7
31	50.4	56.6	6.2	53.6	59.2	5.6
32	49.4	55.5	6.1	52.7	58.1	5.4
33	48.4	54.4	6.0	51.7	57.0	5.3
34	47.5	53.3	5.8	50.7	55.9	5.2

35	46.5	52.3	5.8	49.7	54.8	5.1
36	45.5	51.2	5.7	48.7	53.7	5.0
37	44.6	50.1	5.5	47.7	52.7	5.0
38	43.6	49.0	5.4	46.8	51.6	4.8
39	42.7	47.9	5.2	45.8	50.5	4.7
40	41.7	46.8	5.1	44.8	49.4	4.6
41	40.7	45.7	5.0	43.8	48.3	4.5
42	39.8	44.6	4.8	42.9	47.2	4.3
43	38.8	43.5	4.7	41.9	46.1	4.2
44	37.9	42.5	4.6	40.9	45.1	4.2
45	36.9	41.4	4.5	40.0	44.0	4.0
46	36.0	40.3	4.3	39.0	42.9	3.9
47	35.0	39.2	4.2	38.0	41.8	3.8
48	34.1	38.2	4.1	37.1	40.8	3.7
49	33.2	37.1	3.9	36.1	39.7	3.6
50	32.2	36.1	3.9	35.2	38.6	3.4
51	31.3	35.0	3.7	34.2	37.6	3.4
52	30.4	34.0	3.6	33.3	36.5	3.2
53	29.5	32.9	3.4	32.4	35.5	3.1
54	28.6	31.9	3.3	31.4	34.4	3.0
55	27.7	30.9	3.2	30.5	33.4	2.9
56	26.8	29.9	3.1	29.6	32.4	2.8
57	25.9	28.9	3.0	28.7	31.4	2.7
58	25.0	27.9	2.9	27.8	30.4	2.6
59	24.2	26.9	2.7	26.9	29.4	2.5
60	23.3	25.9	2.6	26.0	28.4	2.4
61	22.4	24.9	2.5	25.1	27.4	2.3
62	21.6	23.9	2.3	24.2	26.4	2.2
63	20.8	23.0	2.2	23.3	25.4	2.1
64	19.9	22.0	2.1	22.5	24.4	1.9
65	19.1	21.1	2.0	21.6	23.4	1.8
66	18.3	20.2	1.9	20.7	22.5	1.8
67	17.5	19.3	1.8	19.9	21.5	1.6
68	16.7	18.4	1.7	19.0	20.6	1.6
69	15.9	17.5	1.6	18.2	19.7	1.5
70	15.2	16.6	1.4	17.4	18.8	1.4
71	14.4	15.8	1.4	16.6	17.9	1.3
72	13.7	15.0	1.3	15.8	17.0	1.2
73	13.0	14.2	1.2	15.0	16.1	1.1
74	12.3	13.4	1.1	14.2	15.3	1.1
75	11.6	12.6	1.0	13.5	14.4	0.9
76	10.9	11.8	0.9	12.7	13.6	0.9

77	10.3	11.1	0.8	12.0	12.8	0.8
78	9.6	10.4	0.8	11.3	12.0	0.7
79	9.0	9.7	0.7	10.6	11.2	0.6
80	8.4	9.1	0.7	9.9	10.5	0.6
81	7.9	8.4	0.5	9.3	9.8	0.5
82	7.3	7.8	0.5	8.6	9.1	0.5
83	6.8	7.2	0.4	8.0	8.4	0.4
84	6.3	6.6	0.3	7.5	7.8	0.3
85	5.8	6.1	0.3	6.9	7.2	0.3
86	5.4	5.7	0.3	6.4	6.7	0.3
87	5.0	5.2	0.2	5.9	6.1	0.2
88	4.6	4.8	0.2	5.5	5.7	0.2
89	4.3	4.4	0.1	5.1	5.2	0.1
90	4.0	4.1	0.1	4.7	4.8	0.1
91	3.6	3.7	0.1	4.3	4.4	0.1
92	3.4	3.4	0.0	3.9	4.0	0.1
93	3.1	3.1	0.0	3.6	3.7	0.1
94	2.8	2.9	0.1	3.3	3.3	0.0
95	2.6	2.6	0.0	3.0	3.1	0.1
96	2.4	2.4	0.0	2.8	2.8	0.0
97	2.3	2.3	0.0	2.6	2.6	0.0
98	2.1	2.1	0.0	2.4	2.4	0.0
99	2.0	2.0	0.0	2.2	2.2	0.0
100	1.8	1.8	0.0	2.1	2.1	0.0

Chapter 6

Life Expectancy of a Child Born in Ireland in the Twenty-First Century

(Co-authored with Rabia Naqvi)

Abstract

Period life expectancies for the Irish population are projected and published by the Central Statistics Office (CSO) and the United Nations (UN). This paper estimates cohort life expectancies at birth in Ireland over the remainder of the 21st century together with 80% and 95% prediction intervals consistent with these official estimates. We report that a female born in Ireland in calendar year 2020 can be expected to live about 92.6 years with an 95% prediction interval around this estimate of 86.8 years to 97.3 years. For males born in 2020, the central estimate is 90.4 years with 95% prediction interval of 83.9 years to 95.2 years. The probability that cohort life expectancies at birth will reach 100 years before the calendar year 2100 is less than 10% for females and less than 2.5% for males.

Introduction

Life expectancies in Ireland have shown a marked increase since statistics on births and deaths were systemically collected after the Registration of Births and Deaths (Ireland) Act of 1863. Over the course of the twentieth century, period life expectancies for females increased by, on average, 0.3 years with the passage of each calendar year (0.25 for males) (Chapter 5, Naqvi and

Whelan (2019)). Mortality improvements were concentrated at the earlier ages in the first decades of the twentieth century but became more evident at later ages as the century progressed in a pattern sometimes referred to as "the aging of mortality improvements" (Chapter 1, Whelan (2008)).

Life expectancies in Ireland are forecast by the Central Statistics Office (CSO) as part of their population projections (CSO (2018)) and also by the United Nations (UN) for Ireland, and for every other country in the world (United Nations (2019)). However, both of these agencies forecast period life expectancies. A period life expectancy is estimated from mortality rates observed at each age over a particular period in the past (usually a calendar year or group of calendar years). The period life expectancy at birth according to the most recent Irish Life Tables published by the CSO relate to the mortality experience observed over the calendar years 2010 to 2012 (CSO (2015)) and show period life expectancy at birth as 82.8 years for females and 78.4 years for males. However, period life expectancies do not give a measure of how long a person will live because, as the CSO makes clear: "Period expectation of life …is therefore not the number of years someone of that age could actually expect to live because death rates are likely to change in the future" (CSO (2015)).

The cohort life expectancy directly addresses the issue of how long a person can be expected to live as it estimates life expectancy not from historic mortality rates but from the (projected) mortality rates the person can be expected to experience as the individual ages. So, for example, a new-born in calendar year 2020 will be aged 50 years in calendar year 2070 so, in estimating the cohort life expectancy, the current mortality rate of a 50-year-old is adjusted to reflect how that mortality rate is expected to change over the next half-century. The resultant projected mortality rates (using this approach to project forward for each age and each future period) are used in the calculation of the cohort life expectancy.

There is generally a significant difference between the life expectancies calculated using the two different approaches, with the cohort life expectancy generally greater than the period life expectancy as mortality rates are expected to continue to decline in the future.

This paper analyses the official mortality projections made by the CSO and the forecasts of life expectancies made by the UN for Ireland. Since 2014, the UN life expectancy forecasts are made using a stochastic approach, so not only are median future period life expectancies at birth reported but also the 80% and 95% prediction intervals. We derive projected mortality rates at each age and for each future calendar year consistent with the UN forecasted life expectancies and apply the resultant mortality rates to project cohort life expectancies together with 80% and 95% prediction bounds.

The novelty of the paper is to report cohort life expectancies at birth in Ireland, together with prediction bounds, for a child born in any calendar year 2020 to 2100.

Methods

The CSO and the UN apply quite different approaches to project period life expectancies for Ireland. The CSO employ an expert panel to advise on short-term and long-term trends in mortality for males and females separately and then apply these trends to project mortality rates and thereby life expectancies. In the most recent projections published in 2018, the CSO assumes that the short-term trend of mortality rates declines by 2.0% per annum for females (2.5% for males) and this will fall linearly to the assumed long-term rate of decline of 1.5% per annum (both sexes) over a 25-year period and then remain declining at 1.5% per annum from then. This pattern of decline was applied to all ages up to age 90 years; from age 100 years no improvements in mortality rates were assumed; and for ages between 90 and 100, the rate of decline was solved by linear interpolation between the assumed rate at 90 years of age and the zero rate assumed at 100 years of age.

In contrast, the UN Population Division employed a stochastic Bayesian hierarchical model to forecast life expectancies (i.e., not mortality rates) in a consistent manner for all 159 countries in the world with a population over 100,000 that are not experiencing an AIDS epidemic, with different parameters for each country drawn from a common world population (Raftery et al. (2014), Raftery et al. (2013)). This stochastic approach leads to many possible trajectories of life expectancies, which allows the estimation of prediction bounds alongside the median forecast. The UN publishes the median life expectancy at birth for each calendar year 2020 to 2100, together with 80% and 95% prediction bounds (United Nations (2019)).

It proved possible to apply the CSO two-parameter model of future mortality rates to get a very close fit to the median and the different prediction bounds produced by the UN stochastic life expectancy model, by simply recalibrating the inputs for short-term and long-term trends. Recalibrating the CSO model in this manner allows us to derive mortality rates at each age and at each future year consistent with the UN period life expectancies. This therefore gives us the building blocks to estimate future cohort life expectancies and their prediction bounds. Also, this approach allows for an interpretation of the different trajectories of the UN stochastic model in terms of different scenarios of short-term and long-term declines of future mortality rates.

Figure 6.1 (Plate 27) highlights how closely the CSO mortality model can be made fit by a judicious selection of the two parameters to the UN median and 95% prediction bounds for the projected period life expectancy at birth for a female and male born in Ireland in any calendar year 2020 to 2100. Similar close approximations were found by optimisation for other prediction bounds.

Table 6.1 gives the two parameters of the CSO model that best fit the UN 50%, 80%, 95% upper and lower bounds and includes, for comparison, the two parameters the CSO used in their latest official projections. We note that both the parameters decrease monotonically as the prediction level decreases, for both males and females. The UN 2019 median period life expectancy forecasts are close to the CSO 2018 projections over the next century, and this is seen in the closeness of their parameters in Table 6.1.

Table 6.1: CSO Model Parameters Corresponding to UN 2019 Forecasts at 50%, 80% and 95% Prediction Intervals of Period Life Expectancy at Birth for Males and Females (and also including CSO 2018 Projection Basis)

	Males		**Females**	
	Short-term Rate	***Long-term Rate***	***Short-term Rate***	***Long-term Rate***
Upper 95%	4.72%	2.41%	3.21%	2.91%
Upper 80%	3.93%	1.98%	2.61%	2.40%
Median	2.76%	1.31%	1.45%	1.55%
Lower 80%	1.14%	0.90%	0.17%	0.97%
Lower 95%	-0.09%	0.78%	-0.57%	0.73%
CSO 2018 Projection	2.50%	1.50%	2.00%	1.50%

Results

The approach of recalibrating the two parameters used in the CSO model to achieve a close fit with the UN projected period life expectancies at birth (median and various prediction bounds) between calendar years 2020 to 2100 gives us a model from which we can derive mortality rates in the future at all ages. Accordingly, by recombining the forecast mortality rates by cohort, we can estimate the cohort life expectancy at birth for each future year. This gives us, as noted earlier, the number of years a person born in that future year is expected to live. We can now also estimate prediction bounds around this central estimate consistent with the UN 2019 model. The results are shown graphically in Figure 6.2 (Plate 28) and, at selected future years of birth, in Table 6.2.

Table 6.2: Male and Female Cohort Life Expectancies for Selected Calendar Years of Birth, 2020-2100, with 50%, 80% and 95% Prediction Intervals Consistent with UN 2019 Forecasts (including CSO 2018 Projection)

	Male					
Calendar Year of Birth	*Lower 95%*	*Lower 80%*	*Median*	*CSO 2018 Projection*	*Upper 80%*	*Upper 95%*
2020	83.88	85.96	89.71	90.38	93.41	95.21
2040	85.34	87.55	91.60	92.42	95.38	97.03
2060	86.74	89.02	93.18	94.08	96.82	98.25
2080	88.06	90.37	94.52	95.44	97.89	99.09
2100	89.29	91.58	95.64	96.54	98.69	99.68
	Female					
Calendar Year of Birth	*Lower 95%*	*Lower 80%*	*Median*	*CSO 2018 Projection*	*Upper 80%*	*Upper 95%*
2020	86.81	88.81	92.51	92.68	95.94	97.31
2040	87.99	90.25	94.22	94.32	97.55	98.73
2060	89.13	91.57	95.62	95.66	98.66	99.64
2080	90.20	92.76	96.75	96.74	99.44	100.22
2100	91.19	93.82	97.65	97.62	99.99	100.61

Table 6.2 shows that a female born in Ireland in 2020 is projected to live, on average, 92.5 years according to the UN and 92.7 years according to the CSO. The 95% prediction bound around this estimate is 86.8 years

to 97.3 years (roughly ±5 years). However, by the end of the 21st century, while it is expected that the average life expectancy for a female is 97.7 years according to the UN and 97.6 years according to the CSO, there is a chance less than 10% that the average life expectancy at birth will be 100 years or over.

Indeed, on the negative side, the table shows that there is a chance, somewhere between 2.5% and 10%, that average life expectancy for females by the end of this century could be lower than that expected in 2020.

Discussion

Projections of the CSO and the UN are remarkably consistent in forecasting that period life expectancies at birth will grow by 0.11 for females and 0.12 for males with the passage of each calendar year over the remainder of the 21st century. This is a considerable slowdown — roughly half — of the gains recorded over the previous century.

Part of the reason for this slowdown is that mortality rates are now so low at the younger ages in Ireland that further percentage declines, even if at the same rate as in the past, will have less of an impact on extending life expectancies. Extension of life expectancies in the future largely depend on declines in mortality at advanced ages, especially at post retirement ages.

The extension of human lifetimes has been attributed to some mix of improvements in nutrition, in income and wealth, in behaviour, in education, in public health, and in medicine, with the mix depending on the country and the time (Riley (2001)). The State, directly or indirectly, plays a key role in shaping many of these factors. Given the importance of reducing mortality rates at advanced ages to extend life expectancies further, the State and its policies on pension and healthcare can be expected to have bigger impact in the 21st century. Amongst other things, there must be increasing resources made available for the treatment of older patients with their associated clinical complexities (Johnson et al. (2018)).

The above mortality projections are, arguably, the best that can be done, but that does not make them reliable. The UN will revise their projections in a year or two and the CSO will do the same following the next census (due in 2021). In the past, projections of life expectancies by official agencies or academics have been poor, generally underestimating increases (Oeppen and Vaupel (2002), Waldron (2005), Keilman (2008)).

So life expectancies are projected to continue to increase throughout the twenty-first century, albeit at about half the pace of the 20th century. A key question is what proportion of the additional years will be in good heath — "Dorian Gray" years — and what proportion maybe in less desirable "Struldbrugg" years. Early evidence is that there is little change in the proportion of "Dorian Gray" years to "Struldbrugg" years over time. A recent study (GBD 2017 DALYs and HALE Collaborators (2018)) shows that there has hardly been a change between ratio of healthy life expectancies to total life expectancies between 1990 and 2017 in Ireland as well as other countries with high incomes and high educational attainment (the ratio being about 86% at birth and 75% at age 65).

Chapter 7

Compensation for Wrongful Injury in Ireland

(Co-authored with Maeve Hally)

Abstract

Compensation for future loss due to wrongful injury in Ireland is currently determined at discount rates that do not take account of current market conditions and on a historic mortality basis. We quantify the impact of assessing damages using a more appropriate discount rate, mortality basis, and method of capitalising the loss. This results in the quantum of damages increasing significantly, and figures are given quantifying the increase by the term of the loss. Total outstanding liabilities of the State Claims Agency now exceed €3 billion, about half of which is in respect of catastrophic birth injuries caused by negligence in the delivery of maternity services. The change in the basis by which compensation is calculated outlined in this paper would increase the estimate of outstanding liabilities by over €1 billion and perhaps closer to €2 billion. We argue the current under-compensation of plaintiffs incentivises the State to settle by way of lump sum and is therefore an obstacle to the required legislation for appropriately indexed periodic payment orders.

Introduction

This paper reviews the legal principles to determine compensation for future loss in wrongful injury cases in Ireland. The judgement in the landmark case, *Gill Russell -v- Health Service Executive* case, is analysed and applied to current circumstances. We show that awards made by Irish courts should be materially higher than at present when this precedent is properly reflected in the determination of lump sum compensation.

The principles of risk minimisation set by the precedent to investment risk can equally be applied to the longevity risk currently borne by the plaintiff (that is the risk the plaintiff lives longer than expected in the lump sum calculation). We explore how this might be achieved now that the long-awaited legislation anticipated to transfer longevity risk to the State proved unsatisfactory. It is shown that reducing the longevity risk to the plaintiff further increases the quantum of damages.

We outline the impact this judgement has on the discount rate, mortality basis, and approach to longevity risk. We estimate that the required change in basis by which compensation is calculated increases the outstanding liabilities of the State by more than €1 billion, and perhaps closer to €2 billion.

The paper is divided into seven sections. First, we outline the rise in claims against the State over the last decade. It is shown that the growth of both claim settlements and the rise in outstanding liabilities has averaged more than 15% per annum since 2010. Second, we overview the principles of how compensation for future loss should be estimated under Irish law. We apply the principles to current market conditions in the subsequent two sections and quantify the extent to which lump sum compensation is currently undercompensating the plaintiff due to outdated investment assumptions. Consistent with legal principles and precedent, this paper shows that the real discount rate for wage-related loss should be -2.5%, as opposed to the +1% discount rate currently used.

Third, we analyse the longevity risk imposed on the plaintiff by the current lump sum form of compensation. The life table, which determines the probability of survival and therefore the likelihood of each future loss being incurred, should, we show, be based on cohort mortality rates and not the period mortality rates generally employed. Also, the method of capitalising the future loss into a lump sum award should make explicit the longevity risk borne by the plaintiff. We show that the current method of allowing for this risk gives a probability of greater than 50% that the lump sum form of compensation will be exhausted before the death of the plaintiff and therefore undercompensates the plaintiff. We quantify the increase in compensation necessary to ensure, at probability of 50% or higher, that the lump sum will not be exhausted.

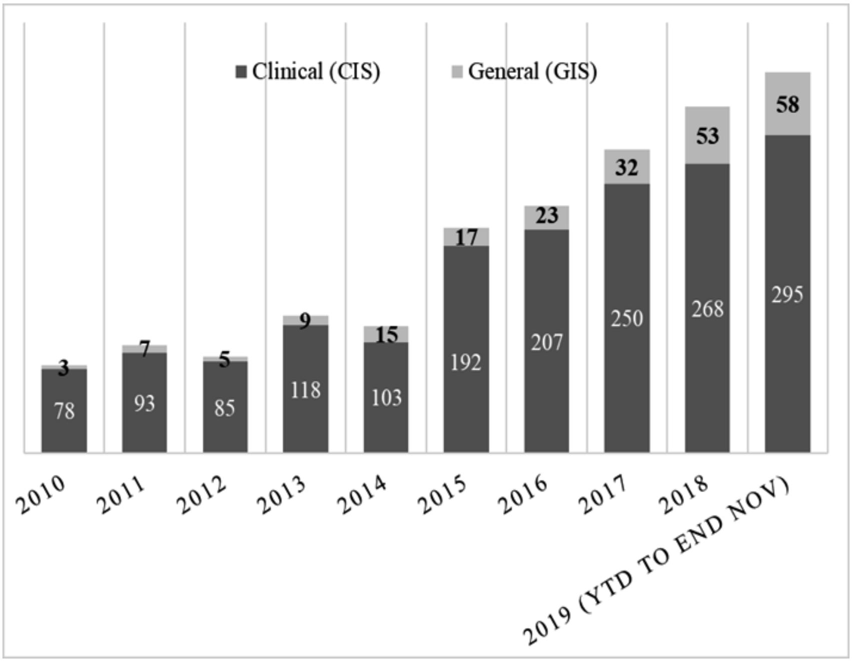
The paper then considers why the long-awaited legal reform to allow periodic payments for the remainder of the plaintiff's lifetime proved inadequate, highlighting how the current practice in capitalising future loss is an obstacle that must first be removed. We quantify the increase in outstanding liabilities to the State when compensation is calculated at current market conditions consistent with legal principles and show the increase in the current outstanding liability exceeds €1 billion and perhaps is closer to €2 billion.

We conclude by reiterating the need for appropriate legislation to effect periodic payment orders to replace lump sum compensation. We also suggest that it might be more cost-effective for the State to invest more in the delivery of sound maternity services when proper account is taken of the cost of maternity claims.

Background

The State Claims Agency (SCA) operates two insurance schemes, the Clinical Indemnity Scheme (CIS) and the General Indemnity Scheme (GIS). The CIS covers all clinical claims against the Health Service Executive, and some other parties. The GIS covers all non-clinical claims against the State, State authorities, and various other bodies such as community and comprehensive schools, the Garda Síochána, and the prison service. Since the start of 2010, total claims settled by the State Claims Agency exceeded €1.9 billion. A total of €1.69 billion was paid-out in respect of the Clinical Indemnity Scheme (CIS), which represented 89% of total payments since 2010. Figure 7.1 illustrates the rising costs of claim settlements each year since 2010.

***Figure 7.1: Total Claims Amounts Paid under the Clinical and General Schemes in € millions, 2010 to November 2019 (includes Damages, Legal Fees and Other Expert Costs)*[1]**



The number of new claims in recent years has been increasing at a faster rate than the number being resolved, so the number of outstanding claims continues to rise. Figures from the Annual Reports and Accounts of the National Treasury Management Agency of which the SCA is a division, show that the estimated total outstanding liabilities to claims under both schemes amounted to €3.15 billion at the end of 2018, up from €783 million in June 2010. In 2011, the Director of the SCA estimated that cases of cerebral palsy at birth, although only 3% of the claims by number, accounted for two-thirds of the CIS liability (Breen (2011), pp.37-38). The most recent accounts which give such a breakdown is the Annual Report and Accounts for 2017, where some 53% of the value of outstanding claims (of both CIS and GIS) were in respect of

[1] Data from Memo prepared by the SCA in answer to Dáil Question on 5 Dec 2019 by Deputy Michael McGrath, www.oireachtas.ie/en/debates/question/2019-12-05/57/.

claims against maternity services (€1.4bn compared to total estimated outstanding claims then of €2.66 billion).

The rate of growth of both claim settlements and the rise in outstanding liabilities has averaged more than 15% per annum since 2010. The greater part of this increase is due to the growth in the number of notified claims, especially claims against maternity units for catastrophic brain-injuries at birth. However, another contributor to the growth is due to a change in how the judiciary determines the lump sum compensation for future pecuniary loss. Future wage or inflation-linked losses were discounted at a 3% per annum at the start of the decade, following the ruling in *Luke Boyne v Bus Átha Cliath and James McGrath* [High Court Record No. 2000/12133P] (see Whelan (2009)). However, the discount rate was contested in 2014 in the case *Gill Russell -v- Health Service Executive* (High Court Record No. 2009/1918P), when it was reduced to 1% per annum for wage-link loss and 1.5% for inflation-linked loss. These discount rates were later upheld by the Court of Appeal (Appeal No. 2015/49). This reduction in discount rate puts a higher value on the present value of future losses and therefore the lump sum compensation. This ruling by the courts required the SCA to raise its estimate of outstanding liabilities by €300 million or about 17%[2] and of course, raises the value of all new claims.

The judgement in *Gill Russell -v- Health Service Executive* is generally summarised by the impact it has on the discount rate, as above. However, a more accurate summary is that the judiciary made explicit the principles by which discount rates are to be determined. In short, the original High Court judgement, developed and clarified by the Court of Appeal, states that the discount rate for inflation-linked loss should be determined using the real yield on index-linked bonds and, for wage-linked loss, this real discount rate should be further reduced.

Principles of Capitalising Damages for Lump Sum Compensation

Compensation for personal injury in Ireland is based on the fundamental principle that as far as possible the wronged party should be restored to the position that he or she was in prior to the incident giving rise to the

[2] NTMA Annual Report and Accounts 2015, p.37.

claim (*restitutio in integrum*). The same principle also applies in the UK, US, and many other jurisdictions (Thornton and Ward (2009)).

Until October 2018, compensation in Ireland for any injury had to be paid by way of a single lump sum. From October 2018, claims for catastrophic injury could be settled by way of annual payments for the remainder of plaintiff's lifetime. A total of six such periodic payment orders had been put in place before the High Court ruled in November 2019 that the legislation was a 'dead letter' as "no judge charged with protecting plaintiffs' best interests could recommend such a scheme" [Judgement in *Jack Hegarty -v- Health Service Executive*, High Court Record No 2015/10520P]. Given that periodic payments orders are currently in abeyance, we shall first consider how precedent has determined lump sums are calculated. We shall reconsider in a later section what amendment to the legislation is required to achieve periodic payment orders that deliver fair compensation.

Once the court determines from the evidence presented the amount, term, and nature of the future loss then it must calculate a present value of the future stream of losses to give a capitalised value. This capitalised value is then added to the amounts determined in respect of losses suffered to the time of trial (past losses) and non-pecuniary losses (e.g., compensation for pain and suffering, life expectancy curtailed, quality of life impaired) to give the overall lump sum compensation. The quantum in respect of future loss comprises most of the ultimate lump sum award for brain-damaged infants.

There are two different approaches to estimating the real return on a lump sum award invested to meet future losses, generally referred to as the 'fair value' approach and the 'best estimate' approach.

The fair value approach takes the view that if there exists a freely traded asset whose proceeds exactly reproduce the future pecuniary loss (i.e., a replicating asset), then the market price of the replicating asset gives the capitalised value. In the situation that there is no freely traded asset, then the fair value approach is to estimate the value of such a replicating asset if it was freely traded on the market. So, suppose that the plaintiff's loss is a series of future payments that rise in line with general inflation for the remainder of their lifetime. If there exists index-linked bonds (ILGS), of very long maturities, issued by the state (or another organisation with a good credit rating), then a judiciously selected portfolio of such bonds can provide a future inflation-linked stream of payments that closely match the future loss. The fair value approach takes the market value of such a portfolio of index-linked bonds to be the amount of the compensatory lump sum for future loss. This solution not only derives a present value for the inflation-linked pecuniary loss but also gives a method to invest the lump sum to restore the plaintiff's lost cash flows.

Alternatively, the best estimate approach estimates the expected real return on an investment portfolio that is deemed appropriate to provide for the future loss given the risk appetite of the plaintiff.

The key difference in the two approaches is how investment risk is treated. The fair value approach is based on minimising the investment risk to the plaintiff and gives the same answer as the best estimate approach when the plaintiff is assumed to be risk averse. The best estimate approach assumes that the plaintiff can tolerate some level of investment risk and constructs an investment strategy that maximises the expected return based on that assumed level of risk. Typically, the best estimate approach gives a higher expected return than the fair value approach and thus a lower level of damages as future loss is discounted at this higher rate. In short, the best estimate approach assumes that the plaintiff can tolerate investment risk to some extent and reduces the lump sum determined by the fair value approach by the extent of investment risk to be borne by the plaintiff.

Before the hearing of *Gill Russell -v- Health Service Executive* in the High Court in 2014 (High Court Record No. 2009/1918P), the precedent on how discount rates were to be determined was set by Mr Justice Finnegan in 2002 in the case of *Luke Boyne v Bus Átha Cliath and James McGrath* [High Court Record No. 2000/12133P]. Here it was established that a prudent investor would invest in a mixed portfolio of higher risk equities and lower risk gilts, the mix reflecting the circumstances of the plaintiff. He judged that a portfolio consisting of 70% in equities and 30% in gilts was prudent for the plaintiff Mr. Boyne and such a portfolio would reasonably mitigate the damages. On the basis of evidence presented, he assessed that the real rate of return on such a portfolio would be 3%, and therefore set 3% as the discount for any loss rising with inflation.

The two different approaches to estimating the real return on a lump sum award were reconsidered in 2014 in the case *Gill Russell -v- Health*

Service Executive. This case determined that the fair value approach is preferable over the previous best estimate approach adopted. The judgement was contested but upheld by the Court of Appeal (Appeal No. 2015/49). The ruling is best summarised in some key quotes from the judgement in the High Court trial and the elaboration and clarifications given by the subsequent ruling in the Court of Appeal.

Finding of Cross J. in High Court:

> I favour the plaintiff's experts' conclusions not because I have any capacity to be an economic forecaster but rather because they have demonstrated that investment in ILGS [Index-linked Gilts] is more risk adverse than any mixed fund. You do not have to be in any sense an expert in economics to come to that conclusion. (para 2.73).

> … I consider that over Gill's lifetime, the price of ILGS will as a matter of probability increase and accordingly, I hold that a figure of 1.5% (i.e. 0.5% being the present price plus 1% to represent the future) is a fair figure for a multiplier on the basis of investment in ILGS. (para 2.65)

Findings of Court of Appeal:

> Quite correctly, in the view of this Court, Cross J. determined that the assessment of the real rate of return is to be made on the assumption that the plaintiff should be entitled to invest his award in as risk free an investment strategy as is available and which will likely meet his future care needs. In particular, we agree with his conclusion that the plaintiff is not to be treated as an ordinary prudent investor for the purposes of calculating the likely return on the investment of his lump sum. In adopting this approach, the High Court judge appropriately adopted the reasoning of the House of Lords in *Wells*. (para 83)

> It follows that we are satisfied that his conclusion that the plaintiff's lump sum should be calculated by reference to ILGS, was well founded on the evidence as was his conclusion that wage inflation in the health care sector is likely to outstrip general inflation in early course and is likely to continue in that vein over his lifetime. (para 160)

In 2017, the Supreme Court refused to allow the HSE leave to appeal against the Court of Appeal Judgment.

So both Courts concluded that the fair value approach, which minimises the investment risk for the plaintiff, is the better of the two approaches to determine the real return on any lump sum award. Both Courts also agreed, based on the evidence presented, that the real rate of return should be estimated with reference to the real return on Index-Linked Gilts (ILGS) issued in euros by a low risk country. The judgement in the *Russell -v HSE* case also distinguished between inflation-linked loss and wage-linked loss, with a lower discount rate to be applied to the later as wages can be expected to increase at a faster rate than prices in the future.

It is now 5 years on from that High Court judgement of Cross J. so it may be opportune to consider again the real rate available on ILGS as the real rate has changed over the intervening years. Also, the allowance for wage escalation should also be reviewed, as the ruling in that case was time limited:

> … this Court is satisfied that the High Court judge's downward adjustment of the real rate of return by 0.5% to take account of future wage inflation, for the purpose of the calculation of the plaintiff's claim for future wage inflation, was appropriate. He was clearly entitled to conclude that wage inflation in general would, over the period of the loss, exceed CPI at a minimum of 1% and that if no adjustment was made, the plaintiff would not receive full compensation. Further, given that wage inflation in the care sector would not fall into line with general wage inflation for a period of approximately five years, that being the opinion of Prof Walsh's [expert witness called by plaintiff], he was entitled to reduce the adjustment required in the real rate of return to 0.5% to take this factor into account. (para 155)

Judgement of the Court of Appeal delivered by Ms Justice Irvine

Box 1 outlines how, in practice, the lump sum compensation for future loss is determined by Irish courts.

Box 1: How Courts in Ireland Determine Lump Sum Compensation for Future Monetary Loss

Damages for future monetary loss are generally computed using a 'multiplicand' and a 'multiplier', with the quantum of loss found by multiplying the two figures. The multiplicand is the estimated monthly (or weekly or annual) loss and the multiplier is the capitalised value of a monthly (or weekly or annual) loss of €1. If expected losses are dependent on different contingencies, reoccur at different frequencies, or increase at different rates, then separate multipliers are computed for each category of loss and the overall capitalised amount is the sum of their products.

The Multiplicand

In an injury case, the monetary loss would include loss of earnings and perquisites of employment, loss of pension benefits, additional healthcare and living expenses arising from injury. The onus is on the plaintiff to take reasonable measures to minimise the loss by, say, finding suitable alternative employment. Accordingly, the calculation is not strictly made on the actual loss but on the loss when minimised. This is qualified somewhat further as an Irish statute[3] stipulates that the hypothecated 'loss' or better, the multiplicand, is not to be reduced by the proceeds of a contract of insurance or, in certain circumstances, by social insurance benefits payable, as a result of the wrongful action (presumably on the justification that plaintiffs provided for these latter benefits themselves).

Sometimes precision is impossible in determining the loss sustained, such as the future loss of earnings for a child incapacitated by an accident long before their career path is clear. Even in these cases, the Irish courts generally impute a loss of earnings from when the child could have been expected to enter the workforce, to be capitalised with a suitable multiplier.

The loss of earnings and other losses determined above are all net of income tax, social insurance contributions or any other deductions that would have been payable by the plaintiff. The offsets are similarly the net receipts in the hand of the plaintiff.[4]

[3] Section 2 of the Civil Liability (Amendment) Act, 1964; Social Welfare Consolidation Act 1993.
[4] Cooke v Walsh (1984) ILRM 208.

So, say the court accepts, on the basis of evidence presented, that the plaintiff has suffered the following monetary loss in the future under different headings (all values in present day terms):

1. Cost of employing a caregiver from now for life: €1000 per week
2. Loss of earning from Age 21 to Age 68: €500 net per week
3. Loss of pension from Age 68 for remainder of life: €250 net per week
4. Cost of aids and appliances (e.g., wheelchair, hoists, car adaptations) from now for life: €100 per week for life

The Multiplier

The multiplier to be applied to the multiplicand is to capitalise the loss of a €1 per week (or other frequency of the loss) over the total period of the loss. Specialist actuaries are retained to determine the multiplier and estimate the lump sum compensation for future loss. The actuary must make assumptions on:

- The probability that each future payment is made. This typically requires assumptions on the mortality rates for the plaintiff, but it could involve other contingencies.
- The amount by which the net loss of €1 in present day terms might increase to by the time of payment. This assessment, in turn, typically requires assumptions on the general level of future inflation, the general level of real salary increases (that is salary increases above inflation), the probability that the salary level of the plaintiff might have changed other than by the general level as a result of, say, promotion.
- The rate discount that must be applied to each future payment so that its present value is determined. This is the assumed return from investing the lump sum.
- The rate and manner of taxation of income and capital gains in the future, both to determine the net future loss and the net proceeds from investing the compensating lump sum to replicate those net future losses.
- Other assumptions, such as investment expenses, loss ceasing on contingencies other than death or reaching a certain age (such as on redundancy).

Determining the Lump Sum

Let us further assume in our example earlier that the plaintiff is a female currently 10 years old. The precedent in such cases is that wage-linked loss is discounted at 1% per annum and inflation-linked loss at 1.5% per annum. So, the loss under headings 1-3 are discounted at 1% per annum while the loss under heading 4 is discounted at 1.5% per annum. Then, allowing for mortality using Irish Life Table 16 as is commonly used (see later), the actuary would calculate the following multiplier under each heading of loss:

		Multiplier (€)
1.	Capitalised cost of employing a caregiver for €1 per week from now for life:	2691
2.	Capitalised value of loss of earning of €1 per week from Age 21 to Age 68:	1718
3.	Capitalised value loss of pension for €1 per week from age 68:	430
4.	Capitalised cost of aids and of €1 per week from now for life:	2304

Hence:

1. Capitalised cost of employing caregiver from now for life:
   1000 x 2691 = 2,691,000
2. Capitalised value of loss of earning from Age 21 to Age 68:
   500 x 1718 = 859,000
3. Capitalised value loss of pension from Age 68:
   250 x 430 = 107,500
4. Capitalised cost of aids from now for life:
   100 x 2304 = <u>230,400</u>

Lump Sum to Compensate for Future Monetary Loss: <u>€3,887,900</u>

Determining the Real Return using the Fair Value Approach at the Present Time

It is not straightforward to construct a portfolio of assets, the proceeds of which will match the plaintiff's future inflation-linked loss. Two problems arise in constructing such a portfolio to replicate future loss:

(i) There are essentially no index-linked bonds linked to future inflation in Ireland.

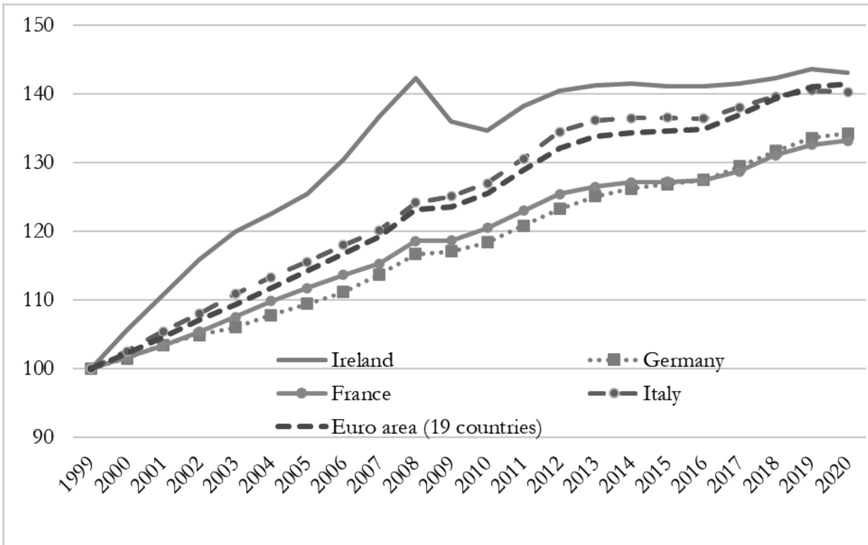
(ii) Index-linked bonds in countries that issue them do not span the maturity range needed to match the plaintiff's loss which might continue for several decades.

We treat each of these issues in turn.

While the market of bonds with proceeds linked to inflation has not developed in Ireland, it has in other countries with the euro as their currency. France, Germany, Italy, and others have issued such bonds with inflation linked to eurozone inflation (the harmonized index of consumer prices excluding tobacco) and the market for index-linked bonds constitutes a growing part of the large euro-denominated bond market. An Irish plaintiff can consider investing in such index-linked bonds with no currency risk. The key risk with such an investment is how inflation across Europe might differ from Irish inflation in the future.

Studies of how inflation differs in different regions with the same currency suggest that inflation rates do not differ very significantly over the long term (Whelan (2005)). So, for instance, when the Irish pound was linked to the UK pound from the political independence of Ireland at the end of 1921 to the breaking of the one-to-one parity between the currencies in early 1979, inflation in Ireland and the UK was very similar year-on-year, with accumulated differences of less than 7% over the entire 58-year period or, equivalently, less than 0.12% per annum. More recently, inflation in Ireland can be compared to the euro area since the euro came into being. Inflation across the eurozone has averaged almost the same from 2000 to the end of 2019, with annualised inflation of 1.6% in Ireland, 1.7% across the euro region, 1.5% in Germany and 1.4% in France (see Figure 7.2). These similarities in inflation over the period are despite the boom and bust in Ireland over the last two decades, not unrelated to the low interest rates caused by the introduction of euro.

***Figure 7.2: Inflation in Ireland, the Euro Area, and Selected Countries, 2000 to 2019*[5]**



Accordingly, it can be reasonably maintained that the average inflation rate in Ireland will be reasonably similar to the eurozone inflation rate over the longer term. Furthermore, over such long periods it is not obvious which region would have slightly higher or slightly lower rates of inflation. While investing in bonds with payments linked to eurozone inflation to match Irish inflation-linked cash flows does involve an element of risk, the risk is of an order of magnitude lower than the risk introduced by investing in equities or other securities.

Accordingly, the strategy of investing in such eurozone inflation-linked bonds is the optimum strategy of all possible strategies in the sense that it minimises the risk in replicating the lost inflation-linked cash flows to the plaintiff. It was accepted in the *Gill Russell -v- Health Service Executive* that ILGS issued in euros and linked to eurozone inflation by France and others constituted the least risk investment portfolio. The overall size of the French Government's outstanding ILGS debt as at end of 2018 was €220 billion. The overall size of the Eurozone Sovereign

[5] OECD Database of National Consumer Price Indices, https://stats.oecd.org/OECDStat_Metadata/ShowMetadata.ashx?Dataset=PRICES_CPI&ShowOnWeb=true&Lang=en

Inflation-Linked Bond market exceeds €660 billion.[6] Note that inflation linkage is to the euro area Harmonised Index of Consumer Prices excluding tobacco.

The longest dated stock linked to euro area inflation currently issued by France is to the year 2047 (Germany is to year 2046). So, at the present time, it not possible to construct a matching portfolio from existing index-linked stock to cover inflation-linked losses extending from the calendar year 2047, which might be necessary if the plaintiffs' losses are expected to continue beyond 2047. However, the associated investment risk can be minimised, as we now outline.

The management agency of the French national debt, Agence France Trésor, undertakes to execute 10% of its issuance programme each year with inflation-linked securities (Agence France Trésor (2014)). With other euro governments also issuing such securities, there will be a considerable ongoing supply of index-linked bonds linked to euro area inflation.

An investment strategy to provide for the inflation-linked losses which fall after the calendar year 2047 consists of a number of steps. Step 1 is to invest that part of the lump sum that is deemed to meet the loss over these years in the 2047 dated French index-linked bond at the current real yield. Step 2 is to sell these index-linked stock holdings and use the proceeds to buy longer dated index-linked stock as soon as such longer dated bonds are issued. By this strategy, the duration of the portfolio can be extended and longer-term losses matched over time. A feature of long-term interest rates or yields (whether real or nominal) is that such interest rates or yields generally show very little change from maturities of 30 years to 40 years and longer. This observation entails that, at the future time when a longer-dated stock is issued, the real yield that the plaintiff sells the 2047 stock at is very close to the real yield that he is simultaneously buying at. In short, he is in effect swapping two securities at a future unknown price — but we know that the prices will be very similar. In market parlance, it is 'hedging the risk' of future price movements of the currently unavailable longer dated stock by investing temporarily in the 2047 stock.

[6] See https://us.spindices.com/indices/fixed-income/sp-eurozone-sovereign-inflation-linked-bond-index

The hedging strategy reduces the future reinvestment risk markedly but does not eliminate it altogether. There is a residual risk. If this residual risk was passed on to a third party, they would charge a risk premium for accepting it. It can be shown that following this investment strategy will lead to a gain to the plaintiff if real yields increase from current levels. Alternatively, if real yields fall from current levels then the plaintiff is exposed to a loss. However, it is the best strategy as it minimises the risk.

The above considerations show that it is a straightforward matter to estimate the appropriate discount rate for inflation-linked loss to a plaintiff. Simply, estimate the average real yield on index-linked stock over the future term of the loss. Table 7.1 shows the real yields available on French sovereign ILGS over different future periods as at end October 2019.

***Table 7.1: Real Yields on Selected French Index-Linked Stock Linked to Euro Inflation Index (excluding Tobacco) at End October 2019*[7]**

Term from Now	Real Yield	Stock
1 Years	-1.6%	France OAT€i 2.25% 2020
5 Years	-1.3%	France OAT€i 0.25% 2024
11 Years	-1.0%	France OAT€i 0.7% 2030
21 Years	-0.8%	France OAT€i 1.8% 2040
28 Years	-0.7%	France OAT€i 0.10% 2047

The real yield varies with the duration. As cerebral palsy claimants tend to have life expectancies of several decades, a gross real yield of the order of -0.75% per annum appears reasonable to use, but it could be lower for those with short life expectancies. This real yield ignores portfolio management costs. An additional allowance of 0.25% to 0.5% per annum for all costs associated with investment — advisory fees,

[7] www.aft.gouv.fr/en/oateuroi-key-figures and prices and real yield calculations by Frankfurt Stock Exchange on 30th October 2019. See http://www.boerse-frankfurt.de/en/bonds/. Note that real yield on the German 0.10% inflation-linked Federal bond 2015 (2046) is -1.1% on 31st October 2019 See https://www.deutsche-finanzagentur.de/en/fact-sheet/sheet-detail/productdata/sheet/DE0001030575/ and https://www.deutsche-finanzagentur.de/en/institutional-investors/federal-securities/inflation-linked-securities/

trading costs, and management costs. Hence, at a conservative estimate, the net real yield to discount future inflation-linked loss is of the order of -1.0% at the present time, after some allowance is made for the costs of implementing the investment strategy.[8]

Discount Rate for Wage-Linked Loss

It is important to distinguish between a wage-related loss and a price-related loss as wage and price indices have exhibited quite different characteristics over time since the industrial revolution. Over the last two hundred years or so, wages have increased faster than inflation, a key factor leading to the dramatic rise of living standards of workers over time in Ireland, UK, Europe, US and the rest of the world. Rising real wages is generally attributed to the productivity gains unleashed since the industrial revolution which ensure that the same inputs of labour, resources, and capital continue to produce more outputs over time. Labour, through increasing real wages, is rewarded for its part in the increase in productivity over time.

In any event, there is overwhelming evidence that wages have increased faster than inflation in the past, in Ireland and elsewhere, and that it is appropriate to make allowance for such differences in the future. As mentioned earlier, an allowance to be made for increases in real wages in the future in Ireland was considered and ruled on in 2014 in the case *Gill Russell -v- Health Service Executive*, later upheld by the Court of Appeal. However, as was made clear in the judgements in that case, the long term assumed real rate of wage increases was reduced to allow for the exigencies at that time.

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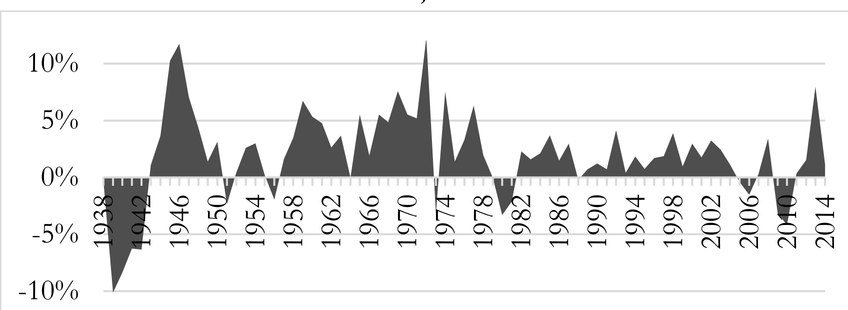
[8] The allowance for such costs was considered recently in the UK in the Government Actuary's advice to the Lord Chancellor on the personal discount rate (Government Actuary UK (2019), see pp. 50-53). Table 9 (p. 52) of this report suggests an adviser fee of 0.25%-0.5% p.a., fund manager fees of 0.25%-0.5% p.a., and platform fees of 0.1%-0.2% p.a. Including an allowance for tax of 0.0%-0.5%, the UK Government Actuary advised an overall allowance of 0.75% per annum. Subsequently, the Lord Chancellor in his reasons for adopting the new -0.25% discount rate for personal injury claims in the England and Wales agreed with the Government Actuary's advice on such charges, stating that "… the Government Actuary's conclusion that a figure of plus 0.75% for tax and expenses is a reasonable one" (paragraph 13). Accordingly, the 0.25% p.a. allowance suggested above for Irish cases is at the lower end of what was recently suggested and adopted in the UK.

There is considerable data, national and international, to show the trends in real wages over long and short periods in the past. The Central Statistics Office (CSO) in Ireland have compiled and published wage or earnings or labour cost indices since the 1930s, other national statistics offices have done the same for their national economy, and bodies such as the International Labour Organisation has collected wage data by occupation around the world since 1924 (the 'October Inquiry').

The CSO has published an historic analysis of wage trends in Ireland from 1938 to 2015 (CSO (2017)), in aggregate and broken down by industries and sectors, occupations, age, and gender. This publication records that real earnings in Ireland (that is, earnings above inflation) grew by an average of 1.9% per annum over the 77 years ending 2015. It varied by decade, ranging from a low of 0.9% real per annum in the 1980s to a high of 4.8% real per annum in the 1970s. The gender pay gap for women in the industrial sector (the only one recorded for such a length of time) fell from 44% in 1943 to 23% in 2014, meaning that the real rate of increase in women's wages in this sector was greater than for men over this period.

Figure 7.3: Real Wage Increases in Ireland for Industrial Workers, Each Year, 1938-2015[9]



The rate of the increase in real earnings in Ireland also varies by sector and occupation. Below, we take an abstract from Table 3.1 in CSO (2017) that highlights how real increases in wages varied by occupation over the thirty years ending 2015.

[9] Data from CSO average weekly earnings data for each year under all industries category (CSO (2017)). This comprises all industrial occupations working in the manufacturing, mining and quarrying, transportable goods, and electricity, water and waste sectors. The mean annual increase was 1.9%.

Table 7.2: Real Average Weekly Earnings in the Industry Sector by Occupational Group in Ireland, 1985-2015[10]

	Real average weekly earnings (€)		
Year	*Managerial & Professional*	*Clerical, Sales & Service Workers*	*Production & Machinery*
1985	801.16	506.08	450.34
2015	1451.89	754.51	685.53
Real Increase per annum, 1985-2015	2.0%	1.3%	1.4%

However, there are issues when applying such historic data wage or earnings indices to estimate the actual wage increases experienced by an individual throughout their working life or for the cost of specialised labour services, such as care givers. Trends in wages indices might not be reliable for four reasons:

(a) First, the composition of general earnings or wage indices might be different to the required occupation.

(b) Second, the composition of the wage index might change over time so, say, greater weight is given to newer occupations with different skills over time.

(c) Third, there can be changes to the skills demanded over time, even in occupations with the same title, so the index is not comparing like-with-like over time.

(d) Fourth, often there are inconsistencies in how the data is collected over time — in respect of bonuses, pension, holiday pay, and other benefits of working.

These issues tend to be compounded when making international comparisons due to currency differences and the possibility that the same job title might not correspond to the same work in different countries.

Academic studies of real wage trends over long period are often structured to remove distortions found in general wage indices.

[10] Figures for real average earnings sourced from Table 3.1 in CSO (2017). The industry sector comprises all working in the manufacturing, mining and quarrying, transportable goods, and electricity, water and waste sectors.

Typically, such studies follow wages in one occupation that has altered little over the very long term (and also their experienced inflation by following the change in prices and composition of the wage-earners consumption basket). Detailed accounts have often been kept of building projects (such as universities or cathedrals), which allows academics to study the long-term trends in skilled (e.g., carpenters) and unskilled labourers wages over time. Clark (2005), for instance, traces the real wage trends for such workers in England for 800 years (1209 to 2004), using some 46,000 wage observations and 110,000 price observations and shows, since the industrial revolution, wages have persistently increased at a higher rate than inflation.

Clark (2005) reports that the annualised real wage increase for craftsmen (labourers) was 1.3% (1.4%) over the two hundred years since 1805, 1.4% (1.6%) over the last hundred years, 2.1% (2.1%) since 1945, 2.7% (2.4%) since 1965 and 2.0% (1.8%) since 1985. Similar findings have been found when studying construction workers real wages in European cities (Allen (2008)) and for wages in the United Kingdom (see, for instance, Feinstein (1995) which includes wages in Ireland prior to 1920). Indeed, there is a considerable body of evidence that real wages have averaged between 1% and 2% above inflation over long periods in the past. Whelan (2002) traces the long history of the wages for carpenters in Ireland over the twentieth century and shows that, over long periods, the average has been between 1% and 2%.

There is an important point to be made about the results of calculating real increases in wages over long periods of time. Put simply, the average real wage rise from different occupations tend to converge to a very similar annualised rate as the time period increases. So, in the long term, despite different wage levels and differing wage trends in the short term, the average increase in real wages for skilled and unskilled men are seen to converge over time. To illustrate why this is the case mathematically, consider Occupation A and Occupation B, with the renumeration from Occupation A being, say, 75% of that of Occupation B. Let us further say that after a period of 50 years that the renumeration for both occupations is the same. This means that the wage rate for Occupation A increased faster than Occupation B, by an accumulated 33% over the 50 years. This translates to an annualised increase of 0.58%. So the annualised rate of the wage increase of Occupation A is just 0.58% higher than that of Occupation B, and this annualised difference will fall as the time period increases. In short, there is a common main driver affecting both wage series that, over time, dominates over any (reasonable) change in relative wages levels. Hence, the annualised rate of increase of both wage series converge to the same value as the time period increases.

An analysis of historic wages in Ireland over the last hundred years or so shows that wages increased faster than inflation over any long-term period. The relationship has varied in the past, by period studied, by sector, by occupation, and by gender. However, across all these variables, it is a fair assessment to summarise the historic statistics as showing that wages exceeded inflation by an average of between 1% and 2% per annum over periods of several decades. Trends in real wages in Ireland in the past are not unique to Ireland — similar trends have been observed in most economies in the world (see, for instance, Officer and Williamson (2012), or Williamson (1992), and earlier cited sources). It appears reasonable to conclude that wage escalation has been about 1.5% higher than general price inflation over the long-term past in Ireland.

Arguments have been advanced by some economists, notably Gordon (2016), suggesting that productivity improvements in the past are difficult to maintain in the future and recent trends are giving warning signs. However, there is somewhat of a consensus that real wage increases in Ireland over the long-term future will be similar to the long-term past according to long-term forecasters. The *Actuarial review of the Social Insurance Fund* assumes that wages will increase at an average of about 1.5% per annum above inflation over the next several decades (Department of Employment Affairs and Social Protection (2017), Department of Social Protection (2012)). Other projections assume salaries will tend to rise by 2% real per annum over the long term (e.g., Pensions Board (2005) and (2006)). Assumptions regarding the real rate of increase in staff nurse wages in the long term were made in Appendix 8 (pp.191-240) in *Report of the Public Service Benchmarking Body* (2007). In this actuarial report, the actuary pointed out that "both historic trends and economic projections point to pay increases of 2% p.a. above inflation" (p.210) and, in addition to these general pay increases, staff nurses would have, on average, promotional increases of about 0.8% per annum (p.214). In a less comprehensive but more up-to-date report, the *Report of the Public Service Pay Commission May 2017*, suggest that general pay increases could reasonably be modelled as 1% above inflation (p.100) increased with allowances for promotional increases, which for nurses appears to be about 0.5% per annum (see commentary on p.105).

These assumptions are in line with actuarial practice in countries such as the UK and US where allowance is typically made that wages will increase faster than inflation over the long-term future, generally by between 1% and 2% per annum (e.g., see actuarial valuations of social security or public service pension schemes in these countries). Courts in these jurisdictions have also had to decide on what is a reasonable allowance to make for future real earnings increases. The Guernsey Court of Appeal and the Judicial Committee of the Privy Council have considered this issue in depth recently in the matter of *Helmont v Simon* [Privy Council Appeal No. 0064 of 2011]. The Judicial Committee of the Privy Council upheld the decision that the economic evidence justified a differential between price and wage inflation of 2%.

Assuming real wages increase at +1.5% per annum on average over the long-term future, then the discount rate used in capitalising wage-linked loss in Irish courts should be -2.5% (that is -0.75% for inflation-linked losses, reduced by 0.25% to allow for portfolio managements costs and reduced by a further 1.5% to allow for the real increase in wages).

Longevity Risk

Each future payment will be made only if the injured party is then alive, so a mortality basis is needed to estimate the survival probability. Accordingly, the part of the lump sum to compensate for future loss is dependent not only on the discount rate but also on the mortality basis assumed.

Longevity risk is the risk that the plaintiff will live longer or shorter than expected (and thus be under- or over-compensated). Longevity risk can usefully be decomposed into three distinct components. First, the mortality basis or life table give average rates of survival for a group. So, even assuming the life table is correct, applying any life table to one individual in the group gives rise to random error, as that particular individual may be the one who dies later or earlier than average. Second, determining the appropriate life table for a group, such as the male or female population of Ireland, requires actuarial judgement as, amongst other things, it involves projecting mortality rates into the long-term future. Third, the plaintiff will typically differ from an average person due to injury and disabilities, so adjustment is required to the life table of the average person. Typically, expert medical opinion is sought by the courts on this third issue to determine what reduction to normal life expectancy, if any, is required for the particular impairments of the plaintiff.

It is possible to estimate statistically the extent of the random error in applying a group average to an individual. It is also possible to give an indication of the size of the risk in projecting mortality rates for the population of Ireland. However, the third risk is obviously specific to the individual's impairments so can only be done, if at all, on a case-by-case basis.

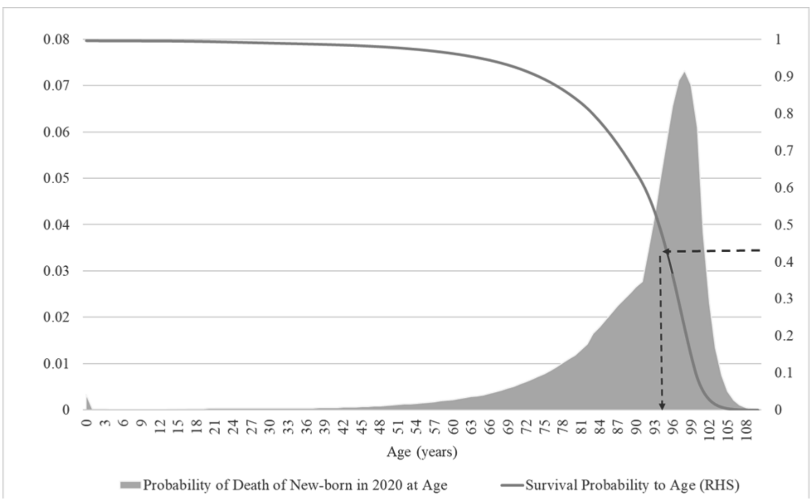
The Central Statistics Office (CSO) publish life tables for the Irish population following each census. The most recent life table is Irish Life Table 16 based on the mortality experience observed over the calendar years 2010 to 2012. These tables give a period life expectancy at birth of 78.4 years for males and 82.8 years for females. These population tables are frequently used as the mortality basis in estimating the present value of future loss court cases in Ireland (Whelan (2009)).

However, period life expectancies do not give a measure of how long a person will live because, as the CSO states: "Period expectation of life …is therefore not the number of years someone of that age could actually expect to live because death rates are likely to change in the future" (CSO (2015)). The cohort life expectancy directly addresses the issue of how long a person can be expected to live as it estimates life expectancy not from historic mortality rates but from the (projected) mortality rates the person can be expected to experience as they go through life. So, for instance, a new-born in calendar year 2020 will be aged 60 years in calendar year 2080 so, in estimating the cohort life expectancy, the current mortality rate of a 60 year-old is adjusted to reflect how that mortality rate is expected to change over the next sixty calendar years. The resultant projected mortality rates are used in the calculation of the cohort life expectancy. There is generally a significant difference between the life expectancies calculated using the two different approaches, with the cohort life expectancy greater than the period life expectancy as mortality rates are forecast to continue to decline in the future.

The CSO project future mortality rates for the population of Ireland as part of an exercise in population and labour force projections undertaken following each census (CSO (2018)). These projected mortality rates are widely used by actuaries and others (e.g., in estimating public and private pensions liabilities) and can be used to estimate cohort life expectancies. Full details of the approach used by the CSO and of alternative approaches are given in Naqvi and Whelan (2019), together with a table of cohort life expectancies in Ireland. The cohort life expectancy for a new-born in Ireland in calendar year 2020 is 90.4 years for a male and 92.7 years for female — some 15% and 12% respectively higher than period life expectancies according to Irish Life Table 16.

Figure 7.4 graphs the probability that a male born in 2020 will survive to each age and the probability of death in each year of age using the most recent mortality projection basis of the CSO.

Figure 7.4: Probability that a New-Born Male in Ireland in 2020 will Die at Each Future age, Together with Probability of Survival to that Age[11]



When an increasing number of similar lives are grouped together then the average lifetime of the group converges to the life expectancy. However, when considering an individual life, one must consider the

[11] Authors' calculations following the methodology employed by the Central Statistics Office (CSO (2018)). For details see Chapter 5 or Naqvi and Whelan (2019). The cohort life table on which the graph is based is shown in Appendix 1.

distribution of the age at death as shown in Figure 7.4. The distribution is negatively skewed, so the mean will be lower than the median. This is a typical feature in human life tables, both period and cohort, with for instance the life expectancy (the mean) of Irish Life Table 16 being 78.4 years for a male at birth but the median being 81.4 years.

The negative skewness of the distribution of the age of death, illustrated in Figure 7.4, is an important consideration when mortality tables are used to estimate the lump sum to compensate a plaintiff for future loss. The cohort life expectancy for a male in Ireland is 90.4 years but the probability that the individual will live longer than 90.4 years is 63%, from the cohort life table tabulated in Appendix 1. Accordingly, a lump sum calculated based on the life expectancy will be adequate for only 37% of individuals. Therefore, the funds available from this lump sum will run out for the majority before they die.

A better alternative to basing the term of the loss on the remaining life expectancy of the plaintiff is to set an explicit probability (or confidence level) that the plaintiff will be adequately compensated. We can then, using the life table, determine the corresponding duration of the loss. So, for instance, if the probability that the plaintiff is not undercompensated is set at, say, 0.5 (and therefore a corresponding 0.5 probability of not overcompensated) then we simply solve for the age in the life table for the term of the loss that matches this probability. This is shown in Figure 7.4, where the probability is selected on the right-hand scale at 0.5 and then we find at what age the survival probability is equal to the given probability. This can be done at various probability levels. Table 7.3 gives the results at selected levels for both males and females.

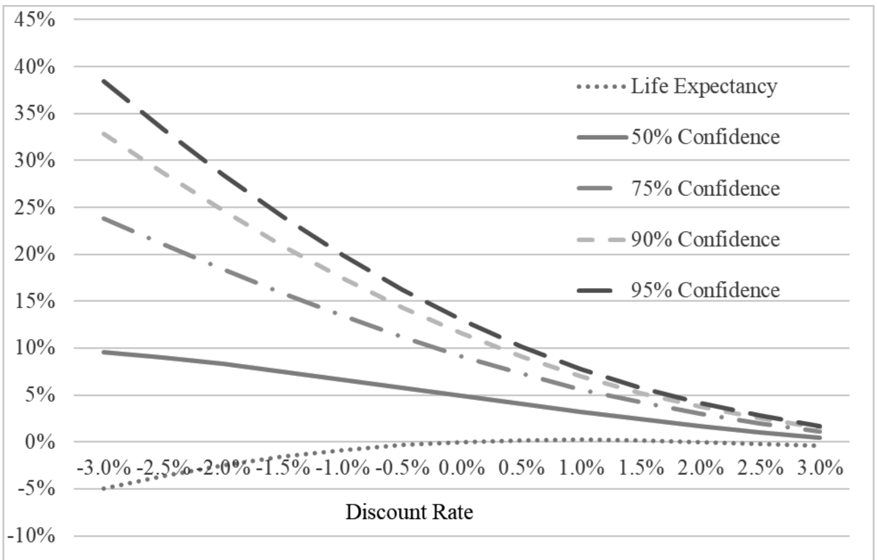
Table 7.3: Duration of Lifetime Loss (in years) of a New-Born in 2020 at Different Confidence Levels to Ensure Not Undercompensated Compared with Life Expectancy[12]

	Life Expectancy	**Probability Not Undercompensated**			
	(Mean)	*50% (Median)*	*75%*	*90%*	*95%*
Male	90.4	94.8	98.6	100.8	102.1
Female	92.7	96.2	99.6	101.7	103.0

[12] Authors' calculations based on the cohort life tables in Appendix 1.

Applying this approach, we can calculate the lump sum required to compensate the individual plaintiff with any associated degree of confidence. Annuity and annuity-certain values are calculated at various discount rates and presented in Figure 7.5. The exercise shows that estimating the loss at the 75% confidence level rather than estimating it using a life annuity increases the present value of the loss by 21% for a new-born male when the discount rate is -2.5%. At the 90% confidence level and a discount rate of -2.5%, the increase the loss above the life annuity approach is 29% for a new-born male. Similar increases are observed for females.

Figure 7.5: Increase in Present Value of an Annuity Certain with Different Confidence Levels above a Life Annuity, for a New-Born Male in Ireland in 2020, at Various Discount Rates[13]



The above methodology allows us to make explicit allowance for the longevity risk arising from random fluctuations in lifetimes. However, it still leaves the risk that the cohort life table employed differs from actual mortality experience that the new-borns in 2020 will experience in the future. The CSO expert group base the cohort life table on its best

[13] Authors' calculations.

estimate of future mortality improvements. However, these forecasts cannot be expected to be that reliable as they involve forecasting the path of mortality improvements for a hundred years and more. It is difficult to forecast medical advances (e.g., antibiotics) or pandemics (e.g., Spanish Flu) which in the past have had a significant impact on mortality rates, either permanently or temporarily In fact, official forecasts of life expectancies in Ireland and elsewhere have tended to be too conservative, with actual improvements exceeding those forecast (Keilman (2008)), Waldron (2005), Oeppen and Vaupel (2002)). This tendency to underestimation is largely due to forecasters predicting a levelling off or slowdown in the rate of mortality improvements while rates of improvement tended, in actuality, to increase in most countries at least until 2011 (Navqi and Whelan (2019)).

The CSO does not give confidence bounds around its central estimate that might give an indication of the inherent uncertainty associated with its projections. However, the Population Division of the United Nations (UN) do forecast period life expectancies at birth for Ireland (and for every other country in the world), together with 80% and 95% prediction bounds for each calendar year 2020 to 2100 (UN (2015)). From the UN period life expectancies, Whelan and Naqvi (2020) derive consistent cohort life expectancies for Ireland with 80% and 95% prediction bounds. These are shown in Table 7.4.

Table 7.4: Male and Female Projected Cohort Life Expectancies in Ireland for New-Born in 2020, with 50%, 80% and 95% Prediction Intervals Consistent with UN 2019 Forecasts (including CSO 2018 projection)[14]

	Lower 95%	*Lower 80%*	*Median*	*CSO 2018 Projection*	*Upper 80%*	*Upper 95%*
Male	83.9	86.0	89.7	90.4	93.4	95.2
Female	86.8	88.8	92.5	92.7	95.9	97.3

[14] Figures sourced from Table 1 in Whelan and Naqvi (2020) (see also Chapter 6).

The figures in Table 7.4 are, naturally, to subject to future revision as they depend on the historic data-driven Bayesian hierarchical model used by the UN, which will change with new data (Raftery et al. (2014)). In short, the figures in Table 7.4 are best viewed as indicative only as it is not possible to be precise about our uncertainty over the future course of mortality improvements. Comparing Table 7.4 with Table 7.3 earlier, suggests that random error associated with applying an average cohort life table to an individual tends to be more significant than estimation errors associated with cohort life expectancies.

Finally, adjustments must be made to the cohort life table so that allowance is made for any increased mortality risk to the plaintiff due to their particular impairments. This adjustment often introduces considerably more uncertainty (and therefore risk) as the studies supporting any adjustment are based on relatively small and heterogenous groups. In cases of cerebral palsy, experts to Irish courts often rely on the percentage reduction to average population life expectancy estimated in a study of a Californian database of persons with cerebral palsy over a 28-year period (Brooks et al. (2014)). This study has considerably less than 20,000 subjects at each age, and sub-divides this number further into ten subgroups based on motor skills and feeding skills and then further sub-divides each subgroup by sex. Inevitably, the sub-divisions ignore commonly associated cognitive and sensory impairments — important factors known to affect mortality rates such as IQ level and vision (e.g., Hutton et al. (2000), Hutton et al. (2006), Hemming et al. (2006), Blair et al. (2001)). The key point is that the adjustment to be made to the population life table to allow for the mortality impact of the plaintiff's impairments is often an issue where evidence is scant and experts can reasonably differ, especially as some mortality impacts might be ameliorated by future care structures which are dependent on the eventual settlement.

There is large uncertainty associated with when an individual will die. The sources of error — the random error associated with the age of death of an individual subject to a life table and the estimation errors in determining the life table — add to the difficulty the plaintiff has in devising a draw-down strategy to ensure s/he will not outlive their financial resources. The analysis in this section is of practical significance to the plaintiff in designing a drawdown strategy so that, with an acceptable degree of certainty, the money will not be exhausted before the plaintiff dies.

The mortality basis frequently used for capitalising future loss in Ireland to date is the most recent period life table of population, adjusted as necessary by medical opinion on the reduction in life expectancy of the plaintiff. The loss is capitalised using a life annuity. As we have shown, this approach tends to undercompensate the plaintiff in two ways. First, cohort rather than period life tables should be used. The resultant cohort life expectancy tends to be 10% to 15% higher than the period life expectancy, the exact uplift depending on sex and age. Second, life expectancies or life annuities should not be used in capitalising the loss, as the individual has a probability greater than 50% of outliving the average life expectancy. To be, say, 75% confident that the plaintiff will not live longer than allowed for in the loss calculations requires a further material increase to the lump sum. For a new-born in Ireland in 2020 the increase is marginally above 20% using a discount rate of -2.5%.

Periodic Payments Orders

The earlier sections highlight the difficulties in converting a lump sum award into a future stream of income that match the expected future outgoes for care costs and other loss. Investing in the risk least portfolio of index-linked gilts still leaves the plaintiff with (i) the small basis risk that Irish inflation will diverge from eurozone inflation, (ii) the reinvestment risk which arises when future proceeds must buy future longer term index-linked bonds that are currently unavailable, and (iii) the risk that future wage increases will exceed the annual average allowed for of 1.5%. Added to those risks must be the significant uncertainty in estimating how long the plaintiff will survive, considered in the previous section. The judiciary in Ireland have long pointed out that due to these difficulties it is an impossible task to determine an award fair to both parties, or, in the words of Ms Justice Irvine:

> To state that the current law in this jurisdiction, which requires the court to award a lump sum intended to compensate the plaintiff for all past and future losses, and in particular future pecuniary loss, is inherently fallible and unjust cannot be disputed. It is also grossly outdated by reference to the approach now adopted by the courts in other Common Law and Civil Law jurisdictions.
>
> *Judgement of Court of Appeal, Russell-v-HSE 2015*

The Irish judiciary would welcome a change in the law so that redress for future loss could be made by way of periodic payments over the future lifetime of the plaintiff. The Law Reform Commission (1996) and the Working Group on Medical Negligence and Periodic Payments (2010) called for such reform to bring the system in Ireland in line with the UK, US, Canada, Australia, and other EU countries. As noted earlier, the law was amended in Ireland so from October 2018 claims for catastrophic injury could be part settled by annual payments for the remainder of plaintiff's lifetime. Here a catastrophic injury is defined as one where the plaintiff is permanently disabled and needs to receive lifelong care (Civil Liability (Amendment) Act 2017). This mode of settlement, known as a Periodic Payment Order (PPO), was targeted to meet the growing number of cerebral palsy claims against the HSE. In fact, the SCA pioneered 'interim' PPOs from 2010, in anticipation of such legislation being put in place for compensation by final PPOs. However, just thirteen months later the High Court ruled that, as drafted, the legislation did not allow full compensation and therefore "no judge charged with protecting plaintiffs' best interests could recommend such a scheme" [Judgement in Jack Hegarty -v- HSE 2015/10520P]. At the time of that judgement in November 2019, the SCA had 83 such catastrophic injury cases where liability has been admitted awaiting final PPO or lump sum settlements.[15]

PPOs once decided by the court are not subject to review in the future in all jurisdictions where they have been introduced, no matter how the needs of the plaintiff subsequently change. However, the payments themselves increase at a pre-agreed rate of indexation. The flaw in the legislation introducing periodic payments in Ireland relates to the indexation applied to the regular payments. All payments for loss of wages, cost of care, cost of medical treatments and aids must be indexed with the Harmonised Index of Consumer Prices for Ireland. This is, of course, an inflation measure, which can be expected to lag wage increases by about 1.5% per annum (see earlier). The consequence of indexing at inflation when a wage rate index is more appropriate is manifest in the long-term from compounding the differences: inflation-linked payments

[15] *Irish Times*, 19 Nov 2019, "Medical Negligence cases set to cost record €374 million next year".

are less than half wage-linked costs after fifty years (assuming an annualised differential of 1.5%).

The rate of indexation of the PPO was obviously a key issue when drafting the legislation. The Working Group on Medical Negligence and Periodic Payments (2010) had make a key recommendation in this regard:

> Provision within the legislation must be made for adequate and appropriate indexation of periodic payments as an essential prerequisite for their introduction as an appropriate form of compensation. In particular, the Group recommends the introduction of earnings and costs-related indices which will allow periodic payments to be index-linked to the levels of earnings of treatment and care personnel and to changes in costs of medical and assistive aids and appliances. This will ensure that plaintiffs will be able to afford the cost of treatment and care into the future.
>
> (*Executive Summary*, p.8)

However, when it came to drafting the legislation, the *Report of the Working Group on Legislation on Periodic Payment Orders* (2015) recommended the index should be the Harmonised Index of Consumer Prices for Ireland (HICP), influenced by an actuarial report commissioned by the SCA (Towers Watson (2014)).[16] The Working Group erroneously state that the actuarial report suggests indexation of the plaintiff's annual award at HICP plus a fixed percentage of 0.5% "to take account of wage increases" (see p. 23 and also p. 21). The actuarial report, *Feasibility study on the introduction of PPOs in Ireland*, models the "indexation matching the claimant needs" — including wage inflation and range up to bespoke medical and living support care cost inflation — at HICP plus 1½% per annum. In short, the Towers Watson report agrees that the appropriate indexation is best modelled at inflation plus

[16] The Working Group decided that it should specify the index in the legislation and not leave it up to the courts to decide (as it was in the UK where a wage index had been adopted by the courts). In making this decision, the Working Group (comprising of senior members of the SCA, Department of Finance and other public servants) expressed itself guided by the interests of the defendants, or in the words of the Report: "the Working Group did not favour leaving the choice of index to the discretion of the court as it could introduce a high degree of uncertainty as to potential financial liabilities both for the State and for the insurance industry ... the index chosen should provide as much certainty as possible for defendants in terms of projected increases in their financial liabilities" (*Report of the Working Group on Legislation on Periodic Payment Orders* (2015), p.19).

1.5% per annum. This actuarial report also shows that introducing PPOs, whether indexed by inflation or a wage index, can be expected to increase market premiums (p.53) and the cost of claims with "significant potential solvency issues for insurers" (p.5). This is consistent with our findings earlier that lump sum compensation is currently reckoned on a basis that is lower than the fair value.

It is a simple matter to amend the legislation so that the indexation of PPOs is either determined by the courts (as in the UK, which deems a wage index appropriate) or a suitable wage index maintained by the CSO. Perhaps one obstacle to this simple remedy is that, if currently implemented, it would have a significant financial impact on the State. PPOs are simply a secure future series of payments rising in line with wages or some other index over some period. It is possible to put a market value on such a stream of future payments. The market value of the PPO, as developed earlier, is considerably greater than the lump sum award currently made by the courts. We may term the difference as the PPO uplift — the value of the PPO is higher than the value that the claim is currently settling. So the State is unlikely to amend the indexation in the current PPO legislation as long as the courts maintain a higher discount rate to capitalise future loss to a lump sum than the ruling market rate.

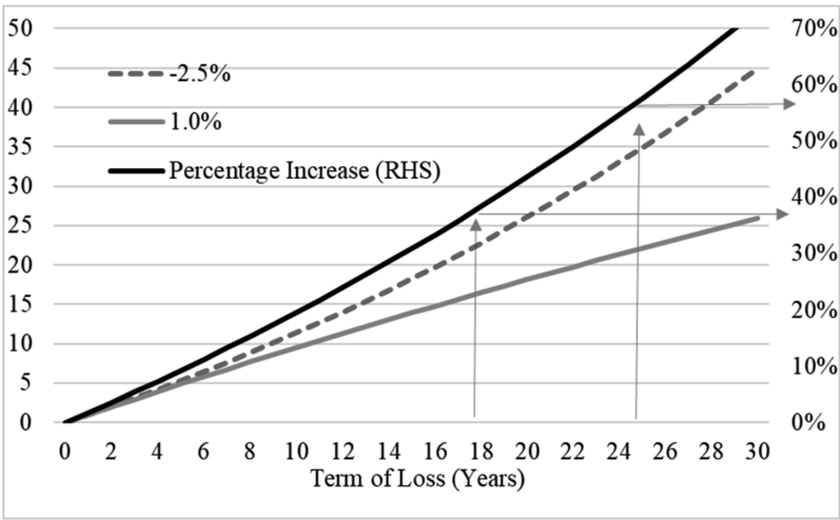
We can estimate the impact of the PPO uplift on the State's current outstanding liability to clinical and general claims. As shown earlier, consistent with legal principles in Ireland, the annualised discount rate for future wage-linked loss should be -2.5% (broken down as -0.75% p.a. real yield on index-linked stock, reduced by c. 0.25% p.a. for investment charges, and reduced by a further 1.5% p.a. to allow for the real rate of salary escalation). Currently awards by the courts are discounting future losses at between +1% per annum (for wage loss) and +1.5% per annum (for inflation-linked loss).

Now a simple but very crude estimate would be to note that the estimated total liability to the State jumped by 17% in 2015 when the discount rate changed from 3% to between 1.5% and 1% following the ruling in the Russell-v-HSE case as detailed in the introductory section. If a change of between 1.5% to 2% in the interest rate leads to a 17% increase in the liability then a change of 3.5% (that is from +1% to -

2.5%) might lead to an increase of double 17%, that is about a third increase. A third increase to the outstanding liability of €3.15 billion is just over €1 billion. This estimate can be expected to underestimate the true figure as present values rise faster than linearly as discount rates fall.

A better estimate is to consider the weighted average duration of the loss. The present value of the loss depends on the duration of the loss. Figure 7.6 graphs the present value against the term of the loss at either discount rate and highlights the factor by which the present value increases when moving from a discount rate of +1.0% to -2.5%.

Figure 7.6: Present Value of an Annuity Certain of 1 per Annum at Discount Rate of -2.5% and +1.0% (LHS) and Percentage Increase in Present Value in Change from +1.0% to -2.5% (RHS)[17]



Now if a change in the discount rate from 3% to between 1.5% and 1.0% increases the aggregate liability by 17%, then, with some elementary computation, we can estimate that the weighted average duration of loss is between 17 and 24 years. Knowing the duration of the loss allows us to estimate the effect of any change in discount rate, as illustrated in Figure 7.6. The change in discount rate from between 1.5% and 1.0% down to -2.5% when the duration of the loss is between 17 and 24 years entails an increase of between 33% to 66%. This in turn equates to an increase of

[17] Authors' calculation.

between €1 to €2 billion on outstanding liabilities of €3.15 billion. These estimates are crude but do give a measure of the State's financial inertia to introducing PPOs. In short, it is difficult to envisage the State amending the PPOs legislation with any urgency when claims against it are currently settling for a fraction of their market value. An incentive for the State to settle by lump sum instead of appropriately indexed PPOs will persist as long as the discount rates for future loss are higher than ruling rates in the market.

Conclusion

Damages inflicted by wrongful or negligent acts can, aside from pain and suffering, be pictured as a series of future costs or losses stretching for the remaining lifetime of the plaintiff, generally rising in line with inflation or wages in the economy. The most appropriate way to compensate the plaintiff is, obviously, to replace that stream of losses with periodic payments that match the amount and rate of increase of the loss. Such simple redress schemes are an important part of tort law in many jurisdictions in the world including the UK, US, and many EU countries. Legislation to achieve this end has not been satisfactorily introduced in Ireland. This paper suggests that one reason for such delay is that lump sum compensation in lieu of such future payments is, and has been, considerably lower than the market value of the stream of payments. Simply, the State which, directly or indirectly is a defendant in many such cases is financially incentivised to delay any legislation until the lump sum awards are increased to the market value of future loss.

This paper demonstrates that the stakes are high when a change is made in how compensation is calculated. First, the discount rate applied to future loss should be reduced to bring it in line with legal precedent and current market conditions, from +1.5% per annum to -1.0% per annum for inflation-linked loss and from +1.0% per annum to -2.5% per annum for wage-linked loss. Second, the lump sum award should no longer be capitalised using the life annuity approach commonly used to date. Instead, to allow appropriately for longevity risk, the lump sum award should be calculated by way of an annuity-certain, the term set so that the plaintiff is not expected to live longer than their compensation allows with a pre-specified degree of confidence. Finally, the mortality basis used, before adjustment for the plaintiff's life-shortening impairments, should be a cohort mortality basis incorporating likely changes in mortality rates over the lifetime of the plaintiff.

The changes if applied to capitalising the loss would have a significant impact on the quantum of awards, increasing with increasing term of the loss. Changing the mortality basis from a period to cohort approach can be expected to increase the term of the loss by about 10% to 15%. Changing how the term of the loss is estimated, from a life expectancy or life annuity approach to the annuity certain with pre-specified confidence level, can increase the term by a further 20% or more. Changing the discount rate can be expected to have the biggest impact, increasing the award by more than one-third if the term exceeds 17 years, and more than double that if it exceeds 25 years (see Figure 7.6). Such changes, we estimate, will increase the State's liability to existing outstanding claims against it by more than €1 billion, and perhaps closer to €2 billion.

Despite the large sums involved, there are only losers when the comes to medical negligence cases. The plaintiff suffers a reduced quality of life, a suffering shared by parents and family of catastrophically damaged infants. Medical and other hospital staff are demoralised (Murphy (2018)). After the trauma of the incident itself follows the prolonged litigation process, giving years of stress and anxiety to all, and involving considerable work by legal teams and experts on either side. The State Claims Agency reports that the monetary costs associated with the legal process in clinical claims amounted to €67 million in 2018 while the awards for that year were €180 million (NTMA Annual Report and Accounts 2018, p. 44).

The State is perhaps misdirecting its attention in trying to reduce the size of each claim rather than reduce the number of claims. Tort law ideally should deter wrongful behaviour through the award of damages. Over the last decade there have been many incidences where the Irish courts have been satisfied that the standard of care in the maternity unit was unacceptably deficient in a manner that led to the injuries and compensation must be paid. Over the last decade there have also been several investigations into the operation of maternity services in Ireland, all highlighting significant scope for improvement. Helps et al. (2020), in a review of the ten national enquiries into maternity services Ireland between 2005 and 2018, report that all ten recommend staffing levels and staff training be increased and nine of them recommend the need for better risk management practices, recommendations reiterated again in the most recent review of maternity services (Health Information and Quality Authority (2020)). Whelan and Hally (2020) show that the rise of claims settlements has been so dramatic over the decade that more is now being paid out by way of claims against the maternity services than it is actually spent in delivering the services and suggest that spending more on maternity services might be cost saving in the long run (see Chapter 8). A way must be found to ensure the HSE priorities the reforms to the maternity services so obviously needed — be it by funding maternity services separately to ensure adequate staffing and training, or by making budget contingent on reform. Also, the Department of Public Expenditure and Reform, under whose remit this falls, should oversee the reform in maternity services and determine its separate budget.

The stakes are also high in non-pecuniary terms when the discount rate and appropriate approach to allow for longevity risk is contested in the Irish courts. For the judiciary, setting a discount rate in line with current market condition and appropriately apportioning longevity risk would remove a key obstacle preventing the modernising of our system to allow compensation by life contingent periodic payments. For maternity and other clinical services, it could be the tipping point when the sums paid out by way of settlements for mismanagement become appreciably larger than the additional costs of operating a sound system. For the State, it means ensuring justice for its most vulnerable citizens.

Appendix 1: *Table A7.1: Cohort Life Table for Male and Females Born in Ireland in 2020 based on CSO Mortality Projection Basis (CSO (2018), Naqvi and Whelan (2019))*

Year	Age	Male			Female		
		Mortality Rate	Probability of New-born in 2020 Dying in Year	Probability of New-born in 2020 Surviving to End of Year	Mortality Rate	Probability of New-born in 2020 Dying in Year	Probability of New-born in 2020 Surviving to End of Year
2020	0	0.00334	0.00334	0.99666	0.00274	0.00274	0.99726
2021	1	0.00015	0.00015	0.99650	0.00015	0.00015	0.99711
2022	2	0.00014	0.00013	0.99637	0.00011	0.00011	0.99700
2023	3	0.00010	0.00010	0.99627	0.00007	0.00007	0.99693
2024	4	0.00008	0.00007	0.99619	0.00006	0.00006	0.99687
2025	5	0.00006	0.00006	0.99613	0.00006	0.00006	0.99681
2026	6	0.00004	0.00004	0.99610	0.00004	0.00004	0.99677
2027	7	0.00004	0.00004	0.99606	0.00005	0.00005	0.99672
2028	8	0.00005	0.00005	0.99601	0.00005	0.00005	0.99667
2029	9	0.00005	0.00005	0.99596	0.00004	0.00004	0.99663
2030	10	0.00005	0.00005	0.99590	0.00004	0.00004	0.99659
2031	11	0.00005	0.00005	0.99585	0.00003	0.00003	0.99656
2032	12	0.00006	0.00006	0.99580	0.00003	0.00003	0.99653
2033	13	0.00008	0.00008	0.99572	0.00004	0.00004	0.99649
2034	14	0.00011	0.00011	0.99561	0.00005	0.00005	0.99644
2035	15	0.00014	0.00014	0.99548	0.00006	0.00006	0.99638
2036	16	0.00017	0.00017	0.99530	0.00007	0.00007	0.99631
2037	17	0.00020	0.00020	0.99510	0.00008	0.00008	0.99623
2038	18	0.00024	0.00024	0.99486	0.00009	0.00009	0.99615
2039	19	0.00028	0.00028	0.99458	0.00010	0.00009	0.99605
2040	20	0.00032	0.00032	0.99426	0.00010	0.00010	0.99595
2041	21	0.00036	0.00036	0.99390	0.00011	0.00011	0.99584
2042	22	0.00039	0.00038	0.99352	0.00012	0.00012	0.99572
2043	23	0.00041	0.00040	0.99311	0.00013	0.00013	0.99559
2044	24	0.00042	0.00042	0.99270	0.00013	0.00013	0.99546
2045	25	0.00043	0.00042	0.99227	0.00014	0.00014	0.99532
2046	26	0.00043	0.00043	0.99185	0.00014	0.00014	0.99518
2047	27	0.00043	0.00043	0.99142	0.00015	0.00015	0.99503
2048	28	0.00043	0.00042	0.99099	0.00016	0.00016	0.99486
2049	29	0.00041	0.00041	0.99058	0.00018	0.00018	0.99469
2050	30	0.00040	0.00039	0.99019	0.00019	0.00019	0.99450
2051	31	0.00039	0.00038	0.98981	0.00020	0.00020	0.99430
2052	32	0.00038	0.00038	0.98943	0.00021	0.00021	0.99409
2053	33	0.00038	0.00038	0.98905	0.00022	0.00022	0.99388
2054	34	0.00039	0.00038	0.98867	0.00022	0.00022	0.99366
2055	35	0.00039	0.00039	0.98828	0.00022	0.00022	0.99344
2056	36	0.00041	0.00040	0.98788	0.00023	0.00023	0.99321
2057	37	0.00042	0.00042	0.98746	0.00024	0.00024	0.99298

		Male			Female		
Year	Age	Mortality Rate	Probability of New-born in 2020 Dying in Year	Probability of New-born in 2020 Surviving to End of Year	Mortality Rate	Probability of New-born in 2020 Dying in Year	Probability of New-born in 2020 Surviving to End of Year
2058	38	0.00045	0.00044	0.98702	0.00025	0.00025	0.99273
2059	39	0.00047	0.00047	0.98655	0.00027	0.00027	0.99246
2060	40	0.00050	0.00050	0.98605	0.00029	0.00029	0.99217
2061	41	0.00054	0.00053	0.98552	0.00032	0.00031	0.99186
2062	42	0.00058	0.00057	0.98495	0.00035	0.00034	0.99151
2063	43	0.00062	0.00061	0.98435	0.00038	0.00037	0.99114
2064	44	0.00066	0.00065	0.98370	0.00041	0.00041	0.99073
2065	45	0.00070	0.00069	0.98301	0.00045	0.00045	0.99028
2066	46	0.00076	0.00074	0.98226	0.00050	0.00049	0.98979
2067	47	0.00082	0.00081	0.98146	0.00055	0.00054	0.98925
2068	48	0.00090	0.00088	0.98058	0.00060	0.00060	0.98865
2069	49	0.00098	0.00096	0.97961	0.00066	0.00066	0.98799
2070	50	0.00107	0.00105	0.97856	0.00073	0.00072	0.98727
2071	51	0.00117	0.00114	0.97742	0.00080	0.00079	0.98648
2072	52	0.00127	0.00124	0.97617	0.00088	0.00087	0.98561
2073	53	0.00138	0.00134	0.97483	0.00098	0.00097	0.98464
2074	54	0.00148	0.00144	0.97339	0.00110	0.00108	0.98356
2075	55	0.00159	0.00155	0.97184	0.00121	0.00119	0.98237
2076	56	0.00171	0.00167	0.97018	0.00132	0.00130	0.98107
2077	57	0.00185	0.00180	0.96838	0.00142	0.00140	0.97967
2078	58	0.00201	0.00194	0.96644	0.00150	0.00147	0.97820
2079	59	0.00216	0.00209	0.96434	0.00155	0.00152	0.97668
2080	60	0.00234	0.00225	0.96209	0.00161	0.00157	0.97511
2081	61	0.00253	0.00243	0.95966	0.00169	0.00165	0.97346
2082	62	0.00275	0.00263	0.95702	0.00182	0.00177	0.97169
2083	63	0.00297	0.00284	0.95418	0.00198	0.00193	0.96976
2084	64	0.00320	0.00305	0.95113	0.00218	0.00211	0.96765
2085	65	0.00345	0.00328	0.94785	0.00239	0.00232	0.96533
2086	66	0.00375	0.00356	0.94429	0.00264	0.00254	0.96279
2087	67	0.00412	0.00389	0.94041	0.00290	0.00279	0.95999
2088	68	0.00453	0.00426	0.93614	0.00318	0.00305	0.95694
2089	69	0.00498	0.00466	0.93148	0.00346	0.00331	0.95364
2090	70	0.00548	0.00510	0.92638	0.00377	0.00359	0.95004
2091	71	0.00603	0.00559	0.92079	0.00412	0.00392	0.94613
2092	72	0.00666	0.00613	0.91466	0.00454	0.00430	0.94183
2093	73	0.00731	0.00668	0.90797	0.00498	0.00469	0.93714
2094	74	0.00798	0.00724	0.90073	0.00543	0.00509	0.93205
2095	75	0.00871	0.00785	0.89288	0.00595	0.00554	0.92651
2096	76	0.00956	0.00854	0.88435	0.00657	0.00608	0.92042
2097	77	0.01056	0.00934	0.87501	0.00734	0.00676	0.91366

		Male			Female		
Year	Age	Mortality Rate	Probability of New-born in 2020 Dying in Year	Probability of New-born in 2020 Surviving to End of Year	Mortality Rate	Probability of New-born in 2020 Dying in Year	Probability of New-born in 2020 Surviving to End of Year
2098	78	0.01161	0.01016	0.86486	0.00820	0.00749	0.90617
2099	79	0.01267	0.01096	0.85390	0.00911	0.00825	0.89792
2100	80	0.01388	0.01185	0.84204	0.01014	0.00911	0.88881
2101	81	0.01536	0.01293	0.82911	0.01138	0.01012	0.87870
2102	82	0.01720	0.01426	0.81485	0.01290	0.01134	0.86736
2103	83	0.02018	0.01644	0.79841	0.01497	0.01298	0.85437
2104	84	0.02242	0.01790	0.78051	0.01683	0.01438	0.84000
2105	85	0.02484	0.01939	0.76112	0.01888	0.01586	0.82413
2106	86	0.02746	0.02090	0.74022	0.02114	0.01742	0.80671
2107	87	0.03026	0.02240	0.71782	0.02360	0.01904	0.78768
2108	88	0.03323	0.02385	0.69397	0.02628	0.02070	0.76698
2109	89	0.03637	0.02524	0.66873	0.02917	0.02237	0.74460
2110	90	0.03965	0.02651	0.64221	0.03227	0.02403	0.72057
2111	91	0.04305	0.02765	0.61457	0.03556	0.02563	0.69495
2112	92	0.05560	0.03417	0.58040	0.04631	0.03218	0.66277
2113	93	0.07169	0.04161	0.53879	0.06023	0.03992	0.62285
2114	94	0.09229	0.04972	0.48907	0.07820	0.04871	0.57414
2115	95	0.11858	0.05799	0.43107	0.10136	0.05819	0.51594
2116	96	0.15207	0.06555	0.36552	0.13110	0.06764	0.44831
2117	97	0.19463	0.07114	0.29438	0.16918	0.07584	0.37246
2118	98	0.24859	0.07318	0.22120	0.21779	0.08112	0.29135
2119	99	0.31689	0.07010	0.15110	0.27968	0.08148	0.20986
2120	100	0.40321	0.06093	0.09018	0.35827	0.07519	0.13467
2121	101	0.42361	0.03820	0.05198	0.38119	0.05134	0.08334
2122	102	0.44303	0.02303	0.02895	0.40342	0.03362	0.04972
2123	103	0.46137	0.01336	0.01559	0.42475	0.02112	0.02860
2124	104	0.47854	0.00746	0.00813	0.44501	0.01273	0.01587
2125	105	0.49448	0.00402	0.00411	0.46406	0.00737	0.00851
2126	106	0.50918	0.00209	0.00202	0.48182	0.00410	0.00441
2127	107	0.52264	0.00105	0.00096	0.49823	0.00220	0.00221
2128	108	0.53490	0.00052	0.00045	0.51327	0.00114	0.00108
2129	109	0.54600	0.00024	0.00020	0.52697	0.00057	0.00051
2130	110	0.55601	0.00011	0.00009	0.53936	0.00027	0.00023
2131	111	0.56498	0.00005	0.00004	0.55050	0.00013	0.00011
2132	112	0.57300	0.00002	0.00002	0.56048	0.00006	0.00005
2133	113	0.58015	0.00001	0.00001	0.56936	0.00003	0.00002
2134	114	0.58649	0.00000	0.00000	0.57724	0.00001	0.00001
2135	115	0.59211	0.00000	0.00000	0.58421	0.00000	0.00000

Please note that probability of new-born in 2020 dying in any future calendar year or surviving to the end of the same year do not sum to 1 after year 2020 as there is a probability that the new-born in 2020 will not survive to the calendar year in question.

Chapter 8

The True Cost to the State of Maternity Services in Ireland

(Co-authored with Maeve Hally and Caoimhe Gaughan)

Abstract

Accounting for the cost of delivery of maternity services in Ireland ignores the cost of claims settlements caused by negligence in delivery. We show that the true cost of maternity services is more than double the generally reported cost when proper account is taken of the associated cost of maternity claims. There must come a tipping point, if it is not already exceeded, when the sums paid out by way of settlements for mismanagement of maternity services become larger than the additional costs of operating a sound service.

Introduction

Maternity services in Ireland support approximately 60,000 deliveries each year through 19 dedicated public maternity units (Health Information and Quality Authority (2020)). Some 15% of the total availing of these services pay for private maternity care but this care is delivered within these public units. The Clinical Indemnity Scheme operated by the State Claims Agency covers all clinical claims made against maternity services in Ireland for both public and private pathways.

The Rising Cost of Clinical Claims

The State Claims Agency (SCA) operates two insurance schemes for the State, the Clinical Indemnity Scheme (CIS) and the General Indemnity Scheme (GIS). The CIS covers all clinical claims against hospitals (including maternity services), the HSE, and some other parties while the GIS covers all non-clinical claims. Of the total €1.9 billion claims settled by the State Claims Agency over the last decade, €1.7 billion (or 89%) was in respect of the Clinical Indemnity Scheme (Whelan and Hally (2020)).

The number of new claims is increasing at a faster rate than the number being resolved in recent years. The rate of growth of both claim settlements and the rise in outstanding liabilities has averaged more than 15% per annum since 2010. At the end of 2019, the estimated outstanding liabilities amounted to €3.63 billion, up from €783 million in June 2010. Outstanding clinical claims comprise three-quarters of this figure "primarily due to the high estimated liability associated with maternity services claims, particularly those arising from the high cost of settling catastrophic brain-injury infant cases."[1] In 2011, the Director of the SCA estimated that such cases of cerebral palsy at birth, while only 3% of the claims by number, accounted for two-thirds of the CIS liability (Breen (2011)). Accordingly, we can estimate that the liability to cerebral palsy cases represent about half of the total outstanding liability (that is two-thirds of the CIS which is three-quarters of the total outstanding liability). This is consistent with the NTMA Report and Annual 2017 which reported that the estimated liability in respect of maternity services claims was €1.38bn compared to total estimated outstanding claims of €2.66 billion (that is 53%). Already, individual settlements for cerebral palsy and associated birth injuries have exceeded €20 million before legal and other costs (*Irish Times*, 5th November 2019).

Cost of Maternity Services and Cost of Claims on Maternity Services

Since 2015 the HSE has implemented "Activity Based Funding", which requires estimates of the cost for each procedure (ignoring capital costs),

[1] National Treasury Management Agency *Annual Reports and Accounts, 2009 to 2019*. See 2019, especially p. 52, 2018, p. 42 and 2009, p. 22.

and annually publish such cost estimates. The most recent figures show that the price range for deliveries varies from €2,418 for a Vaginal Delivery with Minor Complications to €10,313 for a Caesarean Delivery with Major Complications (Healthcare Pricing Office (2020)). Based on the number and type of each delivery and the estimated price per procedure in each year, we estimate that the average cost to the State per delivery was €3,324 between 2015 and 2020 (see Table 8.1). An earlier study put the average cost in 2009 at €2,780 including €1,200 attributable to postnatal bed care costs (Kenny et al. (2015)).

Table 8.1: Number of Deliveries Each Calendar Year in Ireland and Estimated Price per Delivery [2]

Year	No. of Deliveries	Estimated Price Per Delivery
2020	58,718	€3,670.43
2019	58,006	€3,348.25
2018	59,608	€3,409.94
2017	60,496	€3,218.75
2016	62,442	€3,169.49
2015	64,115	€3,128.50
2014	65,608	n/a
2013	65,115	n/a
2012	66,098	n/a
2011	71,231	n/a
2010	72,657	n/a
2009	72,864	€2,780

Discharges from maternity units in Ireland after delivery accounted for about 3% of all acute hospital discharges but, as noted earlier, gave rise to about half of the overall liability to the State in negligence claims. The NTMA accounts for the years 2016 and 2017 show that the outstanding liability for maternity claims increased from €1.09 billion to

[2] Number of deliveries as reported each year in Healthcare Pricing Office, *Activity in Acute Public Hospitals in Ireland Annual Reports, 2009-2019*. Deliveries include live single, multiple and stillbirths. Estimated Price Per Delivery in 2020 calculated from figures published in Healthcare Pricing Office, *ABF 2020 Admitted Patient Price List*. Figures for earlier years were calculated from figures kindly provided to the authors by the Healthcare Pricing Office for those years and, for 2009, by Kenny et al. (2015).

€1.38 billion, that is an increase of €290 million. In addition, a total of €282 million was paid out in 2017, roughly half or €141 million could be for maternity claims giving a total estimate of €431 million. There were 60,496 deliveries in 2017. This gives an average estimated claims cost of €7,124 per delivery in 2017. The estimated claims cost per delivery in 2017 was more than twice the cost per delivery in 2017.

Due to long delays between incident and claim, it is necessary to average over a longer period than one year to see if the pattern is stable. A total of €1.9 billion was paid in claims over the last decade and claims outstanding at the end of the decade increased by about €2.85 billion (that is €3.63 billion as the most recent available figure at end 2019 less €0.78 billion in June 2010). Hence the estimated liabilities over the last decade is €4.75 billion, about half of which is in respect of maternity services or €2.375 billion. The number of deliveries in Ireland was 645,376 over the decade from the start of 2010 to the end of 2019 (see Table 8.1) This gives an estimated claims cost of €3,680 on average per delivery over the last decade. This is higher than the cost to the State of providing the maternity service ignoring capital costs.

In short, the figures show that liabilities arising from negligent birth injuries each year are now greater than the amount actually spent by the State in the day-to-day running of maternity services.

Quality of Maternity Services

There have been several reports published over the last decade investigating the functioning of Irish maternity services and the scope for improvement (see, for instance, Department of Health (2015)). A recent study overviewed the finding of ten of these national inquiries published between 2005 and 2018 and draws attention to the consistent recommendation that staffing levels and staff training be increased (recommended in all reports) and the need for better risk management practices (recommended in 9 out of the 10 reports) (Helps et al. (2020)). Indeed, the Health Information and Quality Authority's more recent overview of maternity services reiterated these recommendations, alongside its recommendation that "The HSE must immediately develop a comprehensive, time-bound and fully costed National Maternity Strategy implementation plan…" (Health Information and Quality Authority (2020), p.118).

The independent investigations also give an assessment of how maternity services have been delivered over the last decade across many of the 19 maternity units in Ireland. For example, the 2014 report on Portlaoise Hospital Maternity Services concludes "poor outcomes that could likely have been prevented were identified and known by the hospital but not adequately and satisfactorily acted upon" and, even at the time of review, "PHMS [Portlaoise Hospital Maternity Services] service cannot be regarded as safe" (Holohan (2014), p.10). These findings follow the warning by the Health Information and Quality Authority the previous year that due to poor records "…it is impossible to assess the performance and quality of the maternity service nationally" (Health Information and Quality Authority (2013), p.123).

Improvements in the provision of maternity services over the last decade have been too slow to stop the rise in the number and size of claims. It is clear that institutional learning from these investigations has been limited. To the national inquiries, we must add the scores of other cases where the Irish courts have been satisfied that the standard of care was unacceptably deficient in a manner that led to injury where compensation is due.

Improving Maternity Services

It is known what must be done to improve the service, the problem is one of implementation. Perhaps the insurer — the SCA since 2002 — should be given a greater role. A case study shows how the withdrawal of insurance from maternity units in Monaghan and Dundalk in 2001 catalysed significant change in the provision of maternity services in that region (Kennedy (2012)). The SCA has alerted hospital authorities to elevated risks, as in the case of Portlaoise Hospital when "… the SCA did indeed raise concerns it had in 2007 and 2008 about maternity services in Portlaoise on the basis of the notifications of incidents it was receiving … the response from the hospital was inadequate to none at all" (Holohan (2014), p.50). Adopting commercial approaches to insurance, including risk assessments and rating techniques, and communicating to hospital management in financial terms would help management better understand the broader financial implications of their decision making. In short, inactions like not increasing staffing or not improving training, currently accounted for as cost-savings, are likely to be raising overall costs when allowance is made for the consequent costs of the increased associated risks. We suggest that the SCA be given greater powers — powers akin to those that commercial insurers can exercise to control and shape the risks borne. Crucially, the SCA must be enabled to signal publicly when the risks are becoming unacceptable in any maternity unit.

Discussion

There are no winners when it comes to medical negligence cases. The plaintiff suffers a reduced quality (and perhaps quantity) of life that no monetary award can make good. The suffering is shared by parents and family, especially in the case of catastrophically damaged infants. The medical and other hospital staff are demoralised. After the trauma of the incident itself follows the prolonged litigation process, giving years of stress and anxiety to all.

The HSE has made a provision of €400 million in its budget to transfer to the SCA for claims against it expected to settle during 2020 (HSE National Service Plan 2020, pp.113-4 and p.122). The same report states that the HSE continues to fail, by a significant margin, to investigate adverse incidents in a timely manner. In 2019, the HSE set as a target that 80% of reviews of serious incidents be completed within 125 calendar days of the occurrence. The actual outcome for 2019 is projected as just 20%. Such delays do not demonstrate an eagerness to learn from such events.

There must come a tipping point when the sums paid out by way of settlements for mismanagement of clinical services become appreciably larger than the additional costs of operating a sound system. Perhaps this tipping point has been reached in the case of maternity services in Ireland.

Epilogue

Reclaiming out of the past
All the good you can use,
Add all the good that you can
And offer it all onward.

Thomas Kinsella
From Songs of Understanding, *Marginal Economy* (2006)

The single greatest achievement of previous generations has been to propagate life. Their most precious gift to the new-born is, to my mind, the ways and means to prolong life. However, this is ultimately a value judgement: what price would *you* pay for an extra year of life for yourself or a loved one? This volume takes a close look at the gift, and how it was augmented in Ireland from one generation to the next. It tells two quite different stories.

The first story begins from around the time when official statistics on deaths were first collected in Ireland in 1864. The high rate of deaths in infancy and childhood starts to fall dramatically year on year. This is the tale of parents witnessing more of their offspring reaching adulthood. The soul-testing tragedy of parents burying their offspring becomes increasingly less routine until it is a rare event (see Table E1). Arguably, with the excision of such suffering from society, it has allowed people to become more compassionate. Undeniably, parental effort has become rewarded more. Parents can reduce the parental load by having fewer children with the confidence that they will reach maturity. This story has continued until today but lost dramatic effect from, say, the 1960s in Ireland, after which further reductions in the already low infant mortality had less impact on the number of deaths.

Table E1: Summary Vital Statistics of Womankind in Ireland[1]

	Hunter-Gatherers	**Ireland, 1821-41**	**Famine Ireland**	**Ireland, 1926**	**Ireland, 2016**
Probability of Child Living to Age 25	0.49	0.64	0.42	0.84	0.99
Probability of Living to Age 50	0.32	0.41	0.17	0.70	0.98
Probability of Living to Age 70	0.14	0.18	0.03	0.43	0.89
Probability of Offspring Dying before Mother	0.55	0.43	0.55	0.27	0.06
LE at Age 0	30	38	22	58	83
LE at Age 25	32	32	23	42	59
LE at Age 50	18	18	11	23	35
LE at Age 70	8	9	6	11	17

The second story overlaps the first. Beginning in the 1970s but so gradual as to be imperceptible at that time, parents in Ireland whose families were raised began to live longer, a trend that accelerated until recent years. Individual lifespans are extending far beyond the time required to rear the next generation. This is a fundamental renegotiation of the equilibrium between our species and nature, beneficial to the individual. This new equilibrium has been emerging now over the last half-century but society remains only dimly aware of its consequences. It has not entered popular consciousness as a triumph of possibilities for the individual after discharging their obligation to our species. If it does enter discourse, it does so in the form of a problem or burden, a "pension time-bomb" or a crisis in healthcare or long-term care provision. Throughout history and prehistory man battled with man over the immediate necessities of life for kith and kin. Now these are abundant, the battleground has moved to the resources to prolong life.

The figures in Table E1 are based on historic mortality rates. For those alive in Ireland today, we must estimate the expected age at death by projecting mortality rates and how they might change in the future.

[1] It is assumed that the mother is 25 years old at the birth of her daughter. Calculations are based on period life tables. See Gurven and Kaplan (2007) for hunter-gatherers (the values were calculated using average parameter values given in Table 2 and using the formula on p. 325). See Boyle and Ó Gráda (1986) for Pre-Famine Ireland (1821-1841) and Famine Ireland (1845-49), with values in Tables A2 and A4 fit and interpolated using the same Siler model as in Gurven and Kaplan (2007). Ireland 1926 and Ireland 2016 are based on official Irish Life Table 1 and 17 for females respectively.

Official projections by the Central Statistics Office and others based on the approach outlined in this book reflect the pace of mortality change in recent decades. Figure E1 shows the current best estimate of the age at death for those alive today in Ireland. The results of this exercise will perhaps surprise many.

Figure E1: Expected Age at Death and Median Age at Death in Ireland at Each Age and by Sex[2]

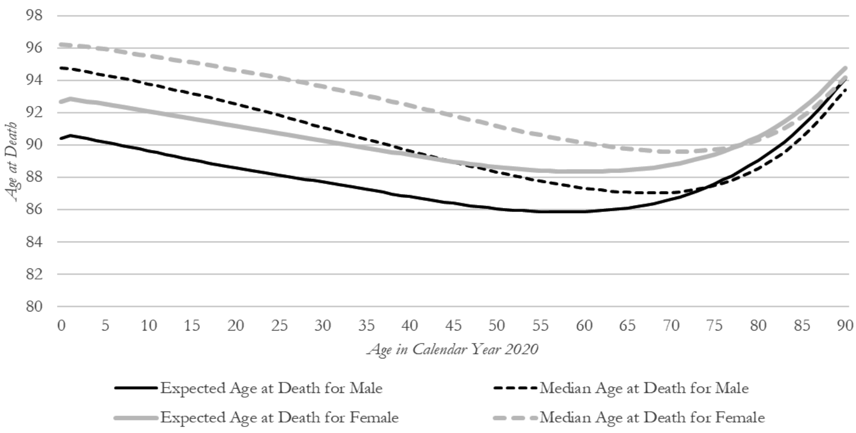


Figure E1 shows that that the average male alive today in Ireland, irrespective of their current age, can expect to celebrate their eighty-sixth birthday and the average female a couple more birthdays after that. These averages, based on cohort life expectancies (see Chapter 5), tend to underestimate how long an individual might live. To my mind, the median age at death is a better measure of how long an individual will live (see Chapter 7). The median age at death is the age that an individual can expect to survive to with a 50% probability. In other words, the median age is the age at which half will die before and half will survive beyond of an original large number of individuals of the same age and sex.

The forecast median age at death for those alive in Ireland, as Figure E1 highlights, is generally higher than the life expectancy. Figure E1

[2] Author's calculations based on the current CSO mortality projection methodology (see Chapter 5).

shows that at least 50% of males alive today at each age in Ireland can expect to live beyond 87 years, and at least 50% of females at each age today can expect to live beyond 89 years. In fact, most alive in Ireland today are projected to live to 90 years and beyond.

The telling of the story by numbers, as is done in this volume, is not the chronicle of the new era of the individual that most want to hear. It singles out no heroes. The simple fact is that we have only a frustratingly vague idea on how the added years to life came about, in Ireland or elsewhere. James Riley's *Rising life expectancy: a global history* (2001) makes a survey of the literature to argue persuasively that mortality decline has come from a mix of six key components: wealth and income, nutrition, public health, education, behaviour, and medicine. Surprisingly, medical advances have played a minor role to date in developed countries. Disentangling the role of each of these factors in the mortality declines of any specific country is complex and speculative. The key point is that the mix has been quite different in different countries, even though the resultant decline in mortality has often been similar. The rapid mortality decline in sub-Saharan Africa since the Second World War relied heavily on biomedicine, through both prevention and treatment, compared with Ireland's and Britain's greater reliance on general improvements in the standard of living. Riley contends that the apportionment of the mortality decline into different factors are country-specific and the results cannot be generalised. In short, there have been many different strategies to reduce mortality in the past and, it may safely be inferred, there will also be many in the future.

Fogel's hypothesis looms large in my night thoughts. The Nobel laureate Robert Fogel put forward a simple theory in his book, *The escape from hunger and premature death, 1700–2100* (2004). He attempts to link nutrition, life expectancies and economic growth through the past and extrapolate trends into the future. Amongst others, he claims that mankind has entered a new phase since 1700 that he terms "technophysio evolution". By technophysio evolution, he means the reshaping of our bodies, which have increased overall mass by a half since 1700, which has given us the strength to fight disease and the energy to reshape our environment. He attempts to relate metrics of lifestyle and environment (such as diet and body mass, real wages, expenditure on healthcare) to developments in life expectancy. Fogel's hypothesis is, simply, that the vast majority of humans were not allowed to eat their fill in agrarian societies before modern times, so never reached their potential. In the last century or two, for the first time since the agrarian revolution, people in many countries acquired enough nutrition to develop the more resistant and reparable human body of the better hunter-gatherer times. The technophysio evolution had a false start in Ireland with the Great Famine but, since that time, it could work its gradual transformation of our bodies and whence to empower our capabilities.

Fogel's hypothesis allows an alarmingly simple perspective on human history. The history of man after hunter-gatherer times in Ireland, from the times of the Céide Fields in 3500 BC to the 1850s, can best be summarised as one of hunger, or the threat of hunger. The malnourished, particularly the young and old, fell easily to the diseases that lay siege to man's settlements since he gave up his nomadic existence. With poor harvests weakening the health of the overall population, an endemic disease could erupt into a general plague or pestilence. And, so, for over five millennia after settling into the farming lifestyle, Irish populations lurched from one sustenance crisis to another. In each intervening year between crises, the weaker young and old succumbed to diseases circulating within the larger settlement or between us and our physically close domesticated animals.

The five millennia of hunger and disease are sometimes described as a new equilibrium between man and nature consequent on the agrarian revolution. With more insight, though, it can be seen as a political failure: a failure of how man governs man. Ever since the first civilisations arose following the agricultural revolution, the power needed to protect the crops and livestock was usurped by those wielding it to pursue their own ambitions over the ambitions of their populace — first by protecting their power from challenges within and without and then in vanity projects such as palaces, pyramids, and plunder. Hence, the consensual form of governance typical in the small hunter-gatherer tribes gave way after the agricultural revolution to political systems where very few held power over a great many.

Such governance systems over the great sweep of history have enriched the few and impoverished the many. States so governed do not sustain economic growth. The rulers or ruling class were all-powerful and typically played the zero-sum game of taking their extravagant wants from their population, and too often curtailing the lives of the unproductive old and weak. Economic historians argue that there is a degree of political power, somewhere between all-powerful and completely powerless, that offers the individual just enough protection from the rulers themselves as well as other individuals. States with this intermediate degree of power that allowed the ruled to pursue their own agenda only became common in the last three centuries (Olson (1993)). It was over the same period that the lot of humanity has appreciably improved.

Ireland (or elsewhere in the UK for that matter) had not fully made the transition to empower the ruled by 1850. Life expectancies at that time were close to record lows in famine-ridden Ireland and in the growing towns of industrialising Britain. The land of Ireland was a farm managed solely in the interests of its owners. The decennial stock-taking of 1851 showed its value was increasing satisfactorily:

> In conclusion, we feel it will be gratifying to your Excellency to find that although the population has been diminished in so remarkable a manner by famine, disease, and emigration between 1841 and 1851, and has been since decreasing, the results of the Irish Census of 1851 are, on the whole, so satisfactory, demonstrating as they do the general advancement of the country. We have shown in the course of our observations that the extent of arable land and the value of farm stock have increased since 1841 — that the worst class of houses is being-replaced by a better — that a smaller proportion of families is dependent on their own manual labour for support — and that the education of the people has favourably progressed.

General Report of the Commissioners,
Census of Ireland for the Year 1851, Part VI, p. lviii.

The lengthening of lifespans over the last century and a half required a significant allocation of resources. Future improvements will, no doubt, require further significant resources, this time directed towards the elderly, where mortality rates are still high. The Irish State already plays key roles in supporting this subgroup — providing a basic income through a state pension and generally providing healthcare and other services. Accordingly, the State will play a significant role in delivering, or failing to deliver, future extensions to lifespans in Ireland. The issue is, as always, fundamentally political. The Covid-19 pandemic, an infection particularly fatal to the elderly, tested the sacrifices a community would make. Despite the opportunity costs, Ireland shielded its aged better than most. This augurs well for future life expectancies as Ireland enters its second century of self-government.

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